Uncertainty, Pessimism, and Economic Fluctuations*

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Abstract. This paper develops a theory of uncertainty-driven business cycles, where uncertainty shocks act as non-inflationary aggregate demand shocks. We regard uncertainty as ambiguity and define the uncertainty (or ambiguity) shock as exogenous variation in the variance of the agents' prior belief over possible models. Under a real business cycle framework featuring ambiguity averse agents and incomplete information, we demonstrate that ambiguity shocks can generate positive co-movements across real quantities together with counter-cyclical labor wedge and labor productivity. After an adverse ambiguity shock, agents behave as if they believe aggregate demand is weaker and more volatile. The perceived weaker aggregate demand creates depressed confidence that makes all real quantities plummet; and the perceived higher volatility increases the use of private information in forecasting, which drives up the cross-sectional dispersion of beliefs. These predictions of the theory are consistent with survey data evidence. Finally, the quantitative potential of our theory is illustrated within a dynamic RBC model.

Keywords. Business cycles, non-inflationary aggregate demand shock, uncertainty, confidence, ambiguity, smooth model of ambiguity.

JEL Classification. E32, E13, D8

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1. Introduction

Recessions are times with heightened uncertainty, and business cycles tend to be uncertainty-driven. As depicted in Figure 1, over the last thirty years, all three recessions experienced by the US economy feature a substantial increase in uncertainty (dashed blue line), which is measured by the cross-sectional dispersion of real GDP forecasts in the Survey of Professional Forecasters (SPF).[1] At the same time, the bulk of the business-cycle fluctuations, namely positive co-movements across real quantities, are disconnected from either productivity or inflation[2]. Then, if uncertainty shocks were the main-driver of the business-cycle fluctuations, they should act as the non-inflationary aggregate demand shocks. Whereas, the theory of uncertainty shock as in Bloom [2009] and Bloom et al. [2018], can only generate positive co-movements across real quantities through incurring a strong pro-cyclical movements in the labor productivity[3]. In other words, uncertainty shocks in Bloom [2009] and Bloom et al. [2018] act as a measured TFP shock instead of the non-inflationary aggregate demand shocks. In this paper, to complement the existing literature of uncertainty shock, we develop a new theory of uncertainty-driven business cycles that accommodates a notion of the non-inflationary aggregate demand shock due to variations in uncertainty.

Notion of Uncertainty. Instead of thinking of uncertainty as risk, we regard uncertainty as ambiguity[4]. When uncertainty refers to risk, uncertainty shocks are defined to be exogenous variations in the variance of underlying shocks and agents are risk averse. In our model, with uncertainty referring to ambiguity, uncertainty (or ambiguity) shocks[5] are defined to be exogenous variations in the variance of agents' prior beliefs about possible models and agents are ambiguity averse. We formalize such notions of uncertainty and ambiguity shocks with the help of the smooth model of ambiguity axiomatized by Klibanoff et al. [2005, 2009]. Within such framework, an adverse ambiguity shock that makes agents more uncertain across different models in their priors creates an increased degree of pessimism about the state of the economy, which maps into a drop in confidence. Such a transmission mechanism from uncertainty to confidence is consistent with survey data evidence. In Figure 1, we proxy confidence with the Sentiment Index in the Michigan Survey of Consumers. Periods with heightened uncertainty are associated with depressed confidence.

Framework and Mechanism. We formalize an otherwise standard real business cycle model that additionally features (a) aggregate demand externalities; (b) ambiguity averse agents (households, workers or firms) and finally (c) incomplete information about ambiguous aggregate fundamentals.

The model features multiple islands differing in productivity, which consists of an aggregate component and an idiosyncratic component. Local agents, including firms and workers, have perfect information about local productivity but incomplete information about the average productivity of all other islands, namely the aggregate productivity of the economy. Therefore, when making economic decisions, local agents need to form expectations over the aggregate productivity because it is the suf-

[1] See Bloom [2009] and Bloom et al. [2018] for more measures of uncertainty and their behavior during the crisis.

[2] In a recent empirical demonstration, Angeletos et al. [2019] identify a business cycle factor, which is a one-dimensional summary of aggregate movements. The business cycle factor turns out to disconnect from either technology or inflation. Moreover see Gali [1999] and Basu et al. [2006] for the former point. Furthermore, see Mavroeidis et al. [2014] for a survey of the empirical evidence on NKPC that corresponds to the latter point, namely the inflation puzzle. Also, see Beaudry and Portier [2014] for some evidence of US business cycles mainly being non-inflationary demand-driven.

[3] In Bloom [2009] and Bloom et al. [2018], with the existence of non-convex adjustment costs, exogenous variations in risk, namely the volatility of underlying shocks, can generate strong co-movements across real quantities mainly through variations in the cross-sectional misallocation of resources.

[4] According to Marinacci [2015], ambiguity refers to subjective uncertainty over probabilities due to lack of ex-ante information to pin down a specific model for the economy in the course of decision-making.

[5] Throughout the paper, we interchange the use of uncertainty shocks with ambiguity shocks.

ficient statistics of the demand conditions or terms of trade within the general equilibrium. Whereas the aggregate productivity of the economy is ambiguous in the sense that agents believe that anything on the real-line can be a candidate for this and they form a normal prior for it, the variance of which is used as a measure of the amount of ambiguity perceived by all agents. Because agents are ambiguity averse, they would form a pessimistic belief about the demand conditions or terms of trade when forming expectations, in the sense that models with a weaker demand are, on average, perceived to be more likely than optimistic models with a stronger demand. In response to an adverse ambiguity shock, the local agents become more uncertain across different possible models of the demand conditions or terms of trade and hence form more depressed expectations about own demand conditions. As a consequence, local production shrinks without any commensurate movement in factor productivities. At the aggregate level, the economy plummets as if there had been a contractionary aggregate demand shock. From this perspective, ambiguity shocks in this paper are nothing more than a particular formulation of **aggregate demand shocks**. Moreover, recall that such a notion of aggregate demand shocks is formulated within the RBC framework. Therefore, we conclude that ambiguity shocks act as non-inflationary aggregate demand shocks in this paper.

The byproducts of our transmission mechanism are the co-movements of confidence and dispersion-based measures of uncertainty. The economy-wide deterioration in the degree of pessimism is directly interpreted as the depressed **confidence**. In addition, the economy features incomplete information about aggregate fundamentals. Therefore, when forming expectations, agents face the trade-off between the use of prior and private information. Note that ambiguity shocks are formulated as shocks to the variance of agents' prior belief about possible models. An adverse ambiguity shock results in a less informative prior. Hence it incentivizes more use of private information in forming expectations about the aggregate demand, making either local or aggregate output respond more to the aggregate productivity of the economy. From the perspective of the professional forecasters, it indicates that there are more to be estimated. Furthermore, they would also use more of their private information when forming expectations about the aggregate productivity of the economy. As a result, output forecasts respond more to private information, which heightens **cross-sectional dispersions of beliefs**.

Incompleteness in information is crucial in our model. If the information was complete, all agents, including firms, workers and households, could not only perfectly coordinate their beliefs but also their actions. With common knowledge of the economy, all agents will have not only perfect information about local productivity but also perfect information about aggregate demand conditions. As a result, there will be no room for ambiguity shocks to have any real impact on the economy. The broader insight is that incomplete information helps us to accommodate a situation where ambiguity is mostly about others' productivity, which is a sufficient statistics of aggregate demand conditions under general equilibrium, rather than own productivity. Therefore, ambiguity is mostly about the short-run aggregate demand rather than the medium to long-run productivity of the economy. The difference in the short-run v.s. the medium- to long-run perspective is crucial in understanding the aggregate demand shocks nature of ambiguity shocks in our model.

Results. The paper starts with a static business cycle model that abstracts out capital accumulation. We demonstrate that an adverse ambiguity shock generates a recession with depressed confidence and heightened cross-sectional dispersion of beliefs if agents are ambiguity averse, and the information is incomplete. At the core of the result is the interplay between incomplete information and dual impacts of ambiguity shocks. An adverse ambiguity shock, on the one hand, makes agents believe that the aggregate fundamental becomes more volatile and, on the other hand, the aggregate fundamental is turning bad when they are ambiguity averse. We discuss the dual impacts of ambiguity shocks using a game theoretic interpretation of the market equilibrium, which resembles the idea in Angeletos and La'O [2009] such that any business cycle model with incomplete information can be transformed into a beauty contest.

We further conduct quantitative evaluations of the impacts of ambiguity shocks within the dynamic RBC framework. Ambiguity shocks can generate positive co-movements in real quantities without commensurate movements in labor productivity similar to confidence shock alias Angeletos et al. [2018] and Huo and Takayama [2015]. What drives the positive co-movements in real quantities are fluctuations in the degree of pessimism about the short-run outlooks of the economy. The model is capable of generating an empirically plausible counter-cyclical labor wedge. Moreover, quantitatively the model can capture cyclical behaviors in cross-sectional dispersions in real GDP forecasts, indicated by the SPF dataset. Finally, the model-implied confidence process closely tracks Sentiment Index in the Michigan Survey of Consumer, manifesting the fact that our theory can also quantitatively capture movements in confidence.

Contributions. The contributions of the paper are fourfolds. First of all, the paper contributes to the uncertainty shocks literature with an alternative formulation and transmission mechanism of uncertainty shocks. Our theory accommodates a notion of non-inflationary aggregate demand shocks out of variations in uncertainty. The transmission mechanism relies on the interplay between ambiguity aversion and incomplete information, rather than the non-convex adjustment costs as in Bloom's theory of uncertainty shock. Our paper further extends the conventional wisdom in understanding the co-movements across confidence, uncertainty, and aggregate economy by arguing that these can be the endogenous outcomes of ambiguity shocks, not only qualitatively but also quantitatively.

Second, the paper also contributes to the confidence shocks literature, such as Angeletos and La'O [2013], Angeletos et al. [2018], Huo and Takayama [2015] and Benhabib et al. [2015], with the empirical linkages between confidence and uncertainty shocks. The confidence shock literature has long been challenged by the lack of empirical counterparts of the higher-order belief shocks in the theory. Our theory reconciles any such concerns by showing that variations in confidence can originate from variations in uncertainty, where we do have a lot of empirical measures in the literature.

Third, we also contribute to the literature with a Bayesian formulation of ambiguity shocks by embedding the smooth model of ambiguity into general equilibrium macro models. Conceptually, it differs with the existing literature in the sense that it is a shock to the amount of ambiguity rather than a shock to agents' taste over ambiguity (Bhandari et al. [2019]) or a mix of both (Ilut and Schneider [2014]). The unique insight for our Bayesian formulation of ambiguity shocks that cannot be shared with the others is that it induces endogenous fluctuations in measured uncertainty, such as dispersion measures.

Finally, methodologically, the paper contributes to the literature with the provision of a linkage between (a) the business cycle models featuring ambiguity aversion and incomplete information and (b) games of incomplete information with ambiguity aversion. The game theoretic interpretation of the business cycle model allows us to build the key economic intuitions behind the mechanism of our paper and to deliver insights into the interplay between ambiguity, ambiguity aversion, and incomplete information.

Layout. The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 sets up the static model without capital. Section 4 characterizes the equilibrium by a formal definition as well as a set of optimality conditions. Section 5 studies the impacts of ambiguity shock within the static model without capital. Section 6 sets up a dynamic RBC model where a couple of quantitative evaluations are conducted. Finally, Section 7 concludes the paper.

2. Related Literature

This paper is broadly related to the literature of expectation-driven business cycles including (a) the news shocks literature, Beaudry and Portier [2004, 2006] and Jaimovich and Rebelo [2009]; (b) the noise shocks literature, Lorenzoni [2009], Barsky and Sims [2012] and Blanchard et al. [2013]; (c) the confidence shocks literature, Angeletos and La'O [2013], Angeletos et al. [2018], Huo and Takayama [2015] and Benhabib et al. [2015]; (d) the misspecification shocks literature, Bhandari et al. [2019]; (e) the non-bayesian ambiguity shocks literature, Ilut and Schneider [2014] and Ilut and Saijo [2018] and finally (f) the uncertainty shocks literature, Bloom [2009], Bidder and Smith [2012], Bloom et al. [2018] and Arellano et al. [2018]. We contribute this line of literature with an alternative "exotic" shock, i.e., the Bayesian formulation of ambiguity shocks, acting as the non-inflationary aggregate demand shocks.

We share the same research spirit with the uncertainty shocks literature in identifying a particular mechanism that enables the second-moment shock to have the first-moment impact. Our paper differs from Bloom [2009] and Bloom et al. [2018] (1) in the notion of uncertainty and uncertainty shocks, (2) in the transmission mechanism of uncertainty shocks and most importantly (3) in the empirical properties of uncertainty shocks. In Bloom [2009] and Bloom et al. [2018] ", uncertainty shocks can only generate realistic positive co-movements across real quantities through inducing strong pro-cyclical movements in aggregate TFP", while in our paper, ambiguity shocks act as non-inflationary aggregate demand shocks. From this perspective, we share the same idea with Bidder and Smith [2012] and Arellano et al. [2018], in which uncertainty shocks can generate aggregate demand-like fluctuations. Bidder and Smith [2012] rely on the interaction between the robust preference and stochastic volatility in generating animal spirits fluctuations. We differ from them in the notion of uncertainty and also the transmission mechanism from animal spirit to aggregate fluctuations. They rely on the news channel as in Jaimovich and Rebelo [2009] while we rely on the confidence channel as in Angeletos and La'O [2013] and Angeletos et al. [2018]. Arellano et al. [2018] rely on financial frictions in generating aggregate-demand like fluctuations. We differ with them in the notion of uncertainty and also in terms of the underlying transmission mechanism which is the hedging behavior of firms in their paper and in our paper it is the variations in the pessimistic beliefs of agents.

Our paper also directly connects to the theory of confidence shocks alias of Angeletos and La'O [2013], Angeletos et al. [2018], Huo and Takayama [2015] and Benhabib et al. [2015] in generating animal-spirit-like aggregate fluctuations, at the core of which is the interplay between ambiguity aversion and incomplete information. The unique insights we provide in this paper are the empirical linkages between confidence and uncertainty.

Some other works also study the implications of ambiguity aversion in the context of business cycle models, but with the different mathematical representation of the preferences such as Ilut and Schneider [2014] and Bhandari et al. [2019]. Apart from the difference mentioned above in the ability to capture fluctuations in dispersion measured uncertainty in general, our paper differs from both of them in a few other aspects. Ilut and Schneider [2014] use the multiple priors preference axiomatized by Gilboa and Schmeidler [1989] to model ambiguity aversion, and the notion of ambiguity shock is in a classical statistics fashion. Bhandari et al. [2019] use the robust preference proposed by Hansen and Sargent

[2001a,b] and focus on time-varying concerns for model misspecification, which can be understood as time-variations in the degree of ambiguity aversion. In our paper, ambiguity aversion is modeled by the (recursive) smooth model of ambiguity axiomatized by Klibanoff et al. [2005, 2009] and the learning process features smooth rule updating proposed by Hanany and Klibanoff [2009] to ensure dynamic consistency. Finally, both of the two works consider ambiguity over the long-run outlook of the economy, i.e., the news- or noise-like formulations, whereas, in our paper, ambiguity is mainly over the short-run outlook of the economy, which allows us to have a notion of aggregate demand fluctuations without relying on nominal rigidity.

A notable exception is Ilut and Saijo [2018], in which ambiguity aversion is modeled as an amplification mechanism of business cycles rather than as devices of exogenous variations. In their paper, recessions are periods of less learning. With an exogenous entropy constraint, reduced learning translates into a broader range of models. Therefore, there is an endogenous counter-cyclical ambiguity. They focus on the aggregate fluctuations and succeed in capturing a couple of salient features in the data. However, their model implications for dispersion measures of uncertainty are ambiguous. Less learning in recession implies less use of private information when making forecasts and decisions. In most of the cases, it implies lower cross-sectional dispersions either in output or output forecasts. We differ from them in generating the right co-movement pattern between various dispersion measures and the aggregate economy unambiguously.

Ambiguity averse preferences, such as recursive multiple priors preference, have been intensively used in the literature to generate asymmetric responses of the economy to shocks in recessions or booms; see for example Epstein and Schneider [2008], Ilut [2012], Ilut et al. [2016], Baqaee [2017] and Zhang [2018]. These papers assume the existence of ambiguity about the precisions of information to provide agents with state-dependent subjective beliefs about precisions of related information. Negative realizations of shocks are associated with a subjective beliefs of higher precisions. Therefore, the economy responds more strongly to shocks in recessions than in booms. In our paper, ambiguity is about the first-moment of aggregate productivity shock, but ambiguity shocks are of second-moment. The interplay between ambiguity aversion and incomplete information in our paper also generates counter-cyclical responses to the aggregate productivity shocks. Finally, ambiguity aversion has been studied in the asset pricing literature such as Ju and Miao [2012] and Collard et al. [2018].

Our paper also relates to the theory of the beauty contest pioneered by Morris and Shin [2002] and then extended by Angeletos and Pavan [2007]. We further extend the theory by allowing for the ambiguity averse preference. Finally, in a broader context, our paper is also related to those studying coordination games with model uncertainty, such as Chen et al. [2016] and Chen and Suen [2016].

3. The Static Model without Capital

In this section, we construct a static general equilibrium model in the vein of Angeletos and La'O [2009]. The model embeds three additional key features in an otherwise standard real business cycle framework: (a) aggregate demand externalities, (b) incomplete information about the ambiguous aggregate state of the economy and finally (c) the smooth model of ambiguity together with ambiguity shocks. We first describe the setup of the model and close up this section by a couple of remarks and interpretations of the setup.

3.1. Physical Environment, Shocks and Information Structure

Geography, markets and timing. The economy consists of a continuum of islands, indexed by $j \in \mathbb{J} = [0,1]$ and a mainland. On each island j, there exists a continuum of firms, indexed by $(h,j) \in \mathbb{H} \times \mathbb{J} = [0,1]^2$ and a continuum of workers, indexed by $(h,j) \in \mathbb{H} \times \mathbb{J} = [0,1]^2$. Island firms and workers interact with each other in the locally competitive labor market for the production of differentiated island commodities indexed by j. These commodities are traded in a centralized market, later-on operated on the mainland, inhabited by a continuum of consumers, indexed by $h \in \mathbb{H} = [0,1]$ and a large number of competitive final good producers inhabit. We assume that consumer h and a continuum of workers $\{(h,j)\,; j \in \mathbb{J}\}$ constitute a large household indexed by $h \in \mathbb{H}$, who owns a continuum of firms $\{(h,j)\,; j \in \mathbb{J}\}$. Thus, we ensure the existence of a representative household at the mainland and a continuum of representative firms and workers on every island.

We focus on the static setup in this section. There is only one period, say period t, which is decomposed into three stages. At stage zero, period t shocks are realized. At stage 1, island-specific competitive labor markets open up. The representative household sends out workers to each island. On island j, firms make labor demand decisions and workers make labor supply decisions on the basis of incomplete information about the ambiguous aggregate state of the economy. At stage 2, on the mainland, the centralized commodities market opens up. All uncertainty, either risk or ambiguity, is resolved. Final good producers produce and the representative household makes consumption decisions upon receiving all transfers from workers and firms on basis of perfect information. In what follows, we abstract from sub-index h without loss of generality.

Households. The utility of the representative household is given by:

$$\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon} dj$$

where γ is the relative risk aversion and ϵ is the inverse Frisch elasticity of labor supply. Note that in our static setup, γ also controls for the income effects of labor supply. The corresponding budget constraint of the household is such that

$$P_t C_t = \int_{\mathbb{J}} W_{j,t} N_{j,t} dj + \int_{\mathbb{J}} \Pi_{j,t} dj$$

where P_t denotes the price of final goods, $\int_{\mathbb{J}} W_{j,t} N_{j,t} dj$ denotes total labor income and finally $\int_{\mathbb{J}} \Pi_{j,t} dj$ denotes the total realized firm profits.

Island firms. Island j firms use labor only for the production of the island j commodity. The production function is given by

$$Y_{j,t} = A_{j,t} N_{j,t}^{1-\alpha} \tag{1}$$

where $A_{j,t}$ is the island-specific productivity and the realized profit is given by

$$\Pi_{j,t} = P_{j,t} Y_{j,t} - W_{j,t} N_{j,t}$$

Here $W_{j,t}$ denotes the nominal wage on island j in period t and $P_{j,t}$ denotes the market price of the island j commodity to be determined at stage 2 when the centralized markets open up. Since it is assumed that it is the representative household that owns the firm, any realized profits are to be transferred back to the household for the purchase of final goods for consumption. Therefore, in the absence of any uncertainty concerns, island j firms care about the consumer valuation of its profits given by

$$\frac{u'(C_t)}{P_t}\Pi_{j,t}$$

where P_t is the price of final goods normalized to 1.

Final-good producers. The competitive final-good sector employs a CES production technology:

$$Y_t = \left(\int_{\mathbb{J}} Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}$$

where θ is elasticities of substitution among island commodities. It also controls the strength of aggregate demand externalities. Therefore, the demand function for the island j commodity is given by

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t$$

where $P_t \equiv \left(\int_{\mathbb{J}} P_{j,t}^{1-\theta} dj\right)^{\frac{1}{1-\theta}}$ denotes the price of final goods that is normalized to 1.

Productivity and ambiguity shocks. Aggregate productivity $a_t \equiv \log A_t$ follows a normal distribution with mean ω_t and variance σ_ζ^2

$$a_t \sim N\left(\omega_t, \sigma_\zeta^2\right)$$

Objectively, the mean of the aggregate productivity shock is zero, i.e. $\omega_t = 0$. However, agents cannot fully "understand" it. Instead, they possess ambiguity about it. Specifically, they believe that anything on the real-line can be a potential candidate for ω_t. And they possess a common mean-zero[6] normal prior belief about $\omega_t \in \mathbb{R}$:

$$\omega_t \sim N\left(0, e^{\psi_t}\right)$$

where ψ_t measures the amount of ambiguity perceived by the agents[7] such that

$$\psi_t = \overline{\psi} + \tau_t \quad \text{with} \quad \tau_t \sim N\left(0, \sigma_\tau^2\right) \tag{2}$$

Here $\overline{\psi}$ denotes the amount of ambiguity perceived by all agents at the ambiguous steady state[8]. And we interpret τ_t as the ambiguity shock, which is normally distributed with mean 0 and variance σ_τ^2.

Island-specific productivity, $a_{j,t} \equiv \log A_{j,t}$, equals to the aggregate productivity plus a idiosyncratic

[6] The objective model ω_t is assumed to be inside the set of possible models of all agents. By assuming this, we rule out any mis-specification concerns and focus on ambiguity. See Peter Hansen and Marinacci [2016] for a detailed discussion of the differences between mis-specification and ambiguity

[7] Maccheroni et al. [2013] propose to use the variance of the ex-ante expected utility of a particular model to quantify the amount of ambiguity in the general information structure, which is shown to be consistent with a quadratic approximation akin to the Arrow-Pratt approximation. Our measure of the amount of ambiguity is consistent with theirs ordinally under thje normality assumption.

[8] Ambiguous steady state refers to the state into which the economy converges to in the absence of any shocks but taking into account the existence of ambiguity.

productivity shock $\iota_{j,t}$:

$$a_{j,t} = a_t + \iota_{j,t}$$

Idiosyncratic productivity shocks $\iota_{j,t}$ are assumed to be i.i.d normally distributed with mean 0 and variance σ_t^2.

Information structure. Denote $\mathcal{I}_{t,0}$, $\mathcal{I}_{j,t,1}$ and $\mathcal{I}_{t,2}$ as the information sets that are available to all agents at stage 0 of period t, are only available to island j agents at stage 1 of period t and are available to all agents at stage 2 of period t, respectively. We define these information sets by

$$\mathcal{I}_{t,0} = \{\psi_t\} \qquad \mathcal{I}_{j,t,1} = \mathcal{I}_{t,0} \cup \{a_{j,t}\} \qquad \mathcal{I}_{t,2} = \cup_j \mathcal{I}_{j,t,1} \cup \{\zeta_t\} \tag{3}$$

A couple of implicit assumptions are made here. First of all, information is symmetric within each island but is asymmetric across islands. Second, the ambiguity shock τ_t happens at the beginning of each period t and is common knowledge to all agents. Third, at stage 1 of period t, island j productivity $a_{j,t}$ is only accessible for island j agents. Therefore, $a_{j,t}$ serves as the private information of island j agents about the aggregate productivity a_t, which is ambiguous. Thus, labor supply and demand decisions on each island are made under incomplete information over the ambiguous aggregate state of the economy. Fourth, $\mathcal{I}_{t,2}$ contains the complete set of local information because commodities prices would perfectly reveal the island-specific productivities. Fifth, all uncertainty, either risk (state uncertainty about a_t) or ambiguity (model uncertainty about ω_t) is fully resolved at stage 2 of period t. Hence, consumption decisions are made under perfect information.[9]

We close the current sub-section by the timeline of our model in Figure 3.

