Fiscal Cyclicality and Market Incompleteness

Andrés Fernández[1]
Central Bank of Chile

Ruy Lama
IMF

Carlos Vegh
Johns Hopkins University and NBER

Guillermo Vuletin
World Bank

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Abstract

Motivated by the fact that government spending and tax policy are procyclical in emerging and developing countries but acyclical/countercyclical in advanced economies, we develop a theory for the joint behavior of optimal (a la Ramsey) tax rates and government spending over the business cycle that relies on financial frictions, captured mainly by various degrees of asset market incompleteness. Departing from the complete markets framework delivers procyclical government spending but not necessarily procyclical tax policy. Procyclical tax policy requires that the ratio of private to public consumption comoves positively with the business cycle. Furthermore, for a given level of market incompleteness, fiscal procyclicality becomes more pronounced as the persistence and volatility of the business cycle increases. Our results hold both in a static model as a well as in a standard dynamic stochastic general equilibrium Ramsey setting. Theoretical results are validated empirically using a large sample of advanced and emerging economies.

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1 Introduction

There is, by now, overwhelming evidence that fiscal policy in developing countries is procyclical (i.e., expansionary in good times and contractionary in bad), compared to developed countries where fiscal policy is generally countercyclical.[2] Figure 1, updated from Reinhart *et al.* (2004), illustrates this stylized fact by plotting the correlation between the cyclical components of real government spending and real GDP for 105 countries (84 developing and 21 developed). Yellow (light) bars denote developing countries while black bars stand for developed countries. The visual message is striking: most of the yellow mass corresponds to positive values (indicating procyclical government spending) while most of the black mass corresponds to negative values (indicating countercyclical government spending). In fact, the average correlation for developing countries is 0.27 compared to -0.12 for developed countries (and both significantly different from zero at the 5 percent level).[3] While the evidence on the tax side has been more difficult to come by due to the need of collecting data on tax *rates* (the policy instrument), Figure 2, updated from Vegh and Vuletin (2015), establishes the procyclicality of tax policy. Indeed, the average correlation between changes in tax rates and real GDP for developing countries is -0.17 (and significantly different from zero at the 1 percent level) and -0.08 for industrial countries (and barely significant at the 10 percent level).[4]

Why would policymakers in developing countries conduct fiscal policy in a procyclical way? This is obviously puzzling since it amounts to making an already volatile business cycle even more pronounced. Furthermore, textbook Keynesian models (and more sophisticated ones as well) tell us that, in recessionary times, countercyclical fiscal policy is called for. A popular explanation for this procyclical puzzle relies on political-economy considerations.[5] In Tornell and Lane (1999), a positive shock may lead to a more-than-proportional increase

[2] For an early study, see Reinhart *et al.* (2004) and, more recently, Frankel *et al.* (2013) and the references therein.

[3] Of course, the question of causality is critical: is real GDP causing fiscal policy or viceversa? Ilzetzki and Vegh (2008) formally show that the causality from real GDP to government spending is indeed statistically significant.

[4] Section 4 shows that we can reject the null hypothesis that these two averages are the same. The sample in this case covers 69 countries.

[5] See, for example, Tornell and Lane (1999), Talvi and Vegh (2005), Alesina *et. al* (2008), and Ilzetzki (2011).

in spending due to various units (e.g., ministries or provinces) staking competing claims on available resources (akin to socially excessive fishing from a common pond). In Alesina *et al.* (2008), the fiscal authority is induced to spend more and/or tax less in good times to prevent it from appropriating rents (the classic starving the Leviathan argument).

The other key explanation that has been put on the table is imperfections in international capital markets. In an early, but unpublished, paper, Riascos and Vegh (2003) show that, in a calibrated model, incomplete markets can explain the procyclicality of government spending, but do not address the taxation side. Cuadra *et al.* (2010) show how the combination of incomplete markets with default risk leads to optimal procyclical fiscal policy (both on the spending and revenue sides). In turn, Bauducco and Caprioli (2014) introduce limited commitment in a small open economy model with exogenous government spending and show how this friction can lead to procyclical fiscal policy on the taxation side.

Our starting point is that while existing arguments based on imperfections in capital markets (like default risk and limited commitment) offer plausible and relevant insights into the procyclical fiscal policy puzzle, they miss a much more fundamental question: is there a role for standard incomplete markets (i.e., access to just a risk-free bond in an uncertain world) in explaining procyclical fiscal policy on both the spending and taxation sides? In fact, Cuadra *et al.* (2010) argue that incomplete markets *per se* cannot explain procyclical tax