3.2. Preferences and Interim Belief Systems

The preference of the representative household is represented by the smooth model of ambiguity proposed in Klibanoff et al. [2005].[10] In addition, with ambiguity averse preferences, Bayesian updating leads to dynamic inconsistency. To restore the dynamic consistency across stages, we employ the smooth rule of updating proposed by Hanany and Klibanoff [2009]. Recall that at stage 2, when all uncertainty is resolved, the model collapses into the standard perfect information business cycle model. In what follows, we formulate the relevant workers' and firms' problems upon carefully describing the preferences and beliefs of all agents at stage 1 of period t.

Worker problem at stage 1. At stage 1 of period t, workers on island j solve the following problem:

$$\max_{N_{j,t}} \int_{\mathbb{R}} \phi \left(\mathbb{E}^{\omega_t}_{j,t,1} \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon} dj \right] \right) \widetilde{f}^{h}_{j,t,1} (\omega_t)\, d\omega_t \tag{4}$$

$$\text{s.t.} \quad P_t C_t = \int_{\mathbb{J}} W_{j,t} N_{j,t} dj + \int_{\mathbb{J}} \Pi_{j,t} dj \tag{5}$$

where $N_{j,t}$ denotes the labor supply of island j workers and $W_{j,t}$ denotes the nominal wage in the island-specific competitive labor market.

Here $\phi(x)$ is a strictly increasing and concave function, whose curvature captures agents' taste about

[9] The assumption that all period t uncertainty is resolved at the second stage of period t is to ensure tractability. However, most of the key messages delivered in this paper do not rely on this particular assumption on information structure.

[10] The smooth model features a two-way separation between the amount of ambiguity (characteristics of subjective belief) and the degree of ambiguity aversion (characteristics of decision makers' tastes).

ambiguity, namely the degree of ambiguity aversion. In addition, $\mathbb{E}^{\omega_t}_{j,t,1}[\cdot]$ denotes the mathematical expectation conditioned on $\mathcal{I}_{t,1}$ under a particular model ω_t for the mean of the aggregate productivity shock. Finally, $\tilde{f}^h_{j,t,1}(\omega_t)$ stands for the posterior belief about possible models ω_t that follows the smooth rule of updating

$$\tilde{f}^h_{j,t,1}(\omega_t) \propto \underbrace{\frac{\phi'\left(\mathbb{E}^{\omega_t}_{j,t,0}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi\int_{\mathbb{J}}\frac{N_{j,t}^{1+\epsilon}}{1+\epsilon}dj\right]\right)}{\phi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi\int_{\mathbb{J}}\frac{N_{j,t}^{1+\epsilon}}{1+\epsilon}dj\right]\right)}}_{\text{Weights}} \underbrace{f\left(a_{j,t}|\omega_t\right) f_t(\omega_t)}_{\text{Bayesian Kernel}} \tag{6}$$

Here $f\left(a_{j,t}|\omega_t\right)$ is the conditional probability density function of $a_{j,t}$ under a particular model ω_t, which is the normal density with mean ω_t and variance $\sigma^2_\zeta + \sigma^2_\iota$, and $f_t(\omega_t)$ stands for the period t prior belief about ω_t, which is the normal density with mean 0 and variance e^{ψ_t}. Relative to the standard Bayesian updating, the smooth rule puts more weight on the model that provides higher marginal incentive to act ex-ante (stage 0) when making comparison to its ex-post (stage 2) counterparts:

$$\phi'\left(\mathbb{E}^{\omega_t}_{t,0}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi\int_{\mathbb{J}}\frac{N_{j,t}^{1+\epsilon}}{1+\epsilon}dj\right]\right) > \phi'\left(\mathbb{E}^{\omega_t}_{t,2}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi\int_{\mathbb{J}}\frac{N_{j,t}^{1+\epsilon}}{1+\epsilon}dj\right]\right)$$

Such a re-weighting process in posterior belief aligns the incentives to act ex-ante and ex-post, which ensures dynamic consistency across stages within a period.

Firm problem at stage 1. Island j firms decides how much labor to hire by solving the following firm problem:

$$\max_{N_{i,j,t}} \int_{\mathbb{R}} \phi\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{C_t^{-\gamma}}{P_t}\left(P_{j,t}Y_{j,t} - W_{j,t}N_{j,t}\right)\right]\right)\tilde{f}^f_{j,t,1}(\omega_t)\,d\omega_t \tag{7}$$

Note that there are a continuum of firms on island j. Therefore, island j firms take $P_{j,t}$ as given when making labor demand decisions. Furthermore, the posterior belief about possible models ω_t of island j firms follows an extended smooth rule of updating given by

$$\tilde{f}^f_{j,t,1}(\omega_t) \propto \underbrace{\frac{\phi'\left(\mathbb{E}^{\omega_t}_{j,t,0}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi\int_{\mathbb{J}}\frac{N_{j,t}^{1+\epsilon}}{1+\epsilon}dj\right]\right)}{\phi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\frac{C_t^{-\gamma}}{P_t}\left(P_{j,t}Y_{j,t} - W_{j,t}N_{j,t}\right)\right]\right)}}_{\text{Weights}} \underbrace{f\left(a_{j,t}|\omega_t\right) f_t(\omega_t)}_{\text{Bayesian Kernel}} \tag{8}$$

Unlike the standard smooth rule of updating, firms' incentives to act are not aligned in their own ex-ante versus ex-post perspectives. Instead, the proposed extended smooth rule of updating aligns the incentives to act of the representative household ex-ante with those of the firms ex-post. Thus, we can ensure the dynamic consistency from the perspective of the representative household. Namely, if we allowed the household to make ex-ante contingency production plans for island firms, they would be respected ex-post by the firms. [11]

[11] Note that we formulate the firms' problem in such a way that the firms are ambiguity averse by themselves. We can justify the above formulation of the firm problem by arguing that firms are maximizing the shareholder value and use the stochastic discount factor of the representative household when evaluating cash flows in different times and states. Therefore, they behave as if they are ambiguity averse by themselves and share the same belief with their shareholders when evaluating the marginal benefit of labor demand. The additional concavity introduced by the ϕ function manifests the former point, and the extended

Finally, we assume that $\phi(x)$ takes the constant absolute ambiguity aversion (CAAA) form for simplicity and tractability:

Assumption 1. (CAAA) We assume $\phi(x) = -\frac{1}{\lambda}e^{-\lambda x}$ where $\lambda \geq 0$ measures the degree of ambiguity aversion of all agents.

3.3. Remarks and Interpretations

We conclude this section by three remarks and interpretations of the three key features of our model.

1. There is no ambiguity about local economic conditions in the sense that firms and workers on island j have the perfect information about the productivity of island j, while they have incomplete information about the aggregate productivity, which is ambiguous. Therefore, if local economic decisions are made solely depending on the expectations about local economic conditions, output or labor will not respond to ambiguity shocks at all. This is the reason why we need aggregate demand externalities in the general equilibrium model.

2. In our model, incomplete information about the ambiguous aggregate productivity a_t can be translated into incomplete information about the ambiguous aggregate demand within general equilibrium environment that features aggregate demand externalities. Then, fluctuations in the amount of ambiguity namely ambiguity shocks τ_t, generate fluctuations in island j agents' subjective beliefs about aggregate demand conditions, which eventually map into fluctuations in aggregate quantities. In this sense, we can formally interpret ambiguity shocks as aggregate demand shocks. More importantly, they are non-inflationary because they are formulated within the RBC framework without relying on NKPC as the transmission mechanism.

3. Ambiguity shock τ_t in our model is a second-moment shock. The key question here is whether such a second-moment shock can generate any first-moment impact at the aggregate level or not. The answer is yes if agents are ambiguity averse. From this perspective, we share the same spirit with Bloom [2009] and Bloom et al. [2018] in identifying possible mechanisms that enable the second-moment shocks to have the first-moment impacts.

4. Equilibrium Characterization

In this section, we define the equilibrium of our model and then derive a set of optimality conditions that jointly describe the equilibrium allocations and beliefs of all relevant agents. Finally, we demonstrate how to characterize the equilibrium associated with these optimality conditions.

smooth rule of updating takes care the latter. Therefore, we can alternatively formulate the firms' problem by

$$\max_{N_{i,j,t}} \int_{\mathbb{R}} \mathbb{E}_{j,t}^{\omega_t}\left[SDF_t\left(P_{j,t}Y_{i,j,t} - W_{j,t}N_{i,j,t}\right)\right] f_{j,t,1}(\omega_t)\, d\omega_t \tag{9}$$

where the stochastic discount factor SDF_t is given by

$$SDF_t \equiv \phi'\left(\mathbb{E}_{j,t,0}^{\omega_t}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon}dj\right]\right)\frac{C_t^{-\gamma}}{P_t}$$

Here the stochastic discount factor does not only take care of households' risk attitude $\frac{C_t^{-\gamma}}{P_t}$ but also their ambiguity attitude $\phi'\left(\mathbb{E}_{j,t,0}^{\omega_t}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon}dj\right]\right)$. The two formulations (7) and (9) are isomorphic to each other.

4.1. Equilibrium Definition

Definition 1 (Equilibrium). *A market equilibrium consists of a set of allocations* $\left\{\{N_{j,t}, Y_{j,t}\}_{j\in\mathbb{J}}, Y_t, C_t\right\}$*; factors and commodities prices* $\left\{\{W_{j,t}\}_{j\in\mathbb{J}}, \{P_{j,t}\}_{j\in\mathbb{J}}, P_t\right\}$*; information sets* $\left\{\mathcal{I}_{t,0}, \{\mathcal{I}_{j,t,1}\}_{j\in J}, \mathcal{I}_{t,2}\right\}$*; exogenous shocks* $\left\{\tau_t, \zeta_t, \{\iota_{j,t}\}_{j\in J}\right\}$*; and finally posterior beliefs about the set of possible models* $\left\{\widetilde{f}^h_{j,t,1}(\omega_t), \widetilde{f}^f_{j,t,1}(\omega_t)\right\}_{j\in\mathbb{J}}$ *such that:*

- *Information sets* $\left\{\mathcal{I}_{t,0}, \{\mathcal{I}_{j,t,1}\}_{j\in\mathbb{J}}, \mathcal{I}_{t,2}\right\}$ *are defined in (24).*
- *At stage* 1*, given factors and commodities prices* $\left\{\{W_{j,t}\}_{j\in\mathbb{J}}, \{P_{j,t}\}_{j\in\mathbb{J}}, P_t\right\}$ *and the posterior beliefs over the set of possible models* $\left\{\widetilde{f}^h_{j,t,1}(\omega_t), \widetilde{f}^f_{j,t,1}(\omega_t)\right\}_{j\in\mathbb{J}}$*,* $\left\{C_t, \{N_{j,t}\}_{j\in\mathbb{J}}\right\}$ *solves the workers' problem (4) and* $\left\{Y_t, \{N_{j,t}, Y_{j,t}\}_{j\in\mathbb{J}}\right\}$ *solves the firms' problem (7).*
- *Posterior beliefs are such that:* $\widetilde{f}^h_{j,t,1}(\omega_t)$ *is given by (6) and* $\widetilde{f}^f_{i,j,t,1}(\omega_t)$ *is given by (8).*
- *Markets clear for island-specific labor, island-specific commodities and final goods with price of the final goods normalized to one, i.e.* $P_t = 1$*.*

4.2. The Optimality Conditions

We can characterize the equilibrium with a set of optimality conditions. First of all, within the island j labor market, the optimal labor supply is governed by the following condition

$$\chi N^{\epsilon}_{j,t} = W_{j,t}\int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t,1}\left[u'(C_t)\right]\widetilde{f}_{j,t,1}(\omega_t)\,d\omega_t \tag{10}$$

Workers on island j equate the subjective valuation of the marginal benefit of labor with marginal the disutility of labor at stage 1. On the other side of the labor market, the optimal labor demand condition is given by

$$W_{j,t}\int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t,1}\left[u'(C_t)\right]\widetilde{f}_{j,t,1}(\omega_t)\,d\omega_t = \left(\int_{\mathbb{R}} \mathbb{E}^{\omega_t}_{j,t,1}\left[u'(C_t)P_{j,t}\right]\widetilde{f}_{j,t,1}(\omega_t)\,d\omega_t\right)\left((1-\alpha)\frac{Y_{j,t}}{N_{j,t}}\right) \tag{11}$$

Firms on island j equate the subjective valuation of the marginal cost of labor with the marginal benefit at stage 1. Unlike expected utility preferences, ambiguity aversion implies that when evaluating marginal effects at stage 1 of period t, firms and workers on the island j employ a distorted posterior belief about the possible models:

$$\widetilde{f}_{j,t,1}(\omega_t) \propto \underbrace{\phi'\left(\mathbb{E}^{\omega_t}_{j,t,0}\left[\frac{C_t^{1-\gamma}-1}{1-\gamma} - \chi\int_{\mathbb{J}}\frac{N^{1+\epsilon}_{j,t}}{1+\epsilon}dj\right]\right)}_{\text{Belief Distortion}}\underbrace{f\left(a_{j,t}|\omega_t\right)f_t(\omega_t)}_{\text{Bayesian Kernel}} \tag{12}$$

It says that whenever a model ω_t generates a lower ex-ante (stage 0) expected utility for the representative household, local agents tend to regard it as the more likely one in their distorted posteriors. Put differently, ambiguity aversion implies a pessimistic belief about the possible models when agents are making decisions.[12]

[12]Note that the firms on island j have the same distorted posterior belief over the set of possible models as island j workers, which is due to the extended smooth rule.

Combing (10) and (11), the labor market equilibrium can be summarized by the following key equation for labor:[13]

$$\chi N_{j,t}^{\epsilon} = \left(\underbrace{\int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t} \left[u'\left(C_t\right) \left(\frac{Y_{j,t}}{Y_t}\right)^{-\frac{1}{\theta}} \right] \widetilde{f}_{j,t,1}\left(\omega_t\right) d\omega_t}_{\text{marginal utility of island } j \text{ commodity}} \right) \left(\underbrace{(1-\alpha)\frac{Y_{j,t}}{N_{j,t}}}_{\text{marginal productivity}} \right) \tag{13}$$

The LHS is the marginal disutility of labor and the RHS is the multiplication of (a) the marginal utility of island j commodity and (b) the marginal productivity of island labor. The key equation says, in equilibrium, the subjective valuation of the private benefit of labor equates the private cost of labor at stage 1. A similar condition also shows up in Angeletos and La'O [2009] and Angeletos et al. [2016]. There are two main differences between ours and theirs. First of all, there is one additional integration of different models due to the existence of ambiguity. Second, agents use a distorted posterior belief due to ambiguity aversion. As will become evident later, these two differences together allow us to build a bridge between variations in ambiguity to confidence, which is the core mechanism of the model.

4.3. Joint Approximation of Allocation and Belief

Using the island production function $Y_{j,t} = A_{j,t}N_{j,t}^{1-\alpha}$ and the market clearing condition for final goods $Y_t = C_t$, we can transform (13) into a fixed point condition for allocation $\left\{Y_{j,t}\right\}_{j\in J}$:

$$\chi Y_{j,t}^{\frac{1+\epsilon}{1-\alpha}-1+\frac{1}{\theta}} = (1-\alpha) A_{j,t}^{\frac{1+\epsilon}{1-\alpha}} \left(\int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t} \left[Y_t^{\frac{1}{\theta}-\gamma} \right] \widetilde{f}_{j,t,1}\left(\omega_t\right) d\omega_t \right) \tag{14}$$

where the distorted posterior belief $\widetilde{f}_{j,t,1}\left(\omega_t\right)$ is given by

$$\widetilde{f}_{j,t,1}\left(\omega_t\right) \propto \underbrace{\phi'\left(\mathbb{E}_{j,t,0}^{\omega_t} \left[\frac{Y_t^{1-\gamma}-1}{1-\gamma} - \chi \int_{\mathbb{J}} \frac{\left(Y_{j,t}/A_{j,t}\right)^{(1+\epsilon)/(1-\alpha)}}{1+\epsilon} dj \right] \right)}_{\text{Belief Distortion}} \underbrace{f\left(a_{j,t}|\omega_t\right) f_t\left(\omega_t\right)}_{\text{Bayesian Kernel}} \tag{15}$$

To ensure the strategic complementarity in productions across islands, we make the following parametric restriction for the static model without capital:[14]

Assumption 2. (Strategic Complementarity) It is assumed that $\frac{1}{\theta} > \gamma$ when there is no capital.

An increase in the output of all other islands $k \neq j \in \mathbb{J}$, on the one hand, raises the demand for island j commodities because households have more labor income from all other islands. This is the so-called aggregate demand externality. However, on the other hand, it also generates upward pressure on the wage rate of island j due to the wealth effect of labor supply. Assumption 2 ensures that the wealth effect of labor supply is so weak that the aggregate demand externality dominates in equilibrium. [15]

[13] A detailed derivation can be found in Appendix A.

[14] To see why this is the case, observe that under perfect information, (14) can be simplified into

$$\chi Y_{j,t}^{\frac{1+\epsilon}{1-\alpha}-1+\frac{1}{\theta}} = \left(\frac{\eta-1}{\eta}\right)(1-\alpha) A_{j,t}^{\frac{1+\epsilon}{1-\alpha}} Y_t^{\frac{1}{\theta}-\gamma}$$

It is straightforward to show $\partial Y_{j,t}/\partial Y_t > 0$ if and only if $\frac{1}{\theta} - \gamma > 0$.

[15] Later in Section 6, we drop this assumption when there exists capital accumulation. Instead we assume $\theta = 1$, i.e., a Cobb-

Definition 2 (**Conditional Log-Normal Equilibrium**)**.** *An allocation* $\left\{Y_{j,t}, Y_t\right\}_{j\in\mathbb{J}}$ *constitutes a conditional log-normal equilibrium if both* $Y_{j,t}|\psi_t$ *and* $Y_t|\psi_t$ *are log-normally distributed.*

The technical complication here is that in equilibrium, the distorted posterior belief $\widetilde{f}_{j,t,1}\left(\omega_t\right)$ is not orthogonal to allocations. They have to be solved simultaneously in equilibrium. As a result, the equilibrium of the economy is the solution to the double fixed point conditions: one solves (14) characterizing the equilibrium cross-sectional allocation $\left\{Y_{j,t}\right\}_{j\in\mathbb{J}}$ conditional on any distorted posterior belief about possible models $\widetilde{f}_{j,t,1}\left(\omega_t\right)$ and the other solves (15) characterizing an equilibrium distorted posterior belief about the possible models conditional on any allocation $\left\{Y_{j,t}\right\}_{j\in\mathbb{J}}$ of the economy.

Lemma 1. The distorted posterior belief $\widetilde{f}_{j,t,1}\left(\omega_t\right)$ can be normally approximated up to second order with sufficient accuracy if allocation $\left\{Y_{j,t}, Y_t\right\}_{j\in\mathbb{J}}$ constitutes a conditional Log-Normal equilibrium.

Proof. See Appendix A. ∎

Lemma 2. Allocation $\left\{Y_{j,t}\left(a_{j,t},\psi_t\right)\right\}_{j\in\mathbb{J}}$ constitutes a conditional log-normal equilibrium if distorted posterior belief over possible models $\widetilde{f}_{j,t,1}\left(\omega_t\right)$ is normal.

Proof. Directly follows Angeletos and La'O [2009]. ∎

The complication can be resolved once we focus on a particular type of equilibrium - conditional log normal equilibrium as defined in Definition 2. On the one hand, the conditional log-normal equilibrium embeds the standard log-normal equilibrium or log-linearized equilibrium as a special case when there are no ambiguity shocks, while, on the other hand, it can be justified, up to an approximation sense, by Lemma 1 and Lemma 2 in a self-fulfilling fashion. The following proposition characterizes the conditional log-normal equilibrium.[16]

Proposition 1 (Equilibrium Characterization)**.** *Under some regularity conditions, there exists a unique approximated symmetric conditional log-normal equilibrium where the allocation* $\left\{Y_{j,t}, Y_t\right\}_{j\in\mathbb{J}}$ *is such that*

$$y_{j,t} \equiv \ln Y_{j,t} = \left(\underbrace{y^* + \overline{h}_y\left(\overline{\psi}\right)}_{\text{Ambiguous SS}}\right) + \underbrace{\kappa_{ya_j}\left(\psi_t,\lambda\right)}_{\text{Use of Private Info.}} \cdot a_{j,t} + \underbrace{\widehat{h}_y\left(\psi_t,\lambda\right)}_{\text{Impact of Amb. Shock}} \tag{16}$$

and

$$y_t \equiv \ln Y_t = \left(\underbrace{y^* + \overline{h}_y\left(\overline{\psi}\right)}_{\text{Ambiguous SS}}\right) + \underbrace{\kappa_{ya_j}\left(\psi_t,\lambda\right)}_{\text{Use of Private Info.}} \cdot a_t + \underbrace{\widehat{h}_y\left(\psi_t,\lambda\right)}_{\text{Impact of Amb. Shock}} \tag{17}$$

where $y^* + \overline{h}_y\left(\overline{\psi}\right)$ *denotes the output level at the ambiguous steady state. And* $\kappa_{ya_j}\left(\psi_t,\lambda\right)$*, the slope of output w.r.t. productivity, is called the use of private information, which is a function of the amount of ambiguity* ψ_t

Dou glas technology for the aggregation of island output $\left\{Y_{j,t}\right\}_{j\in J}$ and $\gamma = 1$, i.e., the log utility for consumption. Such a parameterization also ensures complementarity in production given the existence of the consumption smoothing incentive, which weakens income effect of labor supply.

[16] The approximation turns out to be quite accurate. In Appendix B.1, we conduct a numerical check for accuracy.

and the degree of ambiguity aversion λ. Finally, $\widehat{h}_y(\psi_t;\lambda)$ denotes the impact of the ambiguity shock on output satisfying

$$\widehat{h}_y(\overline{\psi},\lambda)=0$$

Finally, the distorted posterior belief about the possible models is normal with mean μ_t and variance σ_t^2 such that

$$\mu_t=\left(\frac{e^{\psi_t}+g_\sigma(\psi_t,\lambda)}{\sigma_\zeta^2+\sigma_\iota^2+e^{\psi_t}+g_\sigma(\psi_t,\lambda)}\right)x_{j,t}+\left(\frac{\sigma_\zeta^2+\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2+e^{\psi_t}+g_\sigma(\psi_t,\lambda)}\right)g_\mu(\psi_t,\lambda)$$

and

$$\sigma_t^2=\left(\frac{\sigma_\zeta^2+\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2+e^{\psi_t}+g_\sigma(\psi_t,\lambda)}\right)\left(e^{\psi_t}+g_\sigma(\psi_t,\lambda)\right)$$

where the distortion in mean $g_\mu(\psi_t,\lambda)$ and in variance $g_\sigma(\psi_t,\lambda)$ are given by

$$g_\mu(\psi_t,\lambda)=-\lambda\kappa_{J\omega}\left(\frac{1}{1+\lambda\kappa_{J\omega\omega}e^{\psi_t}}\right)e^{\psi_t}\qquad g_\sigma(\psi_t,\lambda)=-\left(\frac{\lambda\kappa_{J\omega\omega}e^{\psi_t}}{1+\lambda\kappa_{J\omega\omega}e^{\psi_t}}\right)e^{\psi_t}\tag{18}$$

with $\kappa_{J\omega}$ and $\kappa_{J\omega\omega}$ being linear functions of κ_{ya_j} whose expressions are given in the Appendix.

Proof. See Appendix A. ■

Equilibrium allocations (16) and (17) are akin to those of Angeletos and La'O [2009]. Log deviations from the ambiguous steady state of island output $\widehat{y}_{j,t}\equiv y_{j,t}-\left(y^*+\overline{h}_y(\overline{\psi})\right)$ and aggregate output $\widehat{y}_t\equiv y_j-\left(y^*+\overline{h}_y(\overline{\psi})\right)$ can be expressed into linear functions of island productivity $a_{j,t}$ and aggregate productivity a_t, respectively. We name the slope of these linear functions $\kappa_{ya_j}(\psi_t,\lambda)$ as the use of private information, which is a function of the amount of ambiguity ψ_t. Furthermore, the intercept term $\widehat{h}_y(\psi_t;\lambda)$ controls the impact of the ambiguity shock on output, given that it is zero when evaluated at the ambiguous steady state. These two terms $\left\{\kappa_{ya_j}(\psi_t,\lambda),\widehat{h}_y(\psi_t;\lambda)\right\}$ are at the core of our analysis. Later in Section 5, we study the impacts of ambiguity shocks through some comparative static analysis of these two terms and demonstrate how ambiguity shocks can possibly generate co-movements across confidence, cross-sectional dispersions in beliefs, and real quantities of the economy.

5. The Impacts of Ambiguity Shocks

In this section, we analyze the impacts of ambiguity shocks by the comparative static analysis of ψ_t. We start by providing a game theoretic interpretation of the equilibrium allocations of our business cycle model. Such a game theoretic interpretation clarifies the main mechanisms of our paper by highlighting the dual impacts of ambiguity shocks. Then, we then demonstrate how to map the dual impacts of ambiguity shocks into fluctuations in confidence, cross-sectional dispersions in beliefs, and real quantities of the economy. Finally, we close up this section with discussions about the interplay between incomplete information and ambiguity aversion.

5.1. The Dual Impacts of Ambiguity Shock

To build up the economic intuitions behind impacts of ambiguity shocks, we demonstrate a game theoretic interpretation of the equilibrium of our business cycle model, which resembles the beauty contest in Morris and Shin [2002] and Angeletos and Pavan [2007], but with a distorted information structure to capture the belief distortions due to ambiguity aversion.

Proposition 2. *The equilibrium allocation* $\left\{Y_{j,t}, Y_t\right\}_{j\in\mathbb{J}}$ *is identical to that of a beauty contest such that*

$$y_{j,t} = \kappa_a a_{j,t} + \kappa_y \mathbb{E}_{j,t}\left[y_t\right]$$

where the coefficients κ_a *and* κ_y *are such that*

$$\kappa_a = \frac{\frac{1+\epsilon}{1-\alpha}}{\frac{1+\epsilon}{1-\alpha} - 1 + \frac{1}{\theta}} \qquad \kappa_y = \frac{\frac{1}{\theta} - \gamma}{\frac{1+\epsilon}{1-\alpha} - 1 + \frac{1}{\theta}} \in (0,1)$$

The information structure is distorted such that

$$\widetilde{a}_{j,t} = \widetilde{a}_t + \widetilde{\iota}_{j,t}, \quad \widetilde{\iota}_{j,t} \sim N\left(0, \sigma_\iota^2\right)$$
$$\widetilde{a}_t \sim N\left(g_\mu\left(\psi_t, \lambda\right), \sigma_\zeta^2 + e^{\psi_t} + g_\sigma\left(\psi_t, \lambda\right)\right)$$

where distortions $\left\{g_\mu\left(\psi_t, \lambda\right), g_\sigma\left(\psi_t, \lambda\right)\right\}$ *are given by (18) and satisfy the following*

$$g_\mu\left(\psi_t, \lambda\right) \leq 0, \quad g_\mu\left(-\infty, \lambda\right) = 0, \quad g_\mu\left(\psi_t, 0\right) = 0, \quad \frac{\partial g_\mu\left(\psi_t, \lambda\right)}{\partial \psi_t} < 0.$$

and

$$g_\sigma\left(\psi_t, \lambda\right) \leq 0, \quad g_\sigma\left(-\infty, \lambda\right) = 0, \quad g_\sigma\left(\psi_t, 0\right) = 0, \quad \frac{\partial\left(e^{\psi_t} + g_\sigma\left(\psi_t, \lambda\right)\right)}{\partial \psi_t} > 0$$

Proof. See Appendix A. ∎

As in the beauty contest, island output $y_{j,t}$ is the linear combination of island productivity $a_{j,t}$ and the island j expectation of aggregate output. The former controls the marginal cost of production on island j and the latter manifests island j's forecast about his their demand conditions. Here κ_y corresponds to the notion of coordination motive in the beauty contest literature. Its magnitude $\kappa_y \in (0,1)$ ensures complementarity in action and uniqueness in allocations, once we fix a distorted information structure.

However, unlike the standard beauty contest, the perceived distribution of aggregate productivity is distorted both in mean and variance due to ambiguity aversion. Figure 2 plots the perceived **as if** distributions[17] of the aggregate fundamental for a low level of ambiguity, i.e., ψ_t is small, and for a high level of ambiguity, i.e., ψ_t is large. It can be shown that with a positive uncertainty (or ambiguity shock), namely an increase in ψ_t, agents become more uncertain about the aggregate fundamental. At the same time, it makes agents become more pessimistic about the aggregate fundamental. We call these the dual impacts of ambiguity shocks.

[17] These distributions are called "as if" because they are the subjective beliefs about aggregate fundamentals that would deliver the same allocations as our baseline model when agents have expected utility preferences.

5.2. Confidence, Belief Dispersion, and Real Quantities

Real Quantities. What are the implications of the dual impacts of ambiguity shocks on real quantities? With the aggregate demand externality, a distorted prior belief about aggregate productivity $\widetilde{a}_t$ can be translated into a distorted prior belief about aggregate demand. Therefore, an adverse ambiguity shock makes all agents believe that the aggregate demand turns bad on average and is more volatile. The former maps into lower output, either of a particular island or at the aggregate, while the latter maps into an increased incentive in the use of private information when making the expectation about aggregate demand, hence when making labor demand and supply decisions. We summarize these results in the following proposition

Proposition 3. *an adverse ambiguity shock that increases the amount of ambiguity ψ_t generates lower aggregate output in the sense that*

$$\frac{\partial \widehat{h}_y(\psi_t, \lambda)}{\partial \psi_t} < 0 \tag{19}$$

if agents are ambiguity averse, i.e. $\lambda > 0$. Moreover, the equilibrium use of private information $\kappa_{ya_j}(\psi_t, \lambda)$ is an increasing function of the amount of ambiguity ψ_t:

$$\frac{\partial \kappa_{ya_j}(\psi_t, \lambda)}{\partial \psi_t} > 0. \tag{20}$$

Proof. See Appendix A. ∎

At the core of understanding (19) is the increased degree of pessimism about aggregate productivity. Two forces at work deepen the agents' degree of pessimism, one fundamental and one strategic. An adverse ambiguity shock, on the one hand, increases the amount of ambiguity perceived by all agents. In response, agents behave in a more pessimistic way as concerns over aggregate productivity. This is the fundamental or direct channel. On the other hand, an adverse ambiguity shock induces all other agents to use more of their private information when making output decisions, which makes aggregate output or demand respond more to the ambiguous aggregate productivity. Therefore, under the aggregate demand externality, it further raises the amount of ambiguity in firms' demand structure, which further increases the degree of pessimism. This is the strategic or indirect channel. An adverse ambiguity shock raises all agents' degree of pessimism through the fundamental and strategic channels, , which eventually drives down the economic activities.

Confidence and Belief Dispersion. What are the implications of the dual impacts of ambiguity shocks on agents' forecasts about the outlook of the aggregate economy? To answer this question, we define the output forecasts of agents on island j as follows:

$$\widetilde{\mathbb{E}}_{j,t}[y_t] \equiv \int_{\mathbb{R}} E_{j,t}^{\omega_t}[y_t]\, \widetilde{f}_{j,t,1}(\omega_t)\, d\omega_t$$

Note that local output forecasts are defined on the basis of the distorted posterior belief about possible models $\widetilde{f}_{j,t,1}(\omega_t)$. Thus, we implicitly assume that ambiguity averse agents would use their subjective pessimistic beliefs to make forecasts. Then we define *confidence* and *belief dispersion* as the cross-sectional mean and dispersion of output forecasts across islands, respectively.[18]

[18]Such a definition is consistent with the practice of most of the survey exercises. For example, in the Michigan Survey of

Definition 3. *Confidence and belief dispersion are defined to be as the cross-sectional mean and dispersion of output forecasts across islands, respectively:*

$$Conf.\left(\psi_t,\lambda\right)\equiv\int_{\mathbb{J}}\widetilde{\mathbb{E}}_{j,t}\left[y_t\right]dj,\qquad B.D.\left(\psi_t,\lambda\right)\equiv\int_{\mathbb{J}}\left(\widetilde{\mathbb{E}}_{j,t}\left[y_t\right]-\int_{\mathbb{J}}\widetilde{\mathbb{E}}_{j,t}\left[y_t\right]dj\right)^2dj$$

Following Definition 3, we can express confidence into the following:

$$Conf.\left(\psi_t,\lambda\right)=y^*+\overline{h}_y\left(\overline{\psi}\right)+\underbrace{\kappa_{ya_j}\left(\psi_t,\lambda\right)}_{+,\uparrow}\int_{\mathbb{J}}\int_{\mathbb{R}}\mathbb{E}_{j,t}^{\omega_t}\left[a_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t dj+\underbrace{\widehat{h}_y\left(\psi_t,\lambda\right)}_{-,\downarrow}$$

where $\int_{\mathbb{J}}\int_{\mathbb{R}}\mathbb{E}_{j,t}^{\omega_t}\left[a_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t dj$ denotes the cross-sectional mean of beliefs about aggregate productivity given by:

$$\int_{\mathbb{J}}\int_{\mathbb{R}}\mathbb{E}_{j,t}^{\omega_t}\left[a_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t dj\equiv\underbrace{\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+e^{\psi_t}+g_\sigma\left(\psi_t,\lambda\right)}\right)}_{+,\downarrow}\underbrace{g_\mu\left(\psi_t,\lambda\right)}_{-,\downarrow}\tag{21}$$

Hit by an adverse ambiguity shock, all agents, in their priors, become more pessimistic about the aggregate fundamental, i.e., $g_\mu\left(\psi_t,\lambda\right)$ decreases. On the other hand, all agents perceive the aggregate fundamental to become more volatile, i.e. $e^{\psi_t}+g_\sigma\left(\psi_t,\lambda\right)$ increases. The former increases the economy-wide pessimism. However, the latter reduces the use of pessimistic priors. In equilibrium, the former dominates the latter, implying that all agents are becoming more pessimistic about aggregate productivity when making decisions. Furthermore, the key here is that all agents understand that the others are more pessimistic about aggregate productivity. They also understand that others understand that there is an increase in economy-wide pessimism. Moreover, they all understand that all the others understand that others understand this, etc. The consequence of such higher-order belief reasoning is the drop in aggregate output, i.e., $\widehat{h}_y\left(\psi_t,\lambda\right)$, which further depresses confidence. Finally, all agents also understand that others all perceive aggregate fundamental as being more volatile. Hence, they understand that all the others would use more of their private information when making output decisions. Therefore, they know that aggregate output will respond more to the aggregate fundamental, i.e., $\kappa_{ya_j}\left(\psi_t,\lambda\right)$ increases. It raises output forecasts' reliance on the pessimistic belief about aggregate productivity, which even further depresses confidence.[19]

At the same time, belief dispersion can be expressed into the following way:

$$B.D.\left(\psi_t,\lambda\right)=\underbrace{\kappa_{ya_j}^2\left(\psi_t,\lambda\right)}_{\uparrow}\underbrace{\left(\frac{\sigma_\zeta^2+e^{\psi_t}}{\sigma_\zeta^2+e^{\psi_t}+\sigma_\iota^2}\right)^2}_{\uparrow}\sigma_\iota^2\tag{22}$$

An adverse ambiguity shock makes firms and workers on all islands believe, in their **as if** subjective priors, that the aggregate fundamental is more volatile, which increases the incentive to use private information when forming expectations about aggregate demand conditions. This maps into an increased

Consumer, consumers are asked about whether they believe output will go up or down in the following 12 months. In a broader context, it is a short-cut for agents' forecasts about the outlooks of the economy.

[19]Note that in our model, island j agents have perfect information about their own productivity. An increase in the amount of ambiguity depresses island j agents' belief about the aggregate productivity without changes in beliefs about their own productivities. These movements in belief are isomorphic to those of a negative confidence shock under the heterogenous prior setup, alias Angeletos et al. [2018].

responsiveness of island output $y_{j,t}$ to island productivity $a_{j,t}$ because it is $a_{j,t}$ that serves as the private information about aggregate demand for island j agents. Upon aggregation, we have aggregate output y_t responds more to aggregate productivity a_t. From the perspective of forecaster j, increase in κ_{ya_j} implies that "there are more to estimate". Moreover, when he estimates the aggregate productivity, he tends to rely more on his private information $a_{j,t}$ because he believes, in his as if subjective prior, that the aggregate fundamental is now more volatile. These two in combination increase the responsiveness of forecaster j's forecast to private information $a_{j,t}$, which eventually leads to a higher cross-sectional dispersion in output forecasts ex-ante, i.e., an increase in belief dispersion. Note that the economy itself does not become more dispersed. It is the increased responsiveness to idiosyncratic shocks that drives up the cross-sectional dispersion. This differentiates our paper to the theory of uncertainty shock as in Bloom [2009] and Bloom et al. [2018], which takes fluctuations in dispersion as model input rather than model output.

We close up the current subsection by a summary of impacts of ambiguity shocks. It demonstrates that an adverse ambiguity shock can generate a recession with depressed confidence and heightened belief dispersion.

Proposition 4 (Impacts of Ambiguity Shocks). *If decision makers are ambiguity averse, i.e., $\lambda > 0$, an adverse ambiguity shock that increases the amount of ambiguity ψ_t generates*

- *lower confidence;*
- *higher belief dispersion;*
- *and finally a lower aggregate output on average.*

Proof. See Appendix A. ∎

5.3. Discussion: Incomplete Information and Ambiguity Aversion

In Section 5.2, we highlight the strategic channel behind the impacts of ambiguity shocks. This is closely related to the assumption of incomplete information in our model. In this subsection, we formally study how variations in the degree of incompleteness in information affect the impacts of ambiguity shocks.

Incomplete information is of primary importance in our paper. When information is complete, i.e., $\sigma_\iota^2 = 0$, island j agents face no uncertainty regarding decisions made by agents on other islands. Then. all agents will have the perfect understanding of the whole economy. Therefore, ambiguity shocks play no role in driving aggregate fluctuations. Corollary 1 summarizes this result.

Corollary 1. The ambiguity shocks have no impacts on aggregate output $\widehat{h}_y(\psi_t, \lambda) = 0$ when information is complete $\sigma_\iota^2/\sigma_\zeta^2 = 0$.

Proof. Straight-forward following the proof of Proposition 1. ∎

Note that the completeness in information only requires that there exists no private information. It does not necessarily imply perfect information.[20] Corollary 1 still holds when there is an information friction as long as there is common knowledge about beliefs within the economy. Unlike our simplified

[20]See Angeletos and Lian [2016] for more formal discussions.

case with $\sigma_\iota^2/\sigma_\zeta^2 = 0$, it still presents the distorted posterior belief about possible models. However, all agents have a common knowledge of over Y_t under complete information. Then (14) transforms into

$$\chi Y_{j,t}^{\frac{1+\epsilon}{1-\alpha}-1+\frac{1}{\theta}} = (1-\alpha)\, A_{j,t}^{\frac{1+\epsilon}{1-\alpha}} Y_t^{\frac{1}{\theta}-\gamma}$$

which leaves no room for ambiguity shocks to have any real impacts. To interpret this, it is the imperfect coordination among agents across islands, possibly due to imperfect communication as in Angeletos and La'O [2013], that matters rather than imperfect information.

The question of interest naturally arises. Does more incompleteness in information amplify or dampen the impacts of ambiguity shocks? We answer these questions by not only varying σ_ι^2 but also σ_ζ^2 while holding the amount of ambiguity constant at the Amb.-SS level. There are two forces at work. On the one hand, more incompleteness in the information, i.e., a larger σ_ι^2 or lower σ_ζ^2, reduces the incentive in the use of private information because private information is becoming less informative, not only about the aggregate state but also about the possible models. This marginally changes degree of pessimism since $g_\mu\left(\overline{\psi}, \lambda\right)$ is a function of both $\kappa_{J\omega}\left(\overline{\kappa}_{ya_j}, \overline{h}_y\right)$ and $\kappa_{J\omega\omega}\left(\overline{\kappa}_{ya_j}, \overline{h}_y\right)$, which are functions of $\overline{\kappa}_{ya_j}$. It can be proved that the mean distortion $g_\mu\left(\overline{\psi}, \lambda\right)$ marginally decreases w.r.t degree of information incompleteness.

Lemma 3. Mean distortion at the Amb.-SS $g_\mu\left(\overline{\psi}, \lambda\right)$ marginally decreases in σ_ι^2 and increases in σ_ζ^2:

$$\frac{\partial g_\mu\left(\overline{\psi}, \lambda\right)}{\partial \sigma_\iota^2} < 0 \qquad\qquad \frac{\partial g_\mu\left(\overline{\psi}, \lambda\right)}{\partial \sigma_\zeta^2} > 0$$

Proof. See Appendix A. ■

On the other hand, fixing the belief distortion $g_\mu\left(\psi_t, \lambda\right) = g_\mu^*$ and $g_\mu\left(\psi_t, \lambda\right) = g_\sigma^*$, more incompleteness in information implies a larger load of belief distortion on island and aggregate output. To see this, express island output $y_{j,t}$ into an infinite sum of a complete hierarchy of all higher-order beliefs of aggregate productivity $\widetilde{a}_t$:

$$y_{j,t} = \kappa_a \left(a_{j,t} + \sum_{n=1}^{\infty} \kappa_y^n \mathbb{E}_{j,t}^n \left[\widetilde{a}_t\right] \right) \tag{23}$$

where higher-order beliefs of island j agents are defined recursively such that

$$\mathbb{E}_{j,t}^1\left[\widetilde{a}_t\right] \equiv \mathbb{E}_{j,t}\left[\widetilde{a}_t\right] \qquad\qquad \mathbb{E}_{j,t}^n\left[\widetilde{a}_t\right] \equiv \mathbb{E}_{j,t}\left[\int_{\mathbb{J}} E_{j,t}^{n-1}\left[\widetilde{a}_t\right] dj\right] \quad \forall n \geq 1$$

It turns out that increased incompleteness in information increases the response of all orders of belief in the pessimistic prior information. Observe that it is the prior of the aggregate fundamental that is pessimistically distorted. Therefore, it must be the case that all orders of beliefs are exposed to a more pessimistic distortion. Therefore, incompleteness in information naturally amplifies the impact of ambiguity at the margin when we fix the pessimistic distortion.

The two forces above interact with each other, indicating that incomplete information is an amplifying mechanism for the impact of ambiguity on aggregate output. Proposition 5 summarizes this result.

Proposition 5. *At the ambiguous steady state (Amb.-SS), a marginal increase in σ_ι^2 that increases the degree of information incompleteness decreases the use of private information and amplifies the impact of ambiguity:*

$$\frac{d\overline{\kappa}_{ya_j}}{d\sigma_\iota^2} < 0 \qquad \frac{d\overline{h}_y}{d\sigma_\iota^2} < 0$$

In contrast, a marginal increase in σ_ι^2 that reduces the degree of information incompleteness increases the use of private information and dampens the impact of ambiguity:

$$\frac{d\overline{\kappa}_{ya_j}}{d\sigma_\zeta^2} > 0 \qquad \frac{d\overline{h}_y}{d\sigma_\zeta^2} > 0$$

Proof. See Appendix A. ■

5.4. A summary

This section has shown that an adverse ambiguity shock generates a recession with depressed confidence and heightened belief dispersion qualitatively. In what follows, we study the impacts of ambiguity shocks quantitatively within an extended dynamic RBC model. We demonstrate that ambiguity shocks in our theory can generate reasonable co-movements patterns in real quantities as well as confidence and belief dispersion in survey data. The quantitative potential of our theory is evaluated by bringing the observable implications into the data.

6. The Dynamic RBC Model: Quantitative Evaluation

In this section, we illustrate the quantitative potential of our theory by studying a dynamic RBC model. We first set up the model. Then, we move to the discussion about our quantitative methodology in the estimation of the conditional log-normal equilibrium. Finally, observable implications of ambiguity shocks, for the real quantities, belief dispersion, as well as confidence, are assessed through a calibrated version of the model.

6.1. Model Setup

To economize on space, we only discuss the additional novelties of the dynamic RBC model relative to the static model in Section 3.

Captial. First of all, in addition to labor, island j firms need to employ capital from the household for the production of the island j commodity:

$$Y_{j,t} = A_{j,t} N_{j,t}^{1-\alpha} K_{j,t}^{\alpha}$$

The realized profits of island j firms are given by

$$\Pi_{j,t} = P_{j,t} Y_{j,t} - W_{j,t} N_{j,t} - R_{j,t} K_{j,t}$$

Capitals are traded at stage 1 of period t in locally competitive capital markets at price $R_{j,t}$. Therefore, the flow budget constraint of the representative household is given by

$$P_t C_t + P_t \int_{\mathbb{J}} K_{j,t+1} - (1-\delta) K_{j,t} dj = \int_{\mathbb{J}} W_{j,t} N_{j,t} dj + \int_{\mathbb{J}} R_{j,t} K_{j,t} dj + \int_{\mathbb{J}} \Pi_{j,t} dj$$

where $\int_{\mathbb{J}} R_{j,t}K_{j,t}dj$ and $\int_{\mathbb{J}} W_{j,t}N_{j,t}dj$ denote the total capital and labor income of all islands, respectively, and $\int_{\mathbb{J}} \Pi_{j,t}dj$ is the transfers of realized profits from all island firms. Here we assume that it is the representative household who owns the capital. And investments are island-specific. Therefore, the capital supply of island j in period $t+1$ is pre-determined at stage 2 of period t by the representative household.

Productivity and ambiguity shocks. Aggregate productivity $a_t \equiv \log A_t$ follows an AR(1) process

$$a_t = \rho a_{t-1} + \zeta_t$$

where $\zeta_t \sim N\left(0, \sigma_\zeta^2\right)$ is the aggregate productivity shock in period t. Island-specific productivity $a_{j,t} \equiv \log A_{j,t}$ equals aggregate productivity plus an idiosyncratic productivity shock $\iota_{j,t}$:

$$a_{j,t} = a_t + \iota_{j,t}$$

The idiosyncratic productivity shock $\iota_{j,t}$ is assumed to be i.i.d normally distributed with mean ω_t and variance σ_t^2. Objectively, the cross-sectional mean of idiosyncratic productivity shocks are zero for all periods, i.e. $\omega_t = 0 \; \forall t > 0$. However, agents inside the economy cannot fully understand it. Instead, they possess some ambiguity over the cross-sectional means of idiosyncratic productivity shocks for all t, i.e., $M \equiv \{\omega_t : \forall t \geq 0\}$.[21]

At the beginning of time, say period 0, all agents have the prior belief that any $\omega_t \in M$ are i.i.d normally distributed with mean 0 and variance $\overline{\sigma}_\omega^2 \equiv e^{\overline{\psi}}$. Here $\overline{\sigma}_\omega^2$ or $\overline{\psi}$ measures the amount of ambiguity that agents possess in the Amb.-SS. Ambiguity in the past does not last forever. As will become evident later, concurrent ambiguity is resolved at stage 2 of that period. Therefore, at stage 0 of any period t, agents inside the economy only possess ambiguity about concurrent and future cross-sectional means of idiosyncratic productivity shocks, i.e., $M_t \equiv \{\omega_{t+k} : \forall k \geq 0\}$. The amount of ambiguity that agents perceived at period t, denoted by ψ_t, is time-varying and governed by an AR(1) process

$$\widehat{\psi}_t \equiv \psi_t - \overline{\psi} = \rho_\psi \widehat{\psi}_{t-1} + \tau_t$$

where τ_t is the ambiguity shock assumed to be normally distributed with mean 0 and variance σ_τ^2. We close the description of the ambiguity process by specifying the common prior beliefs of all agents about $M_t = \{\omega_{t+k} : \forall k \geq 0\}$ at stage 0 of period t:

$$\omega_{t+k} \sim i.i.d \; N\left(0, e^{\psi_{t,t+k}}\right) \quad \forall k \geq 0$$

where the amount of ambiguity that agents perceived about ω_{t+k} at period t, denoted as $\psi_{t,t+k}$, is an increasing affine function of ψ_t:

$$\psi_{t,t+k} = \left(1 - \rho_\psi^k\right)\overline{\psi} + \rho_\psi^k \psi_t$$

The structure of prior ensures a notion of consistency in beliefs about all future ambiguity $\omega_{t+k}, \forall k \geq 1$. It simply says that the period t prior over future cross-sectional means of idiosyncratic productivity shock ω_{t+k} coincides with period $t+k$ prior for it if there are no ambiguity shocks between period t

[21] Note that in the dynamic model, we assume that the ambiguity is about the cross-sectional mean of idiosyncratic productivity shocks. It is isomorphic to the setup where agents are ambiguous about the temporary component of the aggregate productivity shock.

and $t+k$. Put differently,

$$\psi_{t,t+k} = \mathbb{E}_t \left[\psi_{t+k}\right]$$

the ambiguity shock τ_t in the process of the amount of ambiguity ψ_t can be understood as a changing prior process. Moreover, we implicitly assume that an adverse ambiguity shock in period t, i.e., $\tau_t > 0$, makes agents become more uncertain about the entire future ω_{t+k}. However, it is period t biased in the sense that it raises the perceived ambiguity in period t more than that in the future. Furthermore, the increase in ambiguity is mean-reverting such that for ambiguity ω_{t+k} in the very far future $k \to +\infty$, the subjective belief stays in its Amb.-SS belief, i.e. $\lim_{k\to+\infty} \psi_{t,t+k} = \overline{\psi}$.

Information structure. Denote $\mathcal{I}_{t,0}$, $\mathcal{I}_{j,t,1}$ and $\mathcal{I}_{t,2}$ as the information sets that are available to all agents at stage 0 of period t, that are only available to island j agents at stage 1 of period t and that are available to all agents at stage 2 of period t, respectively. Recursively, we can define these information sets by

$$\mathcal{I}_{t,0} = \mathcal{I}_{t-1,2} \cup \{\psi_t\} \qquad \mathcal{I}_{j,t,1} = \mathcal{I}_{t,0} \cup \left\{a_{j,t}\right\} \qquad \mathcal{I}_{t,2} = \cup_j \mathcal{I}_{j,t,1} \cup \{\zeta_t\} \tag{24}$$

Note that all concurrent uncertainty, either risk (state uncertainty over a_t) or ambiguity (model uncertainty over ω_t) is resolved at stage 2 of period t. Hence, consumption-saving decisions are made on the basis of complete information about future cross-sectional means of idiosyncratic shocks. Moreover, since idiosyncratic productivity shocks are i.i.d, $\left\{a_{j,t}\right\}_{j\in\mathbb{J}}$ provides no more information about island j productivity in period $t+1$ than does $\int_j a_{j,t} dj$. Therefore, we can simplify the information set at stage 2 of period t by $\mathcal{I}_{t,2} = \mathcal{I}_{t,0} \cup \left\{\int_{\mathbb{J}} a_{j,t} dj, \omega_t\right\}$. To simplify the notation, we further transform the information structure into

$$\mathcal{I}_{t,0} = \mathcal{I}_{t-1,2} \cup \{\psi_t\} \qquad \mathcal{I}_{j,t,1} = \mathcal{I}_{t,0} \cup \left\{x_{j,t}\right\} \qquad \mathcal{I}_{t,2} = \mathcal{I}_{t,0} \cup \{z_t, \zeta_t\}$$

where $x_{j,t} \equiv \zeta_t + \iota_{j,t}$ denotes the de-facto private information at stage 1 about the aggregate productivity shock and $z_t = \zeta_t + \omega_t$ denotes the de-facto public information at stage 2 about the aggregate productivity shock. Figure 4 displays the timeline and information sets for period t in our dynamic RBC model.

Preferences. Denote $s_{t+1} \equiv \mathcal{I}_{t+1,2} \backslash \mathcal{I}_{t,2}$ as the new information received by agents at stage 2 between two consecutive periods t and $t+1$. We summarize the belief of the representative household at stage 2 by two corresponding Bayesian posteriors: (a) $\pi_M\left(s_{t+1}|\mathcal{I}_{t,2}\right)$, the Bayesian posterior of s_{t+1} at stage 2 of period t under a particular model M, and (2) $\mu\left(M|\mathcal{I}_{t,2}\right)$, the Bayesian posterior over the entire set of possible models $M \in \mathcal{M}$.

Preference of the representative household at stage 2 of period t can be represented by the recursive smooth model of ambiguity proposed by Klibanoff et al. [2009]:

$$\begin{aligned} V_t\left(\mathcal{I}_{t,2}\right) =& u\left(C_t\right) - \chi \int_J \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon} dj \\ &+ \underbrace{\beta\phi^{-1}\left(\int_{\mathcal{M}} \phi\left(\int_{\mathcal{S}_{t+1}} V_{t+1}\left(\mathcal{I}_{t,2}, s_{t+1}\right) d\pi_M\left(s_{t+1}|\mathcal{I}_{t,2}\right)\right) d\mu\left(M|\mathcal{I}_{t,2}\right)\right)}_{\text{Utility Equivalent of the Ambiguous Continuation Value}} \end{aligned}$$

where $\phi(x)$ is some strictly increasing and concave function. The curvature of $\phi(\cdot)$ captures decision makers' taste for ambiguity, namely the degree of ambiguity aversion.[22] The additional concavity in $\phi(x)$ captures the fact that an ambiguous continuation value reduces the utility of the decision maker because ambiguity averse household dislikes mean-preserving spread in expected continuation value due to the existence of the ambiguity.

Denote the value function as $J_t \equiv J\left(\left\{K_{j,t}\right\}, a_{t-1}, z_t, \zeta_t, \psi_t\right)$. Then, we have the following Bellman equation for the representative household at stage 2:

$$J_t = \max_{C_t, \left\{K_{j,t+1}\right\}} u\left(C_t\right) - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon} dj + \beta \phi^{-1}\left(\int_{\mathbb{R}} \phi\left(\mathbb{E}_{t,2}^{\omega_{t+1}}\left[J_{t+1}\right]\right) f_t\left(\omega_{t+1}\right) d\omega_{t+1}\right) \tag{25}$$

subject to

$$P_t C_t + P_t \int_{\mathbb{J}} I_{j,t} dj = \int_{\mathbb{J}} W_{j,t} N_{j,t} didj + \int_{\mathbb{J}} R_{j,t} K_{j,t} dj + \int_{\mathbb{J}} \Pi_{j,t} dj \tag{26}$$

and

$$I_{j,t} = K_{j,t+1} - (1-\delta) K_{j,t} \tag{27}$$

Here $\mathbb{E}_{t,2}^{\omega_{t+1}}[\cdot]$ stands for the mathematical expectation conditioned on $\mathcal{I}_{t,2}$ under a particular model ω_{t+1} for the cross-sectional mean of the idiosyncratic productivity shock tomorrow. And $f_t\left(\omega_{t+1}\right)$ stands for the probability density function for the household's period t prior for the $t+1$ cross-sectional mean of the idiosyncratic productivity shock ω_{t+1}. Since period t knowledge does not reveal any information about ω_{t+1} , prior belief about ω_{t+1} at stage 2 of period t coincides with that at stage 0.

Similar to the static model without capital, at stage 1, the preference of the representative household is given by the smooth model of ambiguity

$$\int_{\Omega_t} \phi\left(\mathbb{E}_{j,t,1}^{\omega_t}\left[J\left(\left\{K_{j,t}\right\}, a_{t-1}, z_t, \zeta_t, \psi_t\right)\right]\right) \widetilde{f}_{j,t,1}^{h}\left(\omega_t\right) d\omega_t$$

with the posterior belief about possible models following the smooth rule of updating to ensure dynamic consistency:

$$\widetilde{f}_{j,t,1}^{h}\left(\omega_t\right) \propto \underbrace{\frac{\phi'\left(\mathbb{E}_{j,t,0}^{\omega_t}\left[J\left(\left\{K_{j,t}\right\}, a_{t-1}, z_t, \zeta_t, \psi_t\right)\right]\right)}{\phi'\left(\mathbb{E}_{j,t,1}^{\omega_t}\left[J\left(\left\{K_{j,t}\right\}, a_{t-1}, z_t, \zeta_t, \psi_t\right)\right]\right)}}_{\text{Weights}} \underbrace{f\left(x_{j,t}|\omega_t\right) f_t\left(\omega_t\right)}_{\text{Bayesian Kernel}}$$

Finally, the firm problem is formulated in a similar fashion as the static model without capital:

$$\int_{\Omega_t} \phi\left(\mathbb{E}_{j,t,1}^{\omega_t}\left[\frac{U'\left(C_t\right)}{P_t}\left(P_{j,t} Y_{j,t} - W_{j,t} N_{j,t} - R_{j,t} K_{j,t}\right)\right]\right) \widetilde{f}_{j,t,1}^{f}\left(\omega_t\right) d\omega_t$$

where the posterior belief about possible models satisfies the extended smooth rule of updating to

[22] Klibanoff et al. [2009] prove that if ϕ^{-1} is Lipschitz and the space of ambiguous model parameters is finite, the recursive smooth model of ambiguity will converge uniformly to the expected utility preferences with true model parameters. In our model, agents are ambiguous over an infinite parameter space $\omega \equiv \{\omega_t : \forall t \geq 0\}$. This prevents the ambiguity from vanishing in the long run through learning.

ensure dynamic consistency:

$$\widetilde{f}^{w}_{j,t,1}(\omega_t) \propto \underbrace{\frac{\phi'\left(\mathbb{E}^{\omega_t}_{j,t,0}\left[J\left(\{K_{j,t}\},a_{t-1},z_t,\zeta_t,\psi_t\right)\right]\right)}{\phi'\left(\mathbb{E}^{\omega_t}_{j,t,1}\left[\ln\left(C_t\right)-\chi\int_{\mathbb{J}}\frac{N^{1+\epsilon}_{j,t}}{1+\epsilon}dj\right]\right)}}_{\text{Weights}}\underbrace{f\left(x_{j,t}|\omega_t\right)f_t\left(\omega_t\right)}_{\text{Bayesian Kernel}}$$

To close up the description of the model, we make the following assumptions on functional forms

Assumption 3. (Log-Exponential) We assume $u(C_t) = \ln C_t$ and $\phi(x) = -\frac{1}{\lambda}e^{-\lambda x}$ with $\lambda \geq 0$.

In what follows, we leave the optimality conditions to the Appendix and directly discuss about our quantitative methodology in the approximation of the conditional log-normal equilibrium, which is closely related to what we have done in Section 4.3.

6.2. Quantitative Methodology

The key feature of the smooth model of ambiguity is that decision makers invoke a distorted (relative to Bayesian posterior) posterior belief about the possible models when evaluating the marginal effects of factor supply and demand. In our RBC extension, the belief distortions at stage 1 and stage 2 are given by

$$D_{t,1}(\omega_t) = e^{-\lambda\mathbb{E}^{\omega_t}_{t,0}[J_t]} \qquad D_{t,2}(\omega_{t+1}) = e^{-\lambda\mathbb{E}^{\omega_{t+1}}_{t,2}[J_{t+1}]} \tag{28}$$

To pin down the equilibrium quantitatively, the expected value functions as functions of today's and tomorrow's cross-sectional means of idiosyncratic productivity shocks, $\left\{\mathbb{E}^{\omega_t}_{t,0}[J_t], \mathbb{E}^{\omega_{t+1}}_{t,2}[J_{t+1}]\right\}$, have to be jointly approximated with policy rules.

Focusing on conditional the log-normal equilibrium as in Definition 2 and denoting the hatted-lower-case variable as the log deviation from Amb. SS, we propose the following policy rules for island employment $\widehat{n}_{j,t}$, output $\widehat{y}_{j,t}$, wage rate $\widehat{w}_{j,t}$ and rental rental rate of capital $\widehat{r}_{j,t}$ at stage 1 of period t:

$$\begin{aligned}
\widehat{y}_{j,t} &= \kappa_{yk}\widehat{k}_t + \kappa_{ya}a_{t-1} + \kappa_{yx}x_{j,t} + \kappa_{yx\psi}x_{j,t}\widehat{\psi}_t + \kappa_{y\psi}\widehat{\psi}_t + \kappa_{y\psi\psi}\widehat{\psi}^2_t \\
\widehat{n}_{j,t} &= \kappa_{nk}\widehat{k}_t + \kappa_{na}a_{t-1} + \kappa_{nx}x_{j,t} + \kappa_{nx\psi}x_{j,t}\widehat{\psi}_t + \kappa_{n\psi}\widehat{\psi}_t + \kappa_{n\psi\psi}\widehat{\psi}^2_t \\
\widehat{w}_{j,t} &= \kappa_{wk}\widehat{k}_t + \kappa_{wa}a_{t-1} + \kappa_{wx}x_{j,t} + \kappa_{wx\psi}x_{j,t}\widehat{\psi}_t + \kappa_{w\psi}\widehat{\psi}_t + \kappa_{w\psi\psi}\widehat{\psi}^2_t \\
\widehat{r}_{j,t} &= \kappa_{rk}\widehat{k}_t + \kappa_{ra}a_{t-1} + \kappa_{rx}x_{j,t} + \kappa_{rx\psi}x_{j,t}\widehat{\psi}_t + \kappa_{r\psi}\widehat{\psi}_t + \kappa_{r\psi\psi}\widehat{\psi}^2_t
\end{aligned}$$

and the following policy rules for consumption $\widehat{c}_t$, investment $\widehat{i}_t$ and capital stock tomorrow $\widehat{k}_{t+1}$ at stage 2 of period t[23]:

$$\begin{aligned}
\widehat{c}_t &= \kappa_{ck}\widehat{k}_t + \kappa_{ca}a_{t-1} + \kappa_{cz}z_t + \kappa_{c\zeta}\zeta_t + \kappa_{c\zeta\psi}\zeta_t\widehat{\psi}_t + \kappa_{cz\psi}z_t\widehat{\psi}_t + \kappa_{c\psi}\widehat{\psi}_t + \kappa_{c\psi\psi}\widehat{\psi}^2_t \\
\widehat{i}_t &= \kappa_{ik}\widehat{k}_t + \kappa_{ia}a_{t-1} + \kappa_{iz}z_t + \kappa_{i\zeta}\zeta_t + \kappa_{i\zeta\psi}\zeta_t\widehat{\psi}_t + \kappa_{iz\psi}z_t\widehat{\psi}_t + \kappa_{i\psi}\widehat{\psi}_t + \kappa_{i\psi\psi}\widehat{\psi}^2_t \\
\widehat{k}_{t+1} &= \kappa_{kk}\widehat{k}_t + \kappa_{ka}a_{t-1} + \kappa_{kz}z_t + \kappa_{k\zeta}\zeta_t + \kappa_{k\zeta\psi}\zeta_t\widehat{\psi}_t + \kappa_{kz\psi}z_t\widehat{\psi}_t + \kappa_{k\psi}\widehat{\psi}_t + \kappa_{k\psi\psi}\widehat{\psi}^2_t
\end{aligned}$$

[23] Here we impose the equilibrium condition: $i_{j,t} = i_t$ and $k_{j,t} = k_t$ directly. The household prefers a balanced investment profile because idiosyncratic productivity shocks are i.i.d.

A couple of implicit assumptions are made here. First of all, the log-deviations of the variables of interest are assumed to be from the Amb.-SS instead of the D-SS. This is because the amount of ambiguity at Amb.-SS $\overline{\psi}$ has a non-negligible first-moment impact in equilibrium when decision makers are ambiguity averse. Second, stage 1 variables are measurable with respect to stage 1 information sets. Since idiosyncratic productivity shocks are i.i.d across time and across islands, past information can be effectively summarized into $\left\{\widehat{k}_t, a_{t-1}, \widehat{\psi}_t\right\}$. Therefore, stage 1 variables, i.e., $\{\widehat{y}_{j,t}, \widehat{n}_{j,t}, \widehat{w}_{j,t}, \widehat{r}_{j,t}\}$, are functions of $\left\{\widehat{k}_t, a_{t-1}, \widehat{\psi}_t, x_{j,t},\right\}$ only. Similar arguments apply to stage 2 variables, i.e., $\left\{\widehat{c}_t, \widehat{i}_t, \widehat{k}_{t+1}\right\}$. Third, fixing the amount of ambiguity $\widehat{\psi}_t$, the policy rules are linear in productivity shocks, either the aggregate or the idiosyncratic one. This corresponds to the standard log-linearization when there is no ambiguity shock at all, i.e. $\psi_t = \overline{\psi}$ for $\forall t$. When there is an ambiguity shock, we allow it to interact with productivity shocks $x_{j,t}$, z_t and ζ_t and approximate the interaction using a linear function.[24] The interaction here reflects the fact that the ambiguity shock is capable of a generating time-varying response to productivity shocks. In addition, if decision makers are ambiguity averse, ambiguity shocks have the first moment impact manifested by the possibly non-linear functions. To capture the non-linearity, we run a quadratic approximation, which reflects fluctuations in the degree of pessimism about the short-run outlooks of the economy.

To approximate the conditional log-normal equilibrium with the proposed policy rules, we first implement a quadratic approximation of the value function

$$
\begin{aligned}
J_t = J^* + \overline{J} &+ \kappa_{Jk}\widehat{k}_t + \kappa_{Ja}a_{t-1} + \kappa_{Jz}z_t + \kappa_{J\zeta}\zeta_t + \kappa_{J\psi}\widehat{\psi}_t \\
&+ \kappa_{Jka}\widehat{k}_t a_{t-1} + \kappa_{Jkz}\widehat{k}_t z_t + \kappa_{Jk\zeta}\widehat{k}_t\zeta_t + \kappa_{Jaz}a_{t-1}z_t + \kappa_{Ja\zeta}a_{t-1}\zeta_t + \kappa_{Jz\zeta}z_t\zeta_t \\
&+ \kappa_{Jk\psi}\widehat{k}_t\widehat{\psi}_t + \kappa_{Ja\psi}a_{t-1}\widehat{\psi}_t + \kappa_{Jz\psi}z_t\widehat{\psi}_t + \kappa_{J\zeta\psi}\zeta_t\widehat{\psi}_t \\
&+ \kappa_{Jkk}\widehat{k}_t^2 + \kappa_{Jaa}a_{t-1}^2 + \kappa_{Jzz}z_t^2 + \kappa_{J\zeta\zeta}\zeta_t^2 + \kappa_{J\psi\psi}\widehat{\psi}_t^2
\end{aligned}
$$

With the value function recursion (25), we can quadratically approximate the expected value functions by

$$
\begin{aligned}
\mathbb{E}^{\omega_t}_{t,0}\left[J_t\right] &= constant_{t-1} + \left(\kappa_{Jz} + \kappa_{Jkz}\widehat{k}_t + \kappa_{Jaz}a_{t-1} + \kappa_{Jz\psi}\widehat{\psi}_t\right)\omega_t + \kappa_{Jzz}\omega_t^2 \\
\mathbb{E}^{\omega_{t+1}}_{t,2}\left[J_{t+1}\right] &= constant_t + \left(\kappa_{Jz} + \kappa_{Jkz}\widehat{k}_{t+1} + \kappa_{Jaz}a_t + \kappa_{Jz\psi}\rho_\psi\widehat{\psi}_t\right)\omega_{t+1} + \kappa_{Jzz}\omega_{t+1}^2
\end{aligned}
$$

where $\{\kappa_{Jz}, \kappa_{Jkz}, \kappa_{Jaz}, \kappa_{Jzz}\}$ are functions of undetermined coefficients in the above proposed policy rules. The quadratic forms in belief distortions $D_{t,1}(\omega_t)$ and $D_{t,2}(\omega_{t+1})$ imply normality in the posterior belief about possible models both at stage 1 and stage 2.

Then, we log-linearize the optimality conditions[25] around the Amb.-SS. While doing this, we take into account that at stage 1 and 2 posterior beliefs about cross-sectional means of idiosyncratic productivity shocks are distorted in a way which is consistent with the estimated belief distortions $D_{t,1}(\omega_t)$ and $D_{t,2}(\omega_{t+1})$[26]. Plugging the proposed policy rules into the log-linearized optimality conditions, we arrive at a large system of undetermined coefficients. Such a huge system of undetermined coefficients can be quantitatively solved with the restriction that $\kappa_{kk} < 1$ to ensure TVC is not violated.[27].

[24] A linear approximation is justifiable since including higher order interaction terms would only have negligible impacts both at the aggregate level and for measuring cross-sectional dispersions

[25] The details of derivations can be found in Appendix

[26] Here we share the same spirit with Ilut and Schneider [2014] and Ilut and Saijo [2018] in dealing with log-linearization with distorted subjective beliefs.

[27] Details for the quantitative analysis can be found in Appendix D. In Appendix B.2, we check the average normalized Euler equation errors, the magnitude of which is of 10^{-4}, indicating that the quantitative methodology works reasonably well.

6.3. Calibration

Table 1 summarizes the parameters used in our baseline model. To stay close to the existing DSGE literature, we choose the discount factor β to be 0.99; the Frisch elasticity of labor supply to be 2; the capital share in production to be 0.36, and the depreciation rate of capital to be 0.015. θ is chosen to be 1 corresponding to the Cobb-Douglas aggregation technology for island commodities, which implies $y_t = \int_J y_{j,t} dj$. Moreover, χ is chosen to be 4.47 to ensure that 1/3 of the time is devoted to working in the deterministic steady state. The persistence of the aggregate productivity shock ρ is chosen to be 0.95, which is a conventional value in the literature. Following Angeletos et al. [2018], the persistence of ambiguity shock is set to $\rho_\psi = 0.75$, which indicates a 2.5 quarter half-life of ambiguity shocks. The standard deviation of idiosyncratic productivity shock is set to $\sigma_\iota = 0.15$, taken from Straub and Ulbricht [2018], which is broadly consistent with empirical estimates using plant-level data. And the degree of ambiguity aversion is set to $\lambda = 11.3$ according to Collard et al. [2018].

It remains to specify the standard deviations of the aggregate productivity shock σ_ζ and the ambiguity shock σ_τ, as well as the amount of ambiguity at Amb.-SS $\overline{\psi}$. We estimate these three parameters to let the model perfectly match standard deviations of output $\sigma(y)$, hours $\sigma(n)$ and labor wedge $\sigma(\Delta_n)$[28]. We arrive at the estimation such that $\sigma_\zeta = 0.00595$, $\sigma_\tau = 0.5826$ and finally $\overline{\psi} = -6.1423$.

6.4. Business-Cycle Moments and Aggregate Co-movements

Business-cycle moments. Table 2 summarizes the key moments of aggregate variables in the US data throughout 1971Q1-2014Q4 (column 1) and in our calibrated baseline model (column 2). The overall empirical fit of the calibrated baseline model is overall good. The baseline model generates enough volatilities for investment and the labor wedge. Besides, the baseline model succeeds in generating a close to zero correlation between output and labor productivity, significant negative correlations between hours and the labor productivity, and finally, significant negative correlations between output and labor wedge. At the core of such an overall fit is the balancing roles between the two aggregate shocks, i.e., the aggregate productivity shock ζ_t and the ambiguity shock ψ_t. Columns 3 and 4 report the business cycle moments for aggregate variables when there are only aggregate productivity shocks by setting $\sigma_\tau = 0$ or ambiguity shocks by setting $\sigma_\zeta = 0$, respectively. When there are only aggregate productivity shocks, the model fails to generate enough volatilities in hours and labor wedge[29] and predicts counterfactually high positive correlations between output (or hours) and labor productivity. In contrast, when there are only ambiguity shocks, the model generates too much volatilities in hours and labor wedge and predicts almost perfectly negative correlations between output (or hours) and labor productivity. In order to capture volatilities of hours and labor wedge, the estimation process naturally runs a balance between the two shocks. Such a balance is the fundamental reason why the baseline model can generate a near-zero correlation between output and labor productivity.

Aggregate co-movements. Impulse response functions of key aggregate variables to an adverse ambiguity shock are reported in Figure 5. The aggregate co-movement patterns are akin to those of the confidence shocks in Angeletos et al. [2018], Huo and Takayama [2015] and Ilut and Saijo [2018], where an adverse ambiguity shock generates drops of aggregate quantities, i.e., output, consumption, hours and investment, while, at the same time, increases in the labor productivity and the labor wedge, in a way consistent with the interpretation of the aggregate demand shock.

[28] The labor wedge is defined in line with Chari et al. [2007]. The key idea is to interpret aggregate data on quantities and prices as wedges in optimality conditions of a text-book RBC model. See Appendix C for detailed derivations.

[29] The labor wedge can arise within the RBC framework if there is information friction, but in a pro-cyclical way.

What drives the co-movements pattern behind the above IRFs is the fluctuations of in the degree of the pessimism for the short-run outlooks of the economy. By construction, an adverse ambiguity shock deepens the degree of the pessimism of all agents about the cross-sectional mean of idiosyncratic productivity shocks for all periods onwards. From the perspective of the firms, such an increased pessimism means a depressed expectation about aggregate demand. In response, firms reduce their demand for labor and capital, generating downward pressures on factor prices. From the household perspective, it further implies a drop in expected permanent income. In response, consumption drops. At the same time, the household also understands that the downward pressures on factor prices only last in the short-run, which restricts the strength of the wealth effect. Therefore, hours and investment decrease in equilibrium, since the relevant substitution effect dominates the opposing wealth effect. In sum, ambiguity shocks generate aggregate co-movements patterns depicted in Figure 5.

6.5. Cross-sectional Dispersions in Beliefs

Professional forecasters in the model are assumed to be ambiguity neutral $\lambda_{SPF} = 0$, which is consistent with the fact that output forecasts do not exhibit systematic pessimistic bias in SPF dataset. Moreover, to capture the fact that the professional forecasters are better informed than private agents inside the economy, we assume that the professional forecasters in our model have one piece of additional private information about the average productivity $\int_J x_{j,t} dj$:

$$s_{j,t} = \int_J x_{j,t} dj + \xi_{j,t} \quad \text{with} \quad \xi_{j,t} \sim N\left(0, \sigma_\xi^2\right)$$

We calibrate the standard deviation of this piece of additional private information σ_ξ^2 to match the standard deviation of the cross-sectional dispersions of output forecasts in SPF data over the period of 1987Q1-2014Q4 resulting in $100\sigma_\xi = 0.52$.

Table 3 reports the standard deviation of the interdecile range of output forecasts and its correlation with output y for the US data throughout 1987Q1-2014Q4 and for our baseline model. Our baseline model captures the cyclical pattern of cross-sectional dispersions in output forecasts. Therefore, we can conclude that ambiguity shock can capture cyclical behaviors of belief dispersion pretty well.

6.6. Estimated confidence v.s. Sentiment Index

To further validate our theory, we address the following question: can our baseline model replicate the cyclical movements in confidence as proxied by the Sentiment Index in the Michigan Survey of Consumer?

To get an answer, we construct model-implied time-series for confidence and compare them with their data counterparts. To construct such a time-series, we first select the sequence of ambiguity and aggregate productivity shocks that can perfectly back-out output and the cross-sectional dispersion of output forecasts in the SPF dataset. With the help of these two sequences of shocks, we can construct the time-series of model-implied confidence according to Definition 3. Figure 6 reports the model-implied time-series for confidence, hours, investment, belief dispersion, and output together with their data counterparts between 1987Q1-2014Q4. Notably, the model-implied confidence process closely tracks the Sentiment Index in the Michigan Survey of Consumers. Recall that we do not use any information on the Sentiment Index in the construction of the time-series for ambiguity and aggregate productivity shocks. Such an empirical fit provides an additional validation to our theory. Moreover, note that simulated time-series of hours and investment also closely track their data counterparts.

7. Conclusion

We develop a theory of uncertainty-driven business cycles that contributes to accommodate the notion of non-inflationary aggregate demand shocks out of variations in uncertainty. The theory also contributes to explain the observed co-movements across confidence, belief dispersion, and the aggregate economy.

Within a simple RBC model without capital, we demonstrate that an adverse ambiguity shock makes ambiguity averse agents behave as if they believe that the aggregate fundamental is turning bad and becoming more volatile. Such dual impacts of ambiguity shocks generate endogenous movements across confidence and uncertainty. When the economy features imperfect coordination due to incomplete information, the dual impacts of ambiguity shocks translate into depressed beliefs about aggregate demand and the increased incentives in the use of private information both when making output decisions and output forecasts - the former map into depressed confidence and the latter map into a heightened belief dispersion. Moreover, aggregate output falls due to the increase in the economy-wide pessimism about aggregate demand. In this sense, ambiguity shocks in our paper are nothing more than a particular formulation of the non-inflationary aggregate demand shock.

Finally, we explore the quantitative potential of our theory within a dynamic RBC model. The ambiguity shock is shown to be capable of generating co-movements across real quantities together with counter-cyclical labor productivity and the labor wedge. Our model is also capable of capturing cyclicalities in the cross-sectional dispersion of the output forecast in the SPF dataset. Moreover, the model-implied time series of confidence closely track the Sentiment Index in the Michigan Survey of Consumer.

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Table 1. *Model Parameters*

Parameters	Value	Role
β	0.99	discount factor
ϵ	0.5	Frisch elasticity=2
α	0.36	capital share
δ	0.015	depreciation rate
θ	1	Cobb-Douglas aggregation
χ	4.24	1/3 hours in deterministic steady state
ρ	0.95	persistence of agg. productivity shock
ρ_ψ	0.75	persistence of ambiguity shock
σ_ι	0.15	std. dev. of idio. productivity shock
λ	11.3	degree of ambiguity aversion
$100\sigma_\zeta$	0.595	std. dev. of agg. productivity shock
σ_τ	0.5826	std. dev. of ambiguity shock
$\overline{\psi}$	-6.1423	amount of ambiguity $e^{\overline{\psi}}$ at Amb.-SS

Table 2. Bandpass-Filtered Moments of Aggregate Variables

	Data(1971Q1-2014Q4)	Baseline Model	A Only	ψ Only
Standard Deviations				
$\sigma(y)$	1.45	1.45	-	-
$\sigma(c)/\sigma(y)$	0.60	0.24	0.26	0.21
$\sigma(n)/\sigma(y)$	1.21	1.21	0.48	1.57
$\sigma(i)/\sigma(y)$	3.76	4.18	4.02	4.29
$\sigma(y/n)/\sigma(y)$	0.58	0.56	0.54	0.57
$\sigma(\Delta_n)/\sigma(y)$	1.50	1.50	0.19	2.00
Correlations				
$\rho(c,y)$	0.88	0.92	0.95	0.90
$\rho(n,y)$	0.88	0.87	0.97	0.99
$\rho(i,y)$	0.95	0.99	0.99	0.99
$\rho(\Delta_n,y)$	-0.75	-0.67	0.18	-0.93
$\rho(c,n)$	0.85	0.74	0.93	0.86
$\rho(c,i)$	0.80	0.88	0.93	0.85
$\rho(i,n)$	0.85	0.90	0.97	0.99
Correlations with Productivity				
$\rho(y,y/n)$	-0.11	-0.14	0.99	-0.99
$\rho(n,y/n)$	-0.57	-0.53	0.97	-0.99

Note: The first column reports moments for US data from 1971Q1 to 2014Q4. The second column reports moments in our baseline model. Column 3 and 4 report moments generated by some models when there are only aggregate productivity shocks or ambiguity shocks, respectively. All moments are band-pass filtered at frequencies of 6-32 quarters. The data is from the Saint–Louis Federal Reserve Economic Database.

Table 3. *Bandpass-Filtered Moments of Belief Divergence*

	Data$_{(1987Q1-2014Q4)}$	Baseline Model
Stddev	0.09	0.09
Correlation with y	-0.42	-0.71

Note: The first column reports moments of belief divergence for US data from 1987Q1 to 2014Q4. The second column reports corresponding moments in our baseline model. All moments are band-pass filtered at frequencies of 6-32 quarters. US aggregate data is from the Saint-Louis Federal Reserve Economic Database and real GDP forecast data is from the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia.

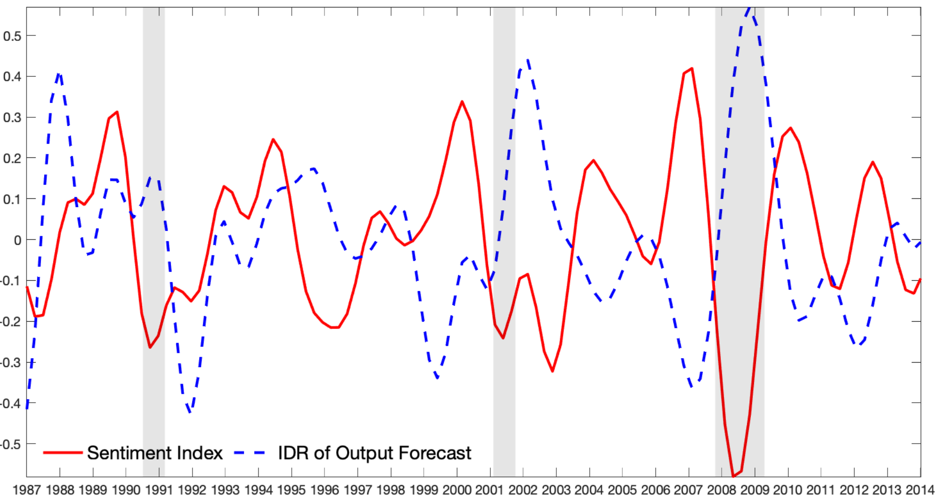


Figure 1. *Confidence and Uncertainty.*

Note: The figure plots consumer sentiment index (red line) and cross-sectional dispersion of real GDP forecasts by professional forecasters (dashed blue line) over the period 1987Q1-2014Q4. Correlation is around -0.41 over the entire period. Both of the time series are bandpass-filtered at frequencies of 6-32 quarters and re-scaled. Consumer sentiment index is from the Michigan Survey of Consumers, and real GDP forecast data is from the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia.

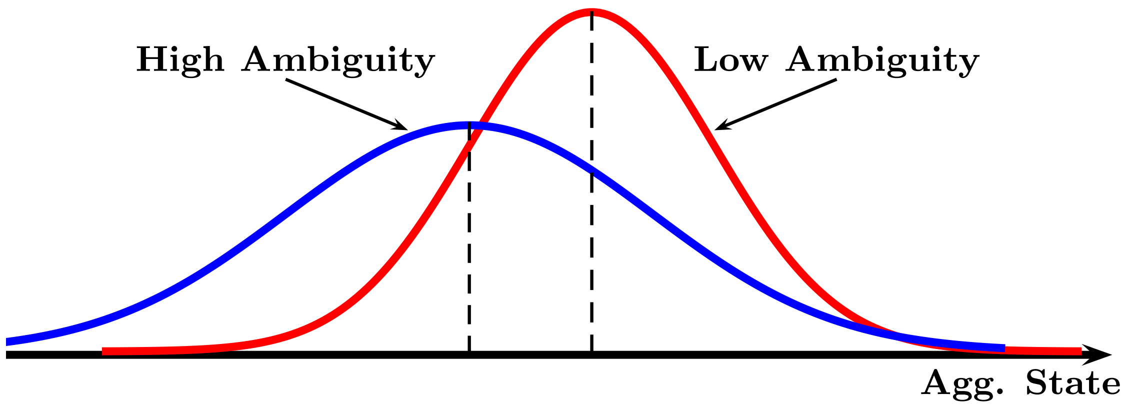


Figure 2. *Dual Impacts of Ambiguity Shock: Main Mechanism*

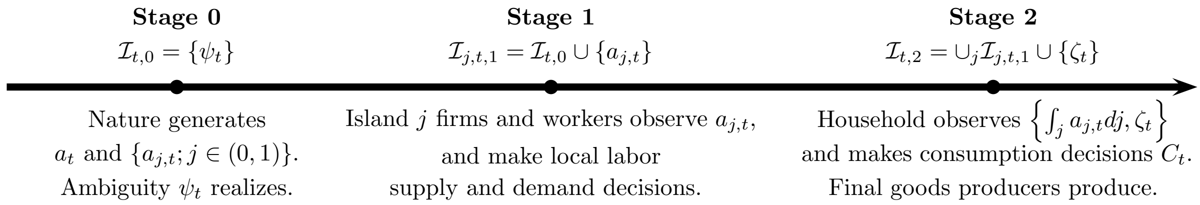


Figure 3. *Timeline for Period t of the Static Model without Capital*

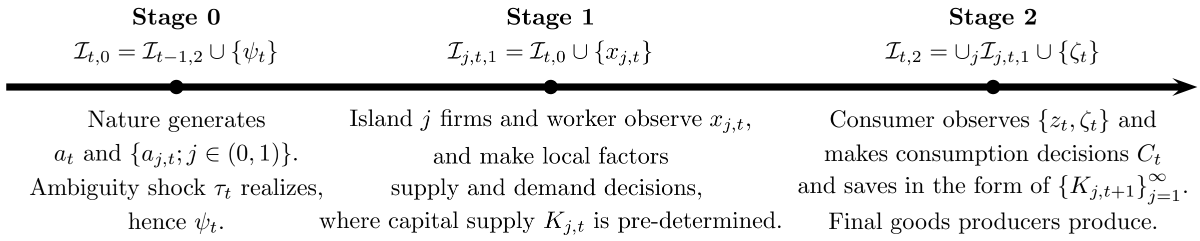


Figure 4. *Timeline for Period t of the Dynamic RBC Model*

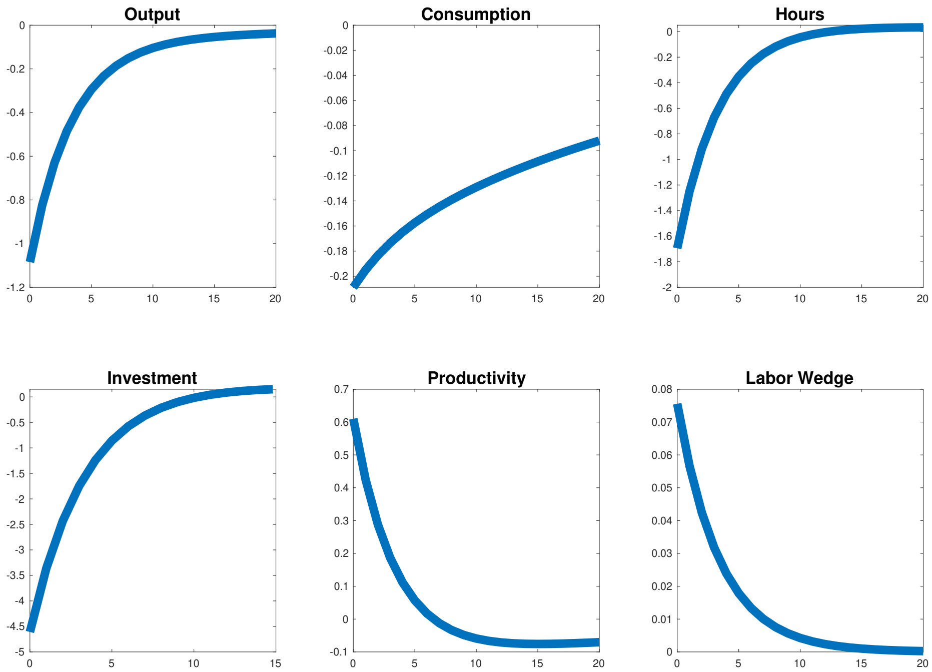


Figure 5. *Impulse responses to one standard deviation of positive ambiguity shock*

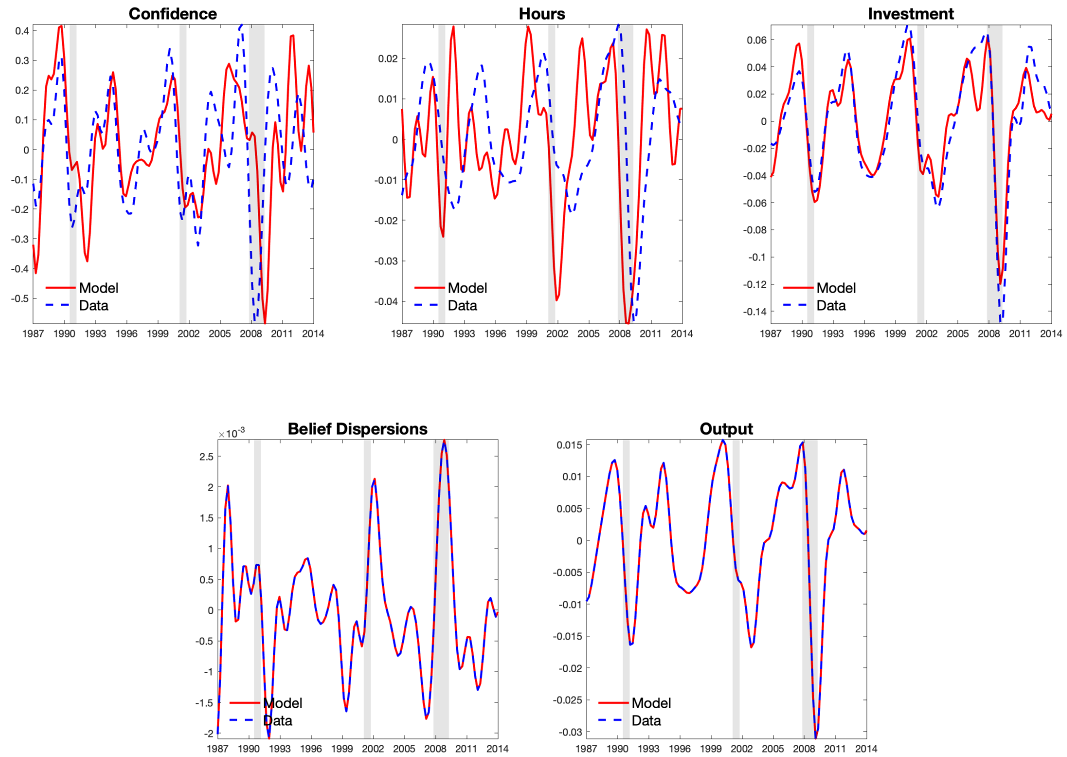


Figure 6. *Estimated Times-Series vs Empirical Proxies*

Note: The figure plots time-series for confidence and hours estimated from our model and their data counterparts between 1987Q1-2014Q4. In the data, confidence is proxied by the Consumer Sentiment Index from the Michigan Survey of Consumer. All moments are band-pass filtered at frequencies of 6-32 quarters. US aggregate data is from the Saint-Louis Federal Reserve Economic Database and real GDP forecast data is from Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia.

Appendix

A. Derivations and Proofs

Derivation of Equation (13). FOC for the island j workers' problem is such that

$$\int_{\mathbb{R}} \phi' \left(\mathbb{E}_{j,t,1}^{\omega_t} \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon} dj \right] \right) \mathbb{E}_{j,t,1}^{\omega_t} \left[C_t^{-\gamma} W_{j,t} - \chi N_{j,t}^{\epsilon} \right] \widetilde{f}_{j,t,1}^{h} (\omega_t)\, d\omega_t = 0$$

Plugging in the expression for $\widetilde{f}_{j,t,1}^{h}(\omega_t)$ given by (6), we arrive at (10) where the distorted posterior belief about possible models can be given by (12). Similar procedures lead to (11). Then in the last step, combining (10) and (11) leads to (13). ∎

Proof of Lemma 1. Under conditional log-normal equilibrium, we have that

$$\begin{aligned} y_{j,t} &= y^* + \overline{h}_y + \kappa_{yx,t} x_{j,t} + \widehat{h}_y (\psi_t) \\ n_{j,t} &= n^* + \overline{h}_n + \kappa_{nx,t} x_{j,t} + \widehat{h}_n (\psi_t) \\ y_t &= \int_{\mathbb{J}} y_{j,t} dj + \frac{1}{2}\left(1 - \frac{1}{\theta}\right) d_{y,j}^2 \end{aligned}$$

where $d_{y,j} \equiv \kappa_{yx,t}^2 \sigma_t^2$ denotes the cross-sectional dispersion in island outputs. We ignore $d_{y,j}$ in the approximation without loss of generality since they are of second order impacts at the aggregate level and have no impact at all on the cyclical behavioursof various dispersion measures.

Define $\overline{S} \equiv e^{s^* + \overline{h}_s}$ for any variable of interest S. It denotes the level of variable S at the ambiguous steady state. A quadratic approximation of utility of the representative household around the ambiguous steady state is then given by

$$\begin{aligned} &\frac{Y_t^{1-\gamma} - 1}{1-\gamma} - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon} dj \\ \approx &\frac{\overline{Y}^{1-\gamma}}{1-\gamma} \left[1 + (1-\gamma)\widehat{y}_t + \frac{1}{2}(1-\gamma)^2 \widehat{y}_t^2 \right] - \frac{1}{1-\gamma} - \chi \frac{\overline{N}^{1+\epsilon}}{1+\epsilon} \int_{\mathbb{J}} \left(1 + (1+\epsilon)\widehat{n}_{j,t} + \frac{1}{2}(1+\epsilon)^2 \widehat{n}_{j,t}^2 \right) dj \\ = &Const. + \overline{Y}^{1-\gamma} \widehat{y}_t + \frac{1}{2}(1-\gamma)\overline{Y}^{1-\gamma} \widehat{y}_t - \int_{\mathbb{J}} \left(\chi \overline{N}^{1+\epsilon} \widehat{n}_{j,t} + \frac{1}{2}(1+\epsilon)\chi \overline{N}^{1+\epsilon} \widehat{n}_{j,t}^2 \right) dj \end{aligned}$$

Further define stage 0 ex-ante expected utility under a particular model ω_t as $\overline{J}_t(\omega_t)$

$$\overline{J}_t(\omega_t) \equiv \mathbb{E}_{t,0}^{\omega_t} \left[\frac{Y_t^{1-\gamma} - 1}{1-\gamma} - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon} dj \right]$$

It turns out that $\overline{J}_t(\omega_t)$ can be quadratically approximated by

$$\overline{J}_t(\omega_t) \approx Const_t + \kappa_{J\omega}(\psi_t, \lambda)\,\omega_t + \frac{1}{2}\kappa_{J\omega\omega}(\psi_t, \lambda)\,\omega_t^2$$

where the coefficients of linear and quadratic terms are given by

$$\begin{aligned} \kappa_{J\omega}(\psi_t, \lambda) &= (Y^*)^{1-\gamma} \left(1 + (1-\gamma)\overline{h}_y \right) \kappa_{ya_j}(\psi_t, \lambda) - \chi (N^*)^{1+\epsilon} \left(1 + (1+\epsilon)\overline{h}_n \right) \kappa_{na_j}(\psi_t, \lambda) \\ \kappa_{J\omega\omega}(\psi_t, \lambda) &= (Y^*)^{1-\gamma} \left(1 + (1-\gamma)\overline{h}_y \right) (1-\gamma)\kappa_{ya_j t}^2(\psi_t, \lambda) - \chi (N^*)^{1+\epsilon} \left(1 + (1+\epsilon)\overline{h}_n \right) (1+\epsilon)\kappa_{na_j}^2(\psi_t, \lambda) \end{aligned}$$

To derive such an approximation, we first approximate $e^{(1-\gamma)\overline{h}_y}$ and $e^{(1+\epsilon)\overline{h}_n}$ by $1 + (1-\gamma)\,\overline{h}_n$ and $1 + (1+\epsilon)\,\overline{h}_n$, respectively. This is justifiable since we want to restrict the Amb.-SS impact of ambiguity to avoid too much statistical sophistication. Then, we ignore the intersection terms between ω_t and $x_{j,t}$ or between ω_t and $\widehat{h}_s(\psi_t)$ with $s \in \{y, n\}$ since those terms are negligible as compared to $\kappa_{J\omega}\omega_t$ at the aggregate level and, at the same time, have no impact on cross-sectional dispersion measures.

Finally, a quadratic approximation of $\overline{J}_t(\omega_t)$ implies that the belief distortion in (15) is of exponential quadratic form. Therefore, the posterior belief about possible models $\widetilde{f}_{j,t,1}(\omega_t)$ is normal. It leads to a normal density with the mean μ_t and the variance σ_t^2 given by

$$\mu_t = \left(\frac{e^{\psi_t} + g_\sigma(\psi_t, \lambda)}{\sigma_\zeta^2 + \sigma_\iota^2 + e^{\psi_t} + g_\sigma(\psi_t, \lambda)}\right) x_{j,t} + \left(\frac{\sigma_\zeta^2 + \sigma_\iota^2}{\sigma_\zeta^2 + \sigma_\iota^2 + e^{\psi_t} + g_\sigma(\psi_t, \lambda)}\right) g_\mu(\psi_t, \lambda) \tag{29}$$

and

$$\sigma_t^2 = \left(\frac{\sigma_\zeta^2 + \sigma_\iota^2}{\sigma_\zeta^2 + \sigma_\iota^2 + e^{\psi_t} + g_\sigma(\psi_t, \lambda)}\right)\left(e^{\psi_t} + g_\sigma(\psi_t, \lambda)\right) \tag{30}$$

where the distortions in mean $g_\mu(\psi_t, \lambda)$ and in variance $g_\sigma(\psi_t, \lambda)$ are given by

$$\begin{aligned} g_\mu(\psi_t, \lambda) &= -\,\lambda\kappa_{J\omega}(\psi_t, \lambda)\left(\frac{e^{\psi_t}}{1 + \lambda\kappa_{J\omega\omega}(\psi_t, \lambda)\, e^{\psi_t}}\right) \\ g_\sigma(\psi_t, \lambda) &= -\left(\frac{\lambda\kappa_{J\omega\omega}(\psi_t, \lambda)\, e^{\psi_t}}{1 + \lambda\kappa_{J\omega\omega}(\psi_t, \lambda)\, e^{\psi_t}}\right) e^{\psi_t} \end{aligned}$$

■

Proof of Proposition 1. Suppose that the conditional log-normal equilibrium is such that:

$$\begin{aligned} y_{j,t} \equiv \ln Y_{j,t} &= y^* + \overline{h}_y(\overline{\psi}) + \kappa_{ya_j}(\psi_t, \lambda) \cdot a_{j,t} + \widehat{h}_y(\psi_t, \lambda) \\ n_{j,t} \equiv \ln N_{j,t} &= n^* + \overline{h}_n(\overline{\psi}) + \kappa_{na_j}(\psi_t, \lambda) \cdot a_{j,t} + \widehat{h}_n(\psi_t, \lambda) \\ y_t \equiv \ln Y_t &= y^* + \overline{h}_y(\overline{\psi}) + \kappa_{ya_j}(\psi_t, \lambda) \cdot a_t + \widehat{h}_y(\psi_t, \lambda) \end{aligned}$$

where we ignore the dispersion adjustment of aggregate output in the approximation without loss of generality since they are of second-order impacts at the aggregate and have no impacts at all on the various dispersion measures. Then at D-SS, we have the following

$$\ln(\chi) + (1+\epsilon)\, n^* = \ln(1-\alpha) + (1-\gamma)\, y^*$$

While, at Amb.-SS, impacts of ambiguity shocks at Amb.-SS denoted by $\overline{h}_s$ $s \in \{n, y\}$ must satisfy the following

$$\overline{h}_n = (1-\gamma)\,\overline{h}_y + \left(\frac{1}{\theta} - \gamma\right) H_y(\overline{\psi}, \lambda) \tag{31}$$

Here $H_y(\overline{\psi}, \lambda)$ denotes the degree of pessimism of island j agents about aggregate output y_t at the

Amb.-SS. Under the proposed conditional log-normal equilibrium, it is given by

$$H_y\left(\overline{\psi},\lambda\right)=\kappa_{ya_j}\left(\overline{\psi},\lambda\right)\left(\int_{\mathbb{R}}\mathbb{E}^{\omega_t}_{j,t,1}\left[a_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t\right)\Big|_{a_{j,t}=0,\psi_t=\overline{\psi}}$$

To understand why this is the case, recall that the ambiguous steady state refers to the state into which the economy converges (a) in the absence of any shocks, i.e., $a_{j,t}=0$, but (b) taking into account the existence of ambiguity, i.e., evaluating $\int_{\mathbb{R}}\mathbb{E}^{\omega_t}_{j,t,1}\left[a_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t$ at $\psi_t=\overline{\psi}\neq-\infty$. Alternatively, we can interpret $H_y\left(\overline{\psi}\right)$ from the perspective of distorted subjective beliefs of all agents. At Amb.-SS, the amount of ambiguity $\overline{\psi}$ plays a non-trivial role in the sense that the subjective belief about aggregate productivity is distorted in the mean. Such a mean distortion must be respected when we evaluate the Amb.-SS, leading to a non-zero term $H_y\left(\overline{\psi}\right)$. Similar arguments can be found in Ilut and Schneider [2014] and Ilut and Saijo [2018] in the context of "worst case" belief due to multiple prior preferences. Following (29), we have that

$$H_y\left(\overline{\psi},\lambda\right)=\kappa_{ya_j}\left(\overline{\psi},\lambda\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2+e^{\overline{\psi}}+g_\sigma\left(\overline{\psi},\lambda\right)}\right)g_\mu\left(\overline{\psi},\lambda\right)\tag{32}$$

where $\kappa_{ya_j}\left(\overline{\psi},\lambda\right)$ denotes the use of private information at the Amb.-SS.

In the next step, we log-linearize (14) around the Amb.-SS:

$$\left(\frac{1+\epsilon}{1-\alpha}-1+\frac{1}{\theta}\right)\widehat{y}_{j,t}=\left(\frac{1+\epsilon}{1-\alpha}\right)a_{j,t}+\left(\frac{1}{\theta}-\gamma\right)\left(\int_{\mathbb{R}}\mathbb{E}^{\omega_t}_{j,t,1}\left[\widehat{y}_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t-H_y\left(\overline{\psi},\lambda\right)\right)\tag{33}$$

Therefore, matching coefficients lead to the following two equilibrium conditions

$$\left[\left(\frac{1+\epsilon}{1-\alpha}\right)-(1-\gamma)\right]\kappa_{ya_j}\left(\psi_t,\lambda\right)=\left(\frac{1+\epsilon}{1-\alpha}\right)-\left(\frac{1}{\theta}-\gamma\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2+e^{\psi_t}+g_\sigma\left(\psi_t,\lambda\right)}\right)\kappa_{ya_j}\left(\psi_t,\lambda\right)\tag{34}$$

and

$$\left[\left(\frac{1+\epsilon}{1-\alpha}\right)-(1-\gamma)\right]\widehat{h}_y\left(\psi_t,\lambda\right)=\left(\frac{1}{\theta}-\gamma\right)H_y\left(\psi_t,\lambda\right)-\left(\frac{1}{\theta}-\gamma\right)H_y\left(\overline{\psi},\lambda\right)\tag{35}$$

by using the fact that under the proposed policy rules, we have that

$$\int_{\mathbb{R}}\mathbb{E}^{\omega_t}_{j,t,1}\left[\widehat{y}_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t=\left[1-\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2+e^{\psi_t}+g_\sigma\left(\psi_t,\lambda\right)}\right)\right]\kappa_{ya_j}\left(\psi_t,\lambda\right)a_{j,t}+H_y\left(\widehat{\psi}_t,\lambda\right)-H_y\left(\overline{\psi},\lambda\right)$$

with

$$H_y\left(\psi_t,\lambda\right)=\kappa_{ya_j}\left(\psi_t,\lambda\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2+e^{\psi_t}+g_\sigma\left(\psi_t,\lambda\right)}\right)g_\mu\left(\psi_t,\lambda\right)\tag{36}$$

In what follows, we first present an auxiliary lemma that is intensively used later in the proofs. It helps us prove that there exists a unique Amb.-SS. Upon establishing the uniqueness of Amb.-SS, we then move on to prove that for any given amount of ambiguity ψ_t, there exists a unique $\kappa_{ya_j}\left(\psi_t,\lambda\right)$ manifesting the use of private information. And, finally, the existence and uniqueness of $\widehat{h}_y$ would be straightforward given all the results we have established, which completes the whole proof.

Lemma 4. For any realized ψ_t, both $\kappa_{J\omega}(\psi_t, \lambda)$ and $\kappa_{J\omega\omega}(\psi_t, \lambda)$ are positive in equilibrium.

Proof. First of all, at the D-SS, it is straightforward to show that $\chi (N^*)^{1+\epsilon} = (1-\alpha)(Y^*)^{1-\gamma}$. Then, at the Amb.-SS, it has to be the case that

$$\frac{(Y^*)^{1-\gamma}\left(1+(1-\gamma)\overline{h}_y\right)}{1-\gamma} - \chi \frac{(N^*)^{1+\epsilon}\left(1+(1+\epsilon)\overline{h}_n\right)}{1+\epsilon} > 0$$

Otherwise, it is better-off to be inactive by choosing $\overline{Y} = \overline{C} = \overline{N} = 0$. Therefore, it must be the case that

$$\frac{\left(1+(1-\gamma)\overline{h}_y\right)}{1-\gamma} - \frac{(1-\alpha)\left(1+(1+\epsilon)\overline{h}_n\right)}{1+\epsilon} > 0 \tag{37}$$

Directly following (34), we know that $\kappa_{ya_j} > 0$. This implies that

$$(1+\epsilon)\kappa_{na_j} - (1-\gamma)\kappa_{ya_j} < 0 \tag{38}$$

since we know

$$(1+\epsilon)\kappa_{na_j} - (1-\gamma)\kappa_{ya_j} = -\left(\frac{1}{\theta} - \gamma\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2 + \sigma_\iota^2 + e^{\psi_t} + g_\sigma(\psi_t, \lambda)}\right)\kappa_{ya_j}$$

Furthermore, it can be shown that, by using $\chi (N^*)^{1+\epsilon} = (1-\alpha)(Y^*)^{1-\gamma}$, we would have that

$$\kappa_{J\omega}(\psi_t, \lambda) = (Y^*)^{1-\gamma}\left[\left(1+(1-\gamma)\overline{h}_y\right)\kappa_{ya_j}(\psi_t, \lambda) - \left(1+(1+\epsilon)\overline{h}_n\right)(1-\alpha)\kappa_{na_j}(\psi_t, \lambda)\right]$$

Then it is straightforward to prove $\kappa_{J\omega}(\psi_t, \lambda) > 0$ given (37) and (38). Then, we know that $g_\mu(\psi_t, \lambda) < 0$, hence $H_y\left(\overline{\psi}, \lambda\right) < 0$. Following (31), we will have that

$$(1-\gamma)\overline{h}_y > (1+\epsilon)\overline{h}_n \tag{39}$$

Since we know that

$$\kappa_{J\omega\omega}(\psi_t, \lambda) = (Y^*)^{1-\gamma}\left[\left(1+(1-\gamma)\overline{h}_y\right)(1-\gamma)\kappa_{ya_jt}^2(\psi_t, \lambda) - \left(1+(1+\epsilon)\overline{h}_n\right)(1-\alpha)(1+\epsilon)\kappa_{na_j}^2(\psi_t, \lambda)\right]$$

it is straightforward to have $\kappa_{J\omega\omega}(\psi_t, \lambda) > 0$ given (38) and (39). ■

In what follows, we prove that there exists a **unique Amb.-SS.**

Define $S = (Y^*)^{1-\gamma}$, $X = \left[\frac{1+\epsilon}{1-\alpha} - (1-\gamma)\right]$, $\Sigma^2 = \frac{e^{\overline{\psi}}}{1+\lambda\overline{\kappa}_{J\omega\omega}}$ and finally $\beta_\iota = \frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2+\Sigma^2}$. Amb.-SS can be characterized by the following four equations:

$$
\begin{aligned}
X\overline{\kappa}_{ya_j} &= \left(\frac{1+\epsilon}{1-\alpha}\right) - \left(\frac{1}{\theta}-\gamma\right)\overline{\kappa}_{ya_j}\beta_\iota && (40)\\
X\overline{h}_y &= -\left(\frac{1}{\theta}-\gamma\right)\overline{\kappa}_{ya_j}\beta_\iota\lambda\overline{\kappa}_{J\omega}\Sigma^2 && (41)\\
\overline{\kappa}_{J\omega} &= S\left[\left(1+(1-\gamma)\overline{h}_y\right)\overline{\kappa}_{ya_j} - \left(1+(1+\epsilon)\overline{h}_n\right)(1-\alpha)\overline{\kappa}_{na_j}\right] && (42)\\
\overline{\kappa}_{J\omega\omega} &= S\left[\left(1+(1-\gamma)\overline{h}_y\right)(1-\gamma)\overline{\kappa}^2_{ya_j} - \left(1+(1+\epsilon)\overline{h}_n\right)(1+\epsilon)(1-\alpha)\overline{\kappa}^2_{na_j}\right] && (43)
\end{aligned}
$$

Focusing on (40) and rearranging leads to the following

$$
\overline{\kappa}_{ya_j} = \frac{\frac{1+\epsilon}{1-\alpha}}{X+\left(\frac{1}{\theta}-\gamma\right)\beta_\iota} \equiv f\left(\overline{\kappa}_{ya_j}, \overline{h}_y\right) \tag{44}
$$

It can be shown that

$$
\frac{\partial f}{\partial \overline{\kappa}_{ya_j}} < 0 \qquad\qquad \frac{\partial f}{\partial \overline{h}_y} < 0
$$

For any given $\overline{h}_y$, the LHS of (40) is increasing in $\overline{\kappa}_{ya_j}$ and the RHS is decreasing in $\overline{\kappa}_{ya_j}$. Together with the boundary conditions, by the intermediate value theorem, we can prove that for any $\overline{h}_y$ there exists a unique $\overline{\kappa}^*_{ya_j}\left(\overline{h}_y\right)$ that satisfies (44). Furthermore, we can show that such a function is decreasing in the sense that

$$
\frac{d\overline{\kappa}_{ya_j}}{d\overline{h}_y} = \frac{\partial f/\partial \overline{h}_y}{1-\partial f/\partial \overline{\kappa}_{ya_j}} < 0
$$

Next focus on (41). The target is to show that given any identified function $\overline{\kappa}^*_{ya_j}\left(\overline{h}_y\right)$, there exists a unique $\overline{h}_y$ that satisfies (41). The existence can easily be proved by boundary conditions. Uniqueness can be ensured under some regularity conditions.

Define LHS and RHS of (41) upon taking $\overline{\kappa}^*_{ya_j}\left(\overline{h}_y\right)$ into account as LHS$\left(\overline{h}_y\right)$ and RHS$\left(\overline{h}_y\right)$. A sufficient condition for uniqueness is that

$$
\frac{d\text{RHS}}{d\overline{h}_y} < \frac{d\text{LHS}}{d\overline{h}_y} = X \quad \text{whenever} \quad \text{LHS} = \text{RHS}
$$

It can be shown that

$$
\frac{d\text{RHS}}{d\overline{h}_y} = -\left(\frac{1}{\theta}-\gamma\right)\lambda\overline{\kappa}_{J\omega}\Sigma^2\frac{d\overline{\kappa}_{ya_j}\beta_\iota}{d\overline{h}_y} + \left[-\left(\frac{1}{\theta}-\gamma\right)\overline{\kappa}_{ya_j}\beta_\iota\right]\frac{d\lambda\overline{\kappa}_{J\omega}\Sigma^2}{d\overline{h}_y}
$$

By using the fact that $\overline{\kappa}^*_{ya_j}\left(\overline{h}_y\right)$ is a solution for (40), we will have that

$$
-\left(\frac{1}{\theta}-\gamma\right)\frac{d\overline{\kappa}_{ya_j}\beta_\iota}{d\overline{h}_y} = X\frac{d\overline{\kappa}_{ya_j}}{d\overline{h}_y}
$$

Further evaluating it at LHS = RHS leads to

$$\left[-\left(\frac{1}{\theta}-\gamma\right)\overline{\kappa}_{ya_j}\beta_\iota\right]=\frac{X\overline{h}_y}{\lambda\overline{\kappa}_{J\omega}\Sigma^2}$$

Therefore, we have that

$$-\frac{1}{X\overline{h}_y}\frac{d\text{RHS}}{d\overline{h}_y}=\frac{X}{\left(\frac{1}{\theta}-\gamma\right)\overline{\kappa}_{ya_j}\beta_\iota}\frac{d\overline{\kappa}_{ya_j}}{d\overline{h}_y}-\frac{1}{\overline{\kappa}_{J\omega}}\frac{\partial\overline{\kappa}_{J\omega}}{\partial\overline{\kappa}_{ya_j}}\frac{d\overline{\kappa}_{ya_j}}{d\overline{h}_y}-\frac{1}{\overline{\kappa}_{J\omega}}\frac{\partial\overline{\kappa}_{J\omega}}{\partial\overline{h}_y}+\lambda\Sigma^2\frac{\partial\overline{\kappa}_{J\omega\omega}}{\partial\overline{h}_y}+\lambda\Sigma^2\frac{\partial\overline{\kappa}_{J\omega\omega}}{\partial\overline{\kappa}_{ya_j}}\frac{d\overline{\kappa}_{ya_j}}{d\overline{h}_y}$$

The followings can be easily proved:

$$\begin{aligned}\frac{\partial\overline{\kappa}_{J\omega}}{\partial\overline{\kappa}_{ya_j}}=&S\left[(1-\gamma)\overline{h}_y-(1+\epsilon)\overline{h}_n\right]=S\left(\frac{1}{\theta}-\gamma\right)\overline{\kappa}_{ya_j}\beta_\iota\lambda\overline{\kappa}_{J\omega}\Sigma^2\\ \frac{\partial\overline{\kappa}_{J\omega}}{\partial\overline{h}_y}=&S\left[(1-\gamma)\overline{\kappa}_{ya_j}-(1+\epsilon)\overline{\kappa}_{na_j}\right]=S\left(\frac{1}{\theta}-\gamma\right)\overline{\kappa}_{ya_j}\beta_\iota\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\overline{\kappa}_{J\omega\omega}}{\partial\overline{\kappa}_{ya_j}}=&2S\left[\left(1+(1-\gamma)\overline{h}_y\right)(1-\gamma)\overline{\kappa}_{ya_j}-\left(1+(1+\epsilon)\overline{h}_n\right)(1+\epsilon)\overline{\kappa}_{na_j}\right]>0\\ \frac{\partial\overline{\kappa}_{J\omega\omega}}{\partial\overline{h}_y}=&S\left[(1-\gamma)^2\overline{\kappa}^2_{ya_j}-(1+\epsilon)^2\overline{\kappa}^2_{na_j}\right]>0\end{aligned}$$

when evaluating at Amb.-SS, i.e., (40) and (41) are satisfied. Some algebra further leads to

$$-\frac{1}{X\overline{h}_y}\frac{d\text{RHS}}{d\overline{h}_y}=A\frac{d\overline{\kappa}_{ya_j}}{d\overline{h}_y}+B$$

where we define the A and B such that

$$A\equiv\frac{X}{\left(\frac{1}{\theta}-\gamma\right)\overline{\kappa}_{ya_j}\beta_\iota}+\lambda\Sigma^2S\left[2(1-\gamma)^2\overline{h}_y\overline{\kappa}_{ya_j}-2(1+\epsilon)\overline{h}_n\overline{\kappa}_{na_j}+(1-\gamma)\overline{\kappa}_{ya_j}-(1+\epsilon)\overline{\kappa}_{na_j}\right]>0$$

and

$$B\equiv-\frac{1}{\overline{\kappa}_{J\omega}}\frac{\partial\overline{\kappa}_{J\omega}}{\partial\overline{h}_y}+\lambda\Sigma^2S\left[(1-\gamma)^2\overline{\kappa}^2_{ya_j}-(1+\epsilon)^2\overline{\kappa}^2_{na_j}\right]<\frac{\lambda e^{\overline{\psi}}S\left[(1-\gamma)^2\overline{\kappa}^2_{ya_j}-(1+\epsilon)^2\overline{\kappa}^2_{na_j}\right]}{1+\lambda e^{\overline{\psi}}\overline{\kappa}_{J\omega\omega}}$$

Therefore, we have that

$$-\overline{h}_yB<\frac{\lambda e^{\overline{\psi}}S\left[-(1-\gamma)^2\overline{\kappa}^2_{ya_j}\overline{h}_y+(1+\epsilon)^2\overline{\kappa}^2_{na_j}(1-\alpha)\overline{h}_n\right]}{1+\lambda e^{\overline{\psi}}\overline{\kappa}_{J\omega\omega}}$$

Finally, we can prove

$$\begin{aligned}&1+\lambda e^{\overline{\psi}}\overline{\kappa}_{J\omega\omega}-\lambda e^{\overline{\psi}}S\left[-(1-\gamma)^2\overline{\kappa}^2_{ya_j}\overline{h}_y+(1+\epsilon)^2\overline{\kappa}^2_{na_j}(1-\alpha)\overline{h}_n\right]\\ =&1+\lambda e^{\overline{\psi}}S\left[(1-\gamma)\overline{\kappa}^2_{ya_j}-(1+\epsilon)(1-\alpha)\overline{\kappa}^2_{na_j}+2(1-\gamma)^2\overline{\kappa}^2_{ya_j}\overline{h}_y-2(1+\epsilon)^2\overline{\kappa}^2_{na_j}(1-\alpha)\overline{h}_n\right]\\ =&1+\lambda e^{\overline{\psi}}S\left\{\left[1+2(1-\gamma)\overline{h}_y\right](1-\gamma)\overline{\kappa}^2_{ya_j}-\left[1+2(1+\epsilon)\overline{h}_n\right](1+\epsilon)(1-\alpha)\overline{\kappa}^2_{na_j}\right\}>0\end{aligned}$$

where the last step can be justified by the fact that $\overline{h}_y$ is relatively small in the sense that $1 + 2(1-\gamma)\overline{h}_y > 0$. Therefore, we arrive at the fact that

$$-\overline{h}_y B < 1$$

which ensures uniqueness since it directly implies that $\frac{d\text{RHS}}{d\overline{h}_y} < X$.

After proving the uniqueness of Amb.-SS, we proceed to prove that the use of private information $\kappa_{ya_j}(\psi_t, \lambda)$ **is unique.** The use of private information κ_{ya_j} is determined by (34). Denote the gap between LHS and RHS of (34) as $f\left(\kappa_{ya_j}\right)$ such that

$$f\left(\kappa_{ya_j}\right) \equiv \left[\left(\frac{1+\epsilon}{1-\alpha}\right) - (1-\gamma)\right]\kappa_{ya_j} - \left(\frac{1+\epsilon}{1-\alpha}\right) + \left(\frac{1}{\theta} - \gamma\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2 + \sigma_\iota^2 + e^{\psi_t} + g_\sigma(\psi_t, \lambda)}\right)\kappa_{ya_j} \quad (45)$$

It can be shown that

- $f\left(\kappa_{ya_j}\right) < 0$ if $\kappa_{ya_j} < 1$
- $f\left(\kappa_{ya_j}\right) > 0$ if $\kappa_{ya_j} > 1 + \frac{(1-\gamma)(1-\alpha)}{(1+\epsilon)-(1-\gamma)(1-\alpha)}$
- $f'\left(\kappa_{ya_j}\right) > 0$

The last item follows the fact that $g_\sigma(\psi_t, \lambda)$ is decreasing in κ_{ya_j} because $g_\sigma(\psi_t, \lambda)$ decreases in $\kappa_{J\omega\omega}$, which is an increasing function of κ_{ya_j}.

Finally, in the last step of the proof, it is straightforward to demonstrate the existence and uniqueness for $\widehat{h}_y(\psi_t, \lambda)$ from (35) given the existence and uniqueness for Amb.-SS and κ_{ya_j}. ∎

Proof of Proposition 2 . It directly follows the comparison between Proof of Proposition 1 and the solution for the beauty contest identified in the proposition. The comparative static analysis of g_μ and g_σ can be proved by showing that $\kappa_{J\omega}$ is increasing and $\kappa_{J\omega\omega}$. is decreasing in ψ_t.

∎

Proof of Proposition 3 . It can be shown that (45) has the following properties regarding its partial derivatives evaluated at the equilibrium, i.e., $f\left(\kappa_{ya_j}\right) = 0$:

- $\frac{\partial f\left(\kappa_{ya_j}\right)}{\partial \kappa_{ya_j}}\Big|_{f\left(\kappa_{ya_j}\right)=0} > 0$
- $\frac{\partial f\left(\kappa_{ya_j}\right)}{\partial \psi_t}\Big|_{f\left(\kappa_{ya_j}\right)=0} < 0$

which are all straightforward!. Therefore, $\kappa_{ya_j}(\psi_t, \lambda)$ is increasing in ψ_t.

Furthermore, it can be shown that $\kappa_{J\omega}$ is increasing in ψ_t since it is increasing in κ_{ya_j}, which is an increasing function of ψ_t. Moreover, note that we can transform (36) into

$$H_y(\psi_t, \lambda) = -\kappa_{ya_j}(\psi_t, \lambda)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2 + \sigma_\iota^2 + e^{\psi_t} + g_\sigma(\psi_t, \lambda)}\right)\left(e^{\psi_t} + g_\sigma(\psi_t, \lambda)\right)\lambda\kappa_{J\omega}$$

Following (34), we know that in equilibrium, it must be the case that $e^{\psi_t} + g_\sigma(\psi_t, \lambda)$ is increasing in ψ_t. Then $H_y(\psi_t, \lambda)$ must be decreasing in ψ_t. Combined with (35), we can prove that $\widehat{h}_y(\psi_t, \lambda)$ is decreasing in ψ_t. ■

Proof of Lemma 3 and Proposition 5. First of all, it can be shown that

$$\begin{aligned}\left(\overline{\kappa}_{J\omega\omega} + \frac{1}{\lambda e^{\overline{\psi}}}\right)\frac{\partial \log \overline{\kappa}_{J\omega}}{\partial \overline{\kappa}_{ya_j}} =& \frac{1}{\overline{\kappa}_{J\omega}}\frac{\partial \overline{\kappa}_{J\omega}}{\partial \overline{\kappa}_{ya_j}}\left(\overline{\kappa}_{J\omega\omega} + \frac{1}{\lambda e^{\overline{\psi}}}\right)\\ =& S\left(\frac{1}{\theta} - \gamma\right)\overline{\kappa}_{ya_j}\beta_\iota\\ =& S\left[(1-\gamma)\kappa_{ya_j} - (1+\epsilon)\kappa_{na_j}\right]\\ <& 2S\left[\left(1+(1-\gamma)\overline{h}_y\right)(1-\gamma)\overline{\kappa}_{ya_j} - \left(1+(1+\epsilon)\overline{h}_n\right)(1+\epsilon)\overline{\kappa}_{na_j}\right]\\ =& \frac{\partial \overline{\kappa}_{J\omega\omega}}{\partial \kappa_{ya_j}}\end{aligned}$$

The inequality follows the same argument as in the proof for unique Amb.-SS. Then we can claim that

$$\frac{\partial \log \overline{\kappa}_{J\omega}}{\partial \overline{\kappa}_{ya_j}} < \frac{\lambda e^{\overline{\psi}}}{1+\lambda\overline{\kappa}_{J\omega\omega}e^{\overline{\psi}}}\frac{\partial \overline{\kappa}_{J\omega\omega}}{\partial \overline{\kappa}_{ya_j}}$$

At Amb.-SS, we have that

$$g_\mu(\overline{\psi}, \lambda) = -\frac{\lambda\overline{\kappa}_{J\omega}e^{\overline{\psi}}}{1+\lambda\overline{\kappa}_{J\omega\omega}e^{\overline{\psi}}} \Rightarrow \log(-g_\mu(\overline{\psi}, \lambda)) = \log\left(\lambda\overline{\kappa}_{J\omega}e^{\overline{\psi}}\right) - \log\left(1+\lambda\overline{\kappa}_{J\omega\omega}e^{\overline{\psi}}\right)$$

Taking the derivative leads to

$$\frac{\partial \log(-g_\mu(\overline{\psi}, \lambda))}{\partial \overline{\kappa}_{ya_j}} = \frac{\partial \log \overline{\kappa}_{J\omega}}{\partial \overline{\kappa}_{ya_j}} - \frac{\lambda e^{\overline{\psi}}}{1+\lambda\overline{\kappa}_{J\omega\omega}e^{\overline{\psi}}}\frac{\partial \overline{\kappa}_{J\omega\omega}}{\partial \overline{\kappa}_{ya_j}} < 0$$

Therefore, we have that

$$\frac{\partial g_\mu(\overline{\psi}, \lambda)}{\partial \overline{\kappa}_{ya_j}} > 0 \tag{46}$$

At the Amb.-SS when the amount of ambiguity is $\overline{\psi}$, the equilibrium is characterized by (40) and (41). We can transform (41) by

$$X\overline{h}_y = \left[\left(\frac{1+\epsilon}{1-\alpha}\right) - X\overline{\kappa}_{ya_j}\right] g_\mu(\overline{\psi}, \lambda) \tag{47}$$

As in the proof of unique Amb.-SS, (40) defines a decreasing function of $\kappa_{yx}\left(\overline{h}_y\right)$. And an increase in σ_ι^2 or a decrease in σ_ζ^2 increases the RHS of (40) for any given $\overline{h}_y$. This indicates that the function $\kappa_{yx}^*\left(\overline{h}_y\right)$ derived from (40) shifted to the left in response to an increase in σ_ι^2 or a decrease in σ_ζ^2. Furthermore, the RHS of (47) increases with $\overline{\kappa}_{ya_j}$ marginally at the point where LHS equals RHS. Following the proof of unique Amb.-SS, the gap between RHS and LHS of (47) is marginally increasing in $\overline{h}_y$ at the point where LHS equals RHS. Then, a marginal increase in $\overline{\kappa}_{ya_j}$ implies a marginal increase in $\overline{y}$. Therefore, (47) defines an increasing function of $\kappa_{yx}^{**}\left(\overline{h}_y\right)$. The decreasing function $\kappa_{yx}^*\left(\overline{h}_y\right)$, the increasing function of

$\kappa_{yx}^{**}\left(\overline{h}_y\right)$ and the left-hand side shift of $\kappa_{yx}^{*}\left(\overline{h}_y\right)$ in combination predict that

$$\frac{d\overline{\kappa}_{ya_j}}{d\sigma_\iota^2} < 0 \qquad\qquad \frac{d\overline{\kappa}_{ya_j}}{d\sigma_{\sigma_\zeta^2}^2} > 0 \tag{48}$$

and

$$\frac{d\overline{h}_y}{d\sigma_\iota^2} < 0 \qquad\qquad \frac{d\overline{h}_y}{d\sigma_{\sigma_\zeta^2}^2} > 0 \tag{49}$$

Finally, (46) and (48) together prove Lemma 3. ∎

B. Accuracy of the Approximation

B.1. Static Model without Capital

Suppose that the allocation constitutes a conditional log-normal equilibrium in the sense that in equilibrium, $y_{j,t} \equiv \ln\left(Y_{j,t}\right)$ takes the following forms

$$y_{j,t} = \overline{y} + \kappa_{ya_j}\left(\psi_t, \lambda\right) a_{j,t} + \widehat{h}_y\left(\psi_t, \lambda\right)$$

Then we can implement a quadratic approximation around the ambiguous steady state (Amb.-SS) over $\overline{J}_t\left(\omega_t\right) \equiv \mathbb{E}_{j,t,0}^{\omega_t}\left[\frac{Y_t^{1-\gamma}-1}{1-\gamma} - \chi\int_{\mathbb{J}}\frac{\left(Y_{j,t}/A_{j,t}\right)^{(1+\epsilon)/(1-\alpha)}}{1+\epsilon}dj\right]$, which gives out the following quadratic functional forms with respect to ω_t:

$$\overline{J}_t\left(\omega_t\right) \approx const_t + \kappa_{J\omega}\left(\psi_t, \lambda\right)\omega_t + \frac{1}{2}\kappa_{J\omega\omega}\left(\psi_t, \lambda\right)\omega_t^2$$

Directly following (15), the approximation implies a normal distorted posterior belief about possible models. This is exactly what Lemma 1 is about. In addition, the fixed point condition (14) combined with normality in a distorted posterior belief would automatically imply a conditional log-normal equilibrium such that

$$y_t = \overline{y} + \kappa_{ya_j}a_t + \widehat{h}_y\left(\psi_t, \lambda\right) + \underbrace{R_t + A_t + D_t}_{\text{uncertainty adjustment}}$$

where the uncertainty adjustment consists of (a) risk adjustment R_t, (b) an ambiguity adjustment and finally (c) a dispersion adjustment. All these adjustments are of second-order impacts at the aggregates and have no impacts at all on the cyclical behaviors of various dispersion measures. Therefore, we ignore these adjustments in the approximated conditional log-normal equilibrium without loss of generality. Equivalently, it corresponds to the usual log-linearization of (14), but around the ambiguous steady state (Amb.-SS) instead of the deterministic steady state (D-SS). In sum, to approximate the conditional log-normal equilibrium, we implement a second-order approximation of the ex-ante expected value function of the representative household and, at the same time, a log-linearization of optimality conditions, both around the Amb.-SS.

How accurate is our approximation? We address this issue into two steps. First of all, we highlight at what points of the analysis approximations are used. Then we analyze in details what are the nature of the approximations and conduct some evaluations.

During the characterization of the approximated conditional log-normal equilibrium, we first conduct a quadratic approximation of the ex-ante expected utility $\overline{J}_t(\omega_t)$ under a particular model ω_t so as to express the belief distortion in (15) into an exponential-quadratic form. Second, given the approximated normal distorted posterior, we log-linearize the optimality conditions, consisting of (1) and (13), around the Amb.-SS. Thus, we ignore uncertainty adjustments who are by nature a couple of second-order terms having negligible impacts on allocations and no impacts at all on various measures of dispersions.

For the second approximation, it is fairly standard in the literature. As a matter of fact, the ignored uncertainty adjustments are of the magnitude $\sigma_\zeta^2 + \sigma_\iota^2 + e^{\psi_t}$. However, the first moment impacts of ambiguity shocks are of the magnitude λe^{ψ_t}, which dominates uncertainty adjustments given the calibrated degree of ambiguity aversion λ. Considering the first approximation, we check its accuracy by comparing the estimated posterior density as in (15) and a semi-true posterior density numerically. To compute the semi-true posterior density, we take the estimated policy functions (16) and (17) as given and compute (15) analytically without using a quadratic approximation of $\overline{J}_t(\omega_t)$. Figure B1 demonstrates such a comparison where the parameters are chosen to be the same as our quantitative evaluations in Section 6. χ is chosen to be 1.895 to have hours in D-SS being 1/3, which is the same calibration strategy of χ in Section 6. Following Angeletos and La'O [2009], γ is chosen to be 0.2 to ensure an empirically plausible income effect of labor supply. Comparisons of estimated- and semi-true posteriors are conducted under two parameterizations for the degree of ambiguity aversion λ: one baseline calibration $\lambda = 11.3$ (the left-hand panel) and one counterfactually extreme calibration $\lambda = 100$ (the right-hand panel). Finally, we normalize the realization of island productivity by setting $a_{j,t} = 0$. This indicates that the Bayesian posterior should have the mean zero. Therefore, the leftward shifts of all four posterior densities manifest degree of pessimism due to ambiguity aversion. Finally, it turns out that the approximation of the distorted posterior about possible models is fairly accurate, not only for the baseline but also for the extreme calibration.

B.2. Dynamic RBC Model

This section evaluates the accuracy of our quantitative methodology for the dynamic RBC model.

Appendix B.1 evaluates the accuracy of our approximation method for the static model without capital. Alternatively, it can be interpreted in the following way through the length of the dynamic RBC model: as long as consumption is approximated with sufficient accuracy, hours, output and factor prices can be approximated with sufficient accuracy. To check whether our quantitative methodology can approximate consumption with sufficient accuracy or not, we compute the normalized Euler equation error over a fine grid of capital ranging from 70% to 130% of the level of capital at the Amb.-SS while holding fixed aggregate and idiosyncratic productivity shocks at the steady state level.

Normalized Euler equation errors are of the magnitude of 10^{-4}. The quantitative methodology works reasonably well. Potentially, improvements can be made by using higher order approximations of the non-linear functions regarding the impact of ambiguity shocks and allowing higher order interactions between uncertainty and productivity shocks.

C. Labor Wedges

Define the marginal rate of intra-temporal substitution between leisure and consumption $MRSN_{j,t}$ and labor productivity $MPL_{j,t}$ on island j by

$$MRSN_{j,t} = \epsilon \widehat{n}_{j,t} + \widehat{c}_t$$

and

$$MPL_{j,t} = \widehat{y}_{j,t} - \widehat{n}_{j,t}$$

Then, we can define the island j labor wedge from the household perspective by the gap between the real wage $\widehat{w}_{j,t}$ and the island j marginal rate of intra-temporal substitution between leisure and consumption $MRSN_{j,t}$:

$$\tau_{j,t}^{nh} \equiv \widehat{w}_{j,t} - MRSN_{j,t}$$

and the island j labor wedge from the firm perspective as the gap between island j labor productivity $MPL_{j,t}$ and the real wage $\widehat{w}_{j,t}$:

$$\tau_{j,t}^{nf} \equiv MPL_{j,t} - \widehat{w}_{j,t}$$

Finally, we define the total labor wedge in the economy by the cross-sectional average of the sum of the two:

$$\tau_t^n \equiv \int_{\mathbb{J}} \left(\tau_{j,t}^{nh} + \tau_{j,t}^{nh} \right) dj$$

D. Details for Quantitative Analysis

D.1. Equilibrium Conditions

The equilibrium can be characterized by the standard TVC and the following conditions:

- labor optimality condition

$$\chi N_{j,t}^{\epsilon} = \left(\int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t} \left[\frac{1}{C_t} \left(\frac{Y_{j,t}}{Y_t} \right)^{-\frac{1}{\theta}} \right] \widetilde{f}_{j,t,1} (\omega_t) \, d\omega_t \right) \left((1-\alpha) \frac{Y_{j,t}}{N_{j,t}} \right)$$

- optimal capital demand condition

$$R_{j,t} \int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t} \left[\frac{1}{C_t} \right] \widetilde{f}_{j,t,1} (\omega_t) \, d\omega_t = \left(\int_{\mathbb{R}} \mathbb{E}_{j,t,1}^{\omega_t} \left[\frac{1}{C_t} \left(\frac{Y_{j,t}}{Y_t} \right)^{-\frac{1}{\theta}} \right] \widetilde{f}_{j,t,1} (\omega_t) \, d\omega_t \right) \left(\alpha \frac{Y_{j,t}}{K_{j,t}} \right)$$

- Euler equation

$$\frac{1}{C_t} = \beta \int_{\mathbb{R}} \mathbb{E}_{t,2}^{\omega_{t+1}} \left[\frac{1}{C_{t+1}} \left((1-\delta) + R_{j,t+1} \right) \right] \widetilde{f}_{t,2} (\omega_{t+1}) \, d\omega_{t+1} \tag{50}$$

- budget constraint

$$Y_t = C_t + \int_J I_{j,t} dj \tag{51}$$

- capital accumulation

$$K_{j,t+1} = (1-\delta) K_{j,t} + I_{j,t} \tag{52}$$

- production function for island commodities

$$Y_{j,t} = A_{j,t} K_{j,t}^{\alpha} N_{j,t}^{1-\alpha} \tag{53}$$

- production function for final goods

$$\log Y_t = \int_{\mathbb{J}} \log Y_{j,t} dj \tag{54}$$

- value function recursion for $J_t \equiv J\left(\left\{K_{j,t}\right\}, a_{t-1}, z_t, \zeta_t, \psi_t\right)$:

$$J_t = \max \quad \ln(C_t) - \chi \int_{\mathbb{J}} \frac{N_{j,t}^{1+\epsilon}}{1+\epsilon} dj + \beta \left(-\frac{1}{\lambda}\right) \ln\left(\int_{\mathbb{R}} e^{-\lambda \mathbb{E}_{t,2}^{\omega_{t+1}}[J_{t+1}]} f_t(\omega_{t+1})\, d\omega_{t+1}\right)$$

- distorted beliefs $\left\{\widetilde{f}_{j,t,1}(\omega_t), \widetilde{f}_{t,2}(\omega_{t+1})\right\}$ at stage 1 and at stage 2, respectively

$$\widetilde{f}_{j,t,1}(\omega_t) \propto e^{-\lambda \mathbb{E}_{t,0}^{\omega_t}[J_t]} f\left(x_{j,t}|\omega_t\right) f_t(\omega_t)$$

  and

$$\widetilde{f}_{t,2}(\omega_{t+1}) \propto e^{-\lambda \mathbb{E}_{t,2}^{\omega_t}[J_{t+1}]} f_t(\omega_{t+1})$$

Observe that in equilibrium, it will be the case that $K_{j,t} = K_t$ for all $t > 0$. This is because at stage 2, there exists no heterogeneity in beliefs about capital return $r_{j,t}$ across islands. Therefore, the capital supply exhibits no heterogeneity. In what follows, we replace all $K_{j,t}$ with K_t for simplicity.

D.2. Solution Method

We propose the following semi-linear policy rules of the conditional log-normal equilibrium for island employment $\widehat{n}_{j,t}$, output $\widehat{y}_{j,t}$, the wage rate $\widehat{w}_{j,t}$ and the rental rental rate of capital $\widehat{r}_{j,t}$ at stage 1 of period t:

$$\begin{aligned}
\widehat{y}_{j,t} &= \kappa_{yk}\widehat{k}_t + \kappa_{ya}a_{t-1} + \kappa_{yx}x_{j,t} + \kappa_{yx\psi}x_{j,t}\widehat{\psi}_t + \kappa_{y\psi}\widehat{\psi}_t + \kappa_{y\psi\psi}\widehat{\psi}_t^2 \\
\widehat{n}_{j,t} &= \kappa_{nk}\widehat{k}_t + \kappa_{na}a_{t-1} + \kappa_{nx}x_{j,t} + \kappa_{nx\psi}x_{j,t}\widehat{\psi}_t + \kappa_{n\psi}\widehat{\psi}_t + \kappa_{n\psi\psi}\widehat{\psi}_t^2 \\
\widehat{w}_{j,t} &= \kappa_{wk}\widehat{k}_t + \kappa_{wa}a_{t-1} + \kappa_{wx}x_{j,t} + \kappa_{wx\psi}x_{j,t}\widehat{\psi}_t + \kappa_{w\psi}\widehat{\psi}_t + \kappa_{w\psi\psi}\widehat{\psi}_t^2 \\
\widehat{r}_{j,t} &= \kappa_{rk}\widehat{k}_t + \kappa_{ra}a_{t-1} + \kappa_{rx}x_{j,t} + \kappa_{rx\psi}x_{j,t}\widehat{\psi}_t + \kappa_{r\psi}\widehat{\psi}_t + \kappa_{r\psi\psi}\widehat{\psi}_t^2
\end{aligned}$$

and the following policy rules for consumption $\widehat{c}_t$, investment $\widehat{i}_t$ and capital stock tomorrow $\widehat{k}_{t+1}$ at stage 2 of period t:

$$\begin{aligned}
\widehat{c}_t &= \kappa_{ck}\widehat{k}_t + \kappa_{ca}a_{t-1} + \kappa_{cz}z_t + \kappa_{c\zeta}\zeta_t + \kappa_{c\zeta\psi}\zeta_t\widehat{\psi}_t + \kappa_{cz\psi}z_t\widehat{\psi}_t + \kappa_{c\psi}\widehat{\psi}_t + \kappa_{c\psi\psi}\widehat{\psi}_t^2 \\
\widehat{i}_t &= \kappa_{ik}\widehat{k}_t + \kappa_{ia}a_{t-1} + \kappa_{iz}z_t + \kappa_{i\zeta}\zeta_t + \kappa_{i\zeta\psi}\zeta_t\widehat{\psi}_t + \kappa_{iz\psi}z_t\widehat{\psi}_t + \kappa_{i\psi}\widehat{\psi}_t + \kappa_{i\psi\psi}\widehat{\psi}_t^2 \\
\widehat{k}_{t+1} &= \kappa_{kk}\widehat{k}_t + \kappa_{ka}a_{t-1} + \kappa_{kz}z_t + \kappa_{k\zeta}\zeta_t + \kappa_{k\zeta\psi}\zeta_t\widehat{\psi}_t + \kappa_{kz\psi}z_t\widehat{\psi}_t + \kappa_{k\psi}\widehat{\psi}_t + \kappa_{k\psi\psi}\widehat{\psi}_t^2
\end{aligned}$$

D.2.1. Quadratic approximation of value function

Under the proposed policy rules, the linear approximation of household period utility around the Amb.-SS is such that

$$\ln\left(C_t\right) - \chi\int_{\mathbb{J}}\frac{N_{j,t}^{1+\epsilon}}{1+\epsilon}dj \tag{55}$$

$$\approx c^* + \overline{h}_c + \kappa_{ck}\widehat{k}_t + \kappa_{ca}a_{t-1} + \kappa_{cz,t}z_t + \kappa_{c\zeta,t}\zeta_t + \widehat{h}_c\left(\psi_t\right) \tag{56}$$

$$-\chi\frac{\overline{N}^{1+\epsilon}}{1+\epsilon}\int_{\mathbb{J}}\left[1 + (1+\epsilon)\left(\kappa_{nk}\widehat{k}_t + \kappa_{na}a_{t-1} + \kappa_{nx,t}x_{j,t} + \widehat{h}_n\left(\psi_t\right)\right)\right]dj \tag{57}$$

Guess that J_t is given by

$$\begin{aligned}
J_t = J^* + \overline{J} &+ \kappa_{Jk}\widehat{k}_t + \kappa_{Ja}a_{t-1} + \kappa_{Jz}z_t + \kappa_{J\zeta}\zeta_t + \kappa_{J\psi}\widehat{\psi}_t \\
&+ \kappa_{Jka}\widehat{k}_t a_{t-1} + \kappa_{Jkz}\widehat{k}_t z_t + \kappa_{Jk\zeta}\widehat{k}_t\zeta_t + \kappa_{Jaz}a_{t-1}z_t + \kappa_{Ja\zeta}a_{t-1}\zeta_t + \kappa_{Jz\zeta}z_t\zeta_t \\
&+ \kappa_{Jk\psi}\widehat{k}_t\widehat{\psi}_t + \kappa_{Ja\psi}a_{t-1}\widehat{\psi}_t + \kappa_{Jz\psi}z_t\widehat{\psi}_t + \kappa_{J\zeta\psi}\zeta_t\widehat{\psi}_t \\
&+ \kappa_{Jkk}\widehat{k}_t^2 + \kappa_{Jaa}a_{t-1}^2 + \kappa_{Jzz}z_t^2 + \kappa_{J\zeta\zeta}\zeta_t^2 + \kappa_{J\psi\psi}\widehat{\psi}_t^2
\end{aligned}$$

Hence we have that the Bellman equation implies the following conditions for the relevant elasticities:

$$\begin{aligned}
\kappa_{Jz} &= \kappa_{cz} - \chi\overline{N}^{1+\epsilon}\kappa_{nx} + \beta\kappa_{Jk}\kappa_{kz} \\
\kappa_{Jkz} &= 2\beta\kappa_{Jkk}\kappa_{kk}\kappa_{kz} \\
\kappa_{Jaz} &= \beta\kappa_{Jka}\kappa_{kz}\rho + 2\beta\kappa_{Jkk}\kappa_{ka}\kappa_{kz} \\
\kappa_{Jz\psi} &= \kappa_{cz\psi} - \chi\overline{N}^{1+\epsilon}\kappa_{nx\psi}\rho_\psi + \beta\kappa_{Jk}\kappa_{kz\psi} + 2\beta\kappa_{Jkk}\kappa_{kz}\kappa_{k\psi} + \beta\kappa_{Jk\psi}\kappa_{kz}\rho_\psi \\
\kappa_{Jzz} &= \beta\kappa_{Jkk}\kappa_{kk}^2 \\
\kappa_{Jk} &= \kappa_{ck} - \chi\overline{N}^{1+\epsilon}\kappa_{nk} + \beta\kappa_{Jk}\kappa_{kk} \\
\kappa_{Jkk} &= \beta\kappa_{Jkk}\kappa_{kk}^2 \\
\kappa_{Jka} &= \beta\kappa_{Jka}\kappa_{kk}\rho + 2\beta\kappa_{Jkk}\kappa_{kk}\kappa_{ka} \\
\kappa_{Jk\psi} &= \kappa_{ck\psi} - \chi\overline{N}^{1+\epsilon}\kappa_{nk\psi} + \beta\kappa_{Jk\psi}\kappa_{kk}\rho_\psi + 2\beta\kappa_{Jkk}\kappa_{kk}\kappa_{k\psi}
\end{aligned}$$

D.2.2. Distorted posterior beliefs

Distorted beliefs are normal with the following kernels

$$\widetilde{f}_{j,t,1}\left(\omega_t\right) \propto \exp\left(-\lambda\left(\left(\kappa_{Jz}+\kappa_{Jkz}\widehat{k}_t+\kappa_{Jaz}a_{t-1}+\kappa_{Jz\psi}\widehat{\psi}_t\right)\omega_t+\kappa_{Jzz}\omega_t^2\right)\right) f\left(x_{j,t}|\omega_t\right) f_t\left(\omega_t\right)$$

$$\widetilde{f}_{t,2}\left(\omega_{t+1}\right) \propto \exp\left(-\lambda\left(\left(\kappa_{Jz}+\kappa_{Jkz}\widehat{k}_{t+1}+\kappa_{Jaz}a_t+\kappa_{Jz\psi}\rho_\psi\widehat{\psi}_t\right)\omega_{t+1}+\kappa_{Jzz}\omega_{t+1}^2\right)\right) f_t\left(\omega_{t+1}\right)$$

D.2.3. Ambiguous Steady State

Amb.-SS can be characterized by

$$\begin{aligned}
\log\chi+(1+\epsilon)\,\overline{n} =& \log(1-\alpha)+\overline{y}-\overline{c}+\overline{H}_n \\
\overline{r} =& \log(\alpha)+\overline{y}-\overline{k}+\overline{H}_r \\
1 =& \beta\exp\left(\overline{r}+\overline{s}_1\right)+\beta(1-\delta)\exp\left(\overline{s}_2\right) \\
\overline{y} =& (1-\alpha)\,\overline{n}+\alpha\overline{k} \\
\overline{C}+\overline{I} =& \overline{Y} \\
\overline{I} =& \delta\overline{K}
\end{aligned}$$

where the auxiliary functions are such that

$$\begin{aligned}
\overline{H}_r =& \frac{1}{\theta}\kappa_{yx}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\left(\frac{-\lambda\kappa_{Jz}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}+\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right) \\
\overline{H}_n =& \left\{\left(\frac{1}{\theta}\kappa_{yx}-\kappa_{cz}\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)+\kappa_{c\zeta}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right\}\left(\frac{-\lambda\kappa_{Jz}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}+\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right) \\
\overline{s}_1 =& \left(\kappa_{rx}-\kappa_{cz}\right)\left(\frac{-\lambda\kappa_{Jz}}{\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right) \\
\overline{s}_2 =& -\kappa_{cz}\left(\frac{-\lambda\kappa_{Jz}}{\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right)
\end{aligned}$$

D.2.4. Log-linearization of optimality conditions

Below we report the log-linearized optimality conditions

- labor optimality condition

$$(1+\epsilon)\,\widehat{n}_{j,t}=\left(1-\frac{1}{\theta}\right)\widehat{y}_{j,t}+\int_{\mathbb{R}}\mathbb{E}_{j,t,1}^{\omega_t}\left[\frac{1}{\theta}\widehat{y}_t-\widehat{c}_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t-H_n\left(\overline{\psi}\right)$$

- optimal capital demand condition

$$\widehat{r}_{j,t}=\left(1-\frac{1}{\theta}\right)\widehat{y}_{j,t}+\int_{\mathbb{R}}\mathbb{E}_{j,t,1}^{\omega_t}\left[\frac{1}{\theta}\widehat{y}_t-\widehat{k}_t\right]\widetilde{f}_{j,t,1}\left(\omega_t\right)d\omega_t-H_r\left(\overline{\psi}\right)$$

- Euler equation

$$-c_t = \beta \exp\left(\bar{r} + \bar{s}_1\right) \int_{\mathbb{R}} \mathbb{E}_{t,2}^{\omega_{t+1}} \left[\widehat{r}_{j,t+1} - \widehat{c}_{t+1}\right] \widetilde{f}_{t,2}\left(\omega_{t+1}\right) d\omega_{t+1} - \beta\left(1-\delta\right) \exp\left(\bar{s}_2\right) \int_{\mathbb{R}} \mathbb{E}_{t,2}^{\omega_{t+1}} \left[\widehat{c}_{t+1}\right] \widetilde{f}_{t,2}\left(\omega_{t+1}\right) d\omega_{t+1}$$

- budget constraint

$$\overline{C}\widehat{c}_t + \overline{I}\widehat{i}_t = \overline{Y}\widehat{y}_t$$

- capital accumulation

$$\overline{K}\widehat{k}_{t+1} = \left(1-\delta\right) \overline{K}\widehat{k}_t + \overline{I}\widehat{i}_t$$

- production function for island commodities

$$\widehat{y}_{j,t} = \rho a_{t-1} + x_{j,t} + \alpha \widehat{k}_t + \left(1-\alpha\right) \widehat{n}_{j,t}$$

- production function for final goods

$$\widehat{y}_t = \int_{\mathbb{J}} \widehat{y}_{j,t} dj$$

D.2.5. Systems of undetermined coefficients

Combining the proposed policy rules and log-linearized optimality conditions, we arrive at the following systems of undetermined coefficients

- blocks of equations for the determination of κ_{J*}

$$\begin{aligned}
\kappa_{Jz} &= \kappa_{cz} - \chi \overline{N}^{1+\epsilon} \kappa_{nx} + \beta \kappa_{Jk} \kappa_{kz} \\
\kappa_{Jkz} &= 2\beta \kappa_{Jkk} \kappa_{kk} \kappa_{kz} \\
\kappa_{Jaz} &= \beta \kappa_{Jka} \kappa_{kz} \rho + 2\beta \kappa_{Jkk} \kappa_{ka} \kappa_{kz} \\
\kappa_{Jz\psi} &= \kappa_{cz\psi} - \chi \overline{N}^{1+\epsilon} \kappa_{nx\psi} \rho_\psi + \beta \kappa_{Jk} \kappa_{kz\psi} + 2\beta \kappa_{Jkk} \kappa_{kz} \kappa_{k\psi} + \beta \kappa_{Jk\psi} \kappa_{kz} \rho_\psi \\
\kappa_{Jzz} &= \beta \kappa_{Jkk} \kappa_{kk}^2 \\
\kappa_{Jk} &= \kappa_{ck} - \chi \overline{N}^{1+\epsilon} \kappa_{nk} + \beta \kappa_{Jk} \kappa_{kk} \\
\kappa_{Jkk} &= \beta \kappa_{Jkk} \kappa_{kk}^2 \\
\kappa_{Jka} &= \beta \kappa_{Jka} \kappa_{kk} \rho + 2\beta \kappa_{Jkk} \kappa_{kk} \kappa_{ka} \\
\kappa_{Jk\psi} &= \kappa_{ck\psi} - \chi \overline{N}^{1+\epsilon} \kappa_{nk\psi} + \beta \kappa_{Jk\psi} \kappa_{kk} \rho_\psi + 2\beta \kappa_{Jkk} \kappa_{kk} \kappa_{k\psi}
\end{aligned}$$

- blocks of equations for the determination of κ_{*k} and κ_{*a}

$$
\begin{aligned}
(1+\epsilon)\,\kappa_{nk} &= \left(1-\frac{1}{\theta}\right)\kappa_{yk} + \left(\frac{1}{\theta}\kappa_{yk} - \kappa_{ck}\right)\\
(1+\epsilon)\,\kappa_{na} &= \left(1-\frac{1}{\theta}\right)\kappa_{ya} + \left(\frac{1}{\theta}\kappa_{ya} - \kappa_{ca}\right)\\
\kappa_{yk} &= (1-\alpha)\,\kappa_{nk} + \alpha\\
\kappa_{ya} &= \rho + (1-\alpha)\,\kappa_{na}\\
\kappa_{rk} &= \kappa_{yk} - 1\\
\kappa_{ra} &= \kappa_{ya}\\
-\kappa_{ck} &= \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{rk} - \kappa_{ck}\right)\kappa_{kk}\\
-\kappa_{ca} &= \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{rk} - \kappa_{ck}\right)\kappa_{ka} + \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{ra} - \kappa_{ca}\right)\rho\\
\overline{C}\kappa_{ck} + \overline{I}\kappa_{ik} &= \overline{Y}\kappa_{yk}\\
\overline{C}\kappa_{ca} + \overline{I}\kappa_{ia} &= \overline{Y}\kappa_{ya}\\
\kappa_{kk} &= (1-\delta) + \delta\kappa_{ik}\\
\kappa_{ka} &= \delta\kappa_{ia}
\end{aligned}
$$

- blocks of equations for the determination of κ_{*x}, κ_{*z} and $\kappa_{*\zeta}$

$$
\begin{aligned}
(1+\epsilon)\,\kappa_{nx} &= \left(1-\frac{1}{\theta}\right)\kappa_{yx} + \left[\left(\frac{1}{\theta}\kappa_{yx} - \kappa_{cz}\right)\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) - \kappa_{c\zeta}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right]\\
&\quad + \left[\left(\frac{1}{\theta}\kappa_{yx} - \kappa_{cz}\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \kappa_{c\zeta}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right]\left(\frac{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp\left(\overline{\psi}\right)} + \lambda\kappa_{Jzz}}\right)\\
\kappa_{yx} &= 1 + (1-\alpha)\,\kappa_{nx}\\
\kappa_{rx} &= \left(1-\frac{1}{\theta}\right)\kappa_{yx} + \frac{1}{\theta}\kappa_{yx}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \frac{1}{\theta}\kappa_{yx}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\left(\frac{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp\left(\overline{\psi}\right)} + \lambda\kappa_{Jzz}}\right)\\
-\kappa_{cz} &= \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{rk} - \kappa_{ck}\right)\kappa_{kz}\\
-\kappa_{c\zeta} &= \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{rk} - \kappa_{ck}\right)\kappa_{k\zeta} + \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{ra} - \kappa_{ca}\right)\\
\overline{C}\kappa_{cz} + \overline{I}\kappa_{iz} &= \overline{Y}\kappa_{yx}\\
\overline{C}\kappa_{c\zeta} + \overline{I}\kappa_{i\zeta} &= 0\\
\kappa_{kz} &= \delta\kappa_{iz}\\
\kappa_{k\zeta} &= \delta\kappa_{i\zeta}
\end{aligned}
$$

- blocks of equations for the determination of $\kappa_{*x\psi}$, $\kappa_{*z\psi}$, and $\kappa_{*\zeta\psi}$

$$
\begin{aligned}
(1+\epsilon)\,\kappa_{nx\psi} &= \left(1-\frac{1}{\theta}\right)\kappa_{yx\psi} + \left[\left(\frac{1}{\theta}\kappa_{yx\psi}-\kappa_{cz\psi}\right)\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) - \kappa_{c\zeta\psi}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right] \\
&\quad + \left[\left(\frac{1}{\theta}\kappa_{yx}-\kappa_{cz}\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \kappa_{c\zeta}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right] \frac{\frac{1}{\exp(\overline{\psi})}\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}}{\left(\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}+\frac{1}{\exp(\overline{\psi})}+\lambda\kappa_{Jzz}\right)^2} \\
&\quad + \left[\left(\frac{1}{\theta}\kappa_{yx\psi}-\kappa_{cz\psi}\right)\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \kappa_{c\zeta\psi}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right]\left(\frac{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}+\frac{1}{\exp(\overline{\psi})}+\lambda\kappa_{Jzz}}\right) \\
\kappa_{yx\psi} &= (1-\alpha)\,\kappa_{nx\psi} \\
\kappa_{rx\psi} &= \left(1-\frac{1}{\theta}\right)\kappa_{yx\psi} + \frac{1}{\theta}\kappa_{yx\psi}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \frac{1}{\theta}\kappa_{yx}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\frac{\frac{1}{\exp(\overline{\psi})}\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}}{\left(\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}+\frac{1}{\exp(\overline{\psi})}+\lambda\kappa_{Jzz}\right)^2} \\
&\quad + \frac{1}{\theta}\kappa_{yx\psi}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\left(\frac{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2}+\frac{1}{\exp(\overline{\psi})}+\lambda\kappa_{Jzz}}\right) \\
-\kappa_{cz\psi} &= \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{rk}-\kappa_{ck}\right)\kappa_{kz\psi} \\
-\kappa_{c\zeta\psi} &= \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{rk}-\kappa_{ck}\right)\kappa_{k\zeta\psi} \\
\overline{C}\kappa_{cz\psi}+\overline{I}\kappa_{iz\psi} &= \overline{Y}\kappa_{yx\psi} \\
\overline{C}\kappa_{c\zeta\psi}+\overline{I}\kappa_{i\zeta\psi} &= 0 \\
\kappa_{kz\psi} &= \delta\kappa_{iz\psi} \\
\kappa_{k\zeta\psi} &= \delta\kappa_{i\zeta\psi}
\end{aligned}
$$

- blocks of equations for the determination of $\kappa_{*\psi}$ and $\kappa_{*\psi\psi}$

$$
\begin{aligned}
\kappa_{y\psi} &= (1-\alpha)\,\kappa_{n\psi} \\
\kappa_{y\psi\psi} &= (1-\alpha)\,\kappa_{n\psi\psi} \\
\overline{C}\kappa_{c\psi}+\overline{I}\kappa_{i\psi} &= \overline{Y}\kappa_{y\psi} \\
\overline{C}\kappa_{c\psi\psi}+\overline{I}\kappa_{i\psi\psi} &= \overline{Y}\kappa_{y\psi\psi} \\
\kappa_{k\psi} &= \delta\kappa_{i\psi} \\
\kappa_{k\psi\psi} &= \delta\kappa_{i\psi\psi}
\end{aligned}
$$

$$
\begin{aligned}
(1+\epsilon)\,\kappa_{n\psi} =& \kappa_{y\psi} - \kappa_{c\psi} \\
&+ \left[\left(\frac{1}{\theta}\kappa_{yx} - \kappa_{cz}\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \kappa_{c\zeta}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right]\left(\frac{-\lambda\kappa_{Jz\psi}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}}\right) \\
&+ \left[\left(\frac{1}{\theta}\kappa_{yx\psi} - \kappa_{cz\psi}\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \kappa_{c\zeta\psi}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right]\left(\frac{-\lambda\kappa_{Jz}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}}\right) \\
&+ \left[\left(\frac{1}{\theta}\kappa_{yx} - \kappa_{cz}\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \kappa_{c\zeta}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right]\left(\frac{-\lambda\kappa_{Jz}\frac{1}{\exp(\overline{\psi})}}{\left(\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}\right)^2}\right) \\
(1+\epsilon)\,\kappa_{n\psi\psi} =& \kappa_{y\psi\psi} - \kappa_{c\psi\psi} \\
&+ \left[\left(\frac{1}{\theta}\kappa_{yx\psi} - \kappa_{cz\psi}\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \kappa_{c\zeta\psi}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right]\left(\frac{-\lambda\kappa_{Jz\psi}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}}\right) \\
&+ \left[\left(\frac{1}{\theta}\kappa_{yx\psi} - \kappa_{cz\psi}\right)\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right) + \kappa_{c\zeta\psi}\left(\frac{\sigma_\zeta^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\right]\left(\frac{-\lambda\kappa_{Jz}\frac{1}{\exp(\overline{\psi})}}{\left(\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}\right)^2}\right) \\
\kappa_{r\psi} =& \kappa_{y\psi} + \frac{1}{\theta}\kappa_{yx}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\left(\frac{-\lambda\kappa_{Jz\psi}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}}\right) \\
&+ \frac{1}{\theta}\kappa_{yx\psi}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\left(\frac{-\lambda\kappa_{Jz}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}}\right) \\
&+ \frac{1}{\theta}\kappa_{yx}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\left(\frac{-\lambda\kappa_{Jz}\frac{1}{\exp(\overline{\psi})}}{\left(\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}\right)^2}\right) \\
\kappa_{r\psi\psi} =& \kappa_{y\psi\psi} + \frac{1}{\theta}\kappa_{yx\psi}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\left(\frac{-\lambda\kappa_{Jz\psi}}{\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}}\right) \\
&+ \frac{1}{\theta}\kappa_{yx\psi}\left(\frac{\sigma_\iota^2}{\sigma_\zeta^2+\sigma_\iota^2}\right)\left(\frac{-\lambda\kappa_{Jz}\frac{1}{\exp(\overline{\psi})}}{\left(\frac{1}{\sigma_\zeta^2+\sigma_\iota^2} + \frac{1}{\exp(\overline{\psi})} + \lambda\kappa_{Jzz}\right)^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
-\kappa_{c\psi} &= \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{rk}-\kappa_{ck}\right)\kappa_{k\psi}+\left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{r\psi}-\kappa_{c\psi}\right)\rho_\psi \\
&+\beta\overline{R}\exp\left(\overline{s}_1\right)\left[\left(\kappa_{rx}-\kappa_{cz}\right)\rho_\psi\left(\frac{-\lambda\kappa_{Jz\psi}}{\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right)+\left(\kappa_{rx\psi}-\kappa_{cz\psi}\right)\rho_\psi\left(\frac{-\lambda\kappa_{Jz}}{\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right)\right] \\
&+\beta\overline{R}\exp\left(\overline{s}_1\right)\left[\left(\kappa_{rx}-\kappa_{cz}\right)\left(\frac{-\lambda\kappa_{Jz}\frac{1}{\exp\left(\overline{\psi}\right)}}{\left(\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}\right)^2}\right)\right]\rho_\psi \\
&+\beta\left(1-\delta\right)\exp\left(\overline{s}_2\right)\left[-\kappa_{cz}\rho_\psi\left(\frac{-\lambda\kappa_{Jz\psi}}{\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right)-\kappa_{cz\psi}\rho_\psi\left(\frac{-\lambda\kappa_{Jz}}{\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right)\right] \\
&+\beta\left(1-\delta\right)\exp\left(\overline{s}_2\right)\left[-\kappa_{cz}\left(\frac{-\lambda\kappa_{Jz}\frac{1}{\exp\left(\overline{\psi}\right)}}{\left(\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}\right)^2}\right)\right]\rho_\psi \\
-\kappa_{c\psi\psi} &= \left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{rk}-\kappa_{ck}\right)\kappa_{k\psi\psi}+\left(\beta\overline{R}\exp\left(\overline{s}_1\right)\kappa_{r\psi\psi}-\kappa_{c\psi\psi}\right)\rho_\psi^2 \\
&+\beta\overline{R}\exp\left(\overline{s}_1\right)\left(\kappa_{rx\psi}-\kappa_{cz\psi}\right)\rho_\psi^2\left(\frac{-\lambda\kappa_{Jz\psi}}{\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right) \\
&+\beta\left(1-\delta\right)\exp\left(\overline{s}_2\right)\left[-\kappa_{cz\psi}\rho_\psi^2\left(\frac{-\lambda\kappa_{Jz\psi}}{\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}}\right)\right] \\
&+\beta\overline{R}\exp\left(\overline{s}_1\right)\left[\left(\kappa_{rx\psi}-\kappa_{cz\psi}\right)\rho_\psi\left(\frac{-\lambda\kappa_{Jz}\frac{1}{\exp\left(\overline{\psi}\right)}}{\left(\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}\right)^2}\right)\right]\rho_\psi \\
&+\beta\left(1-\delta\right)\exp\left(\overline{s}_2\right)\left[-\kappa_{cz\psi}\rho_\psi\left(\frac{-\lambda\kappa_{Jz}\frac{1}{\exp\left(\overline{\psi}\right)}}{\left(\frac{1}{\exp\left(\overline{\psi}\right)}+\lambda\kappa_{Jzz}\right)^2}\right)\right]\rho_\psi
\end{aligned}
$$

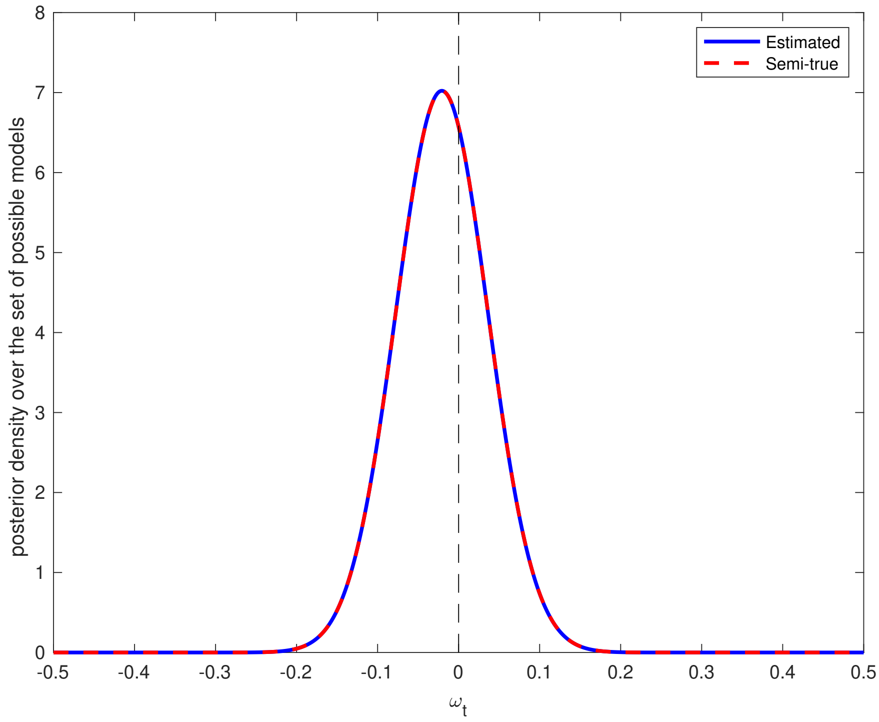
Note that we run a quadratic approximation around Amb.-SS whenever we encounter non-linear terms in signal extraction problems in stage 1 and the non-linear impacts of uncertainty.

D.2.6. The algorithm

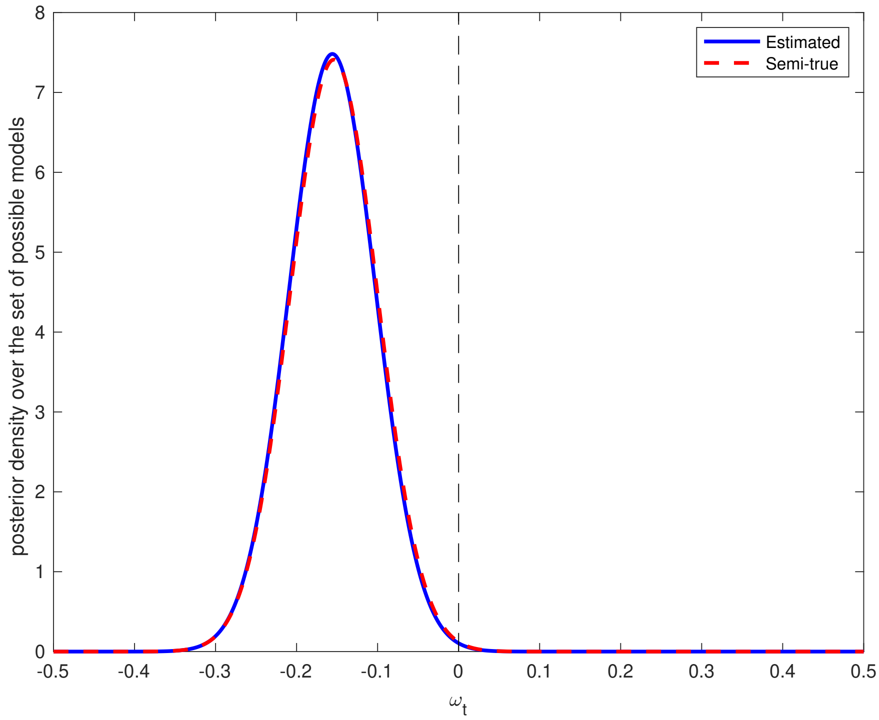
We describe the algorithm to solve this system of undermined coefficients below.

1. Guess that the following vectors $\left\{\overline{N},\overline{C}/\overline{Y},\beta\overline{R}\exp\left(\overline{s}_1\right)\right\}$ at the ambiguous steady state coincide with those at the deterministic steady state.
2. Given $\left\{\overline{N},\overline{C}/\overline{Y},\beta\overline{R}\exp\left(\overline{s}_1\right)\right\}$, solve for κ_{*k} and κ_{*a}. We restrict $\kappa_{kk}<1$ to ensure that TVC is satisfied.
3. Jointly solve for κ_{J*}, κ_{*x}, κ_{*z} and $\kappa_{*\zeta}$ given the solution for κ_{*k} and κ_{*a}.
4. Solve for $\kappa_{*x\psi}$, $\kappa_{*z\psi}$ and $\kappa_{*\zeta\psi}$ given the solution for κ_{*k}, κ_{*a}, κ_{J*}, κ_{*x}, κ_{*z} and $\kappa_{*\zeta}$.
5. Solve for $\kappa_{*\psi}$ and $\kappa_{*\psi\psi}$ given the solution for κ_{*k}, κ_{*a}, κ_{J*}, κ_{*x}, κ_{*z}, $\kappa_{*\zeta}$, $\kappa_{*x\psi}$, $\kappa_{*z\psi}$ and $\kappa_{*\zeta\psi}$.
6. Solve for the Amb.-SS and compute new levels of $\left\{\overline{N},\overline{C}/\overline{Y},\beta\overline{R}\exp\left(\overline{s}_1\right)\right\}$.

7. Check for convergence for $\left\{\overline{N}, \overline{C}/\overline{Y}, \beta\overline{R}\exp\left(\bar{s}_1\right)\right\}$. If there is convergence, stop. Otherwise, return to Step 2.



(a) Baseline Calibration $\lambda = 11.3$



(b) Extreme Calibration $\lambda = 100$

Figure B1. *Accuracy of the Approximation*

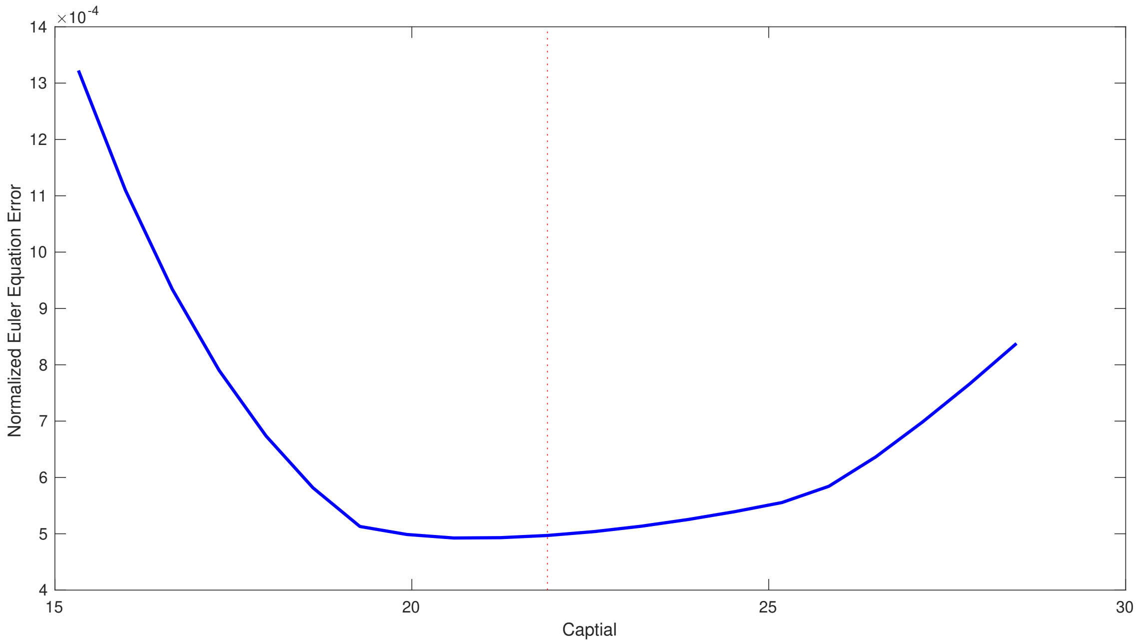


Figure B2. *Normalized Euler Equation Error as Functions of K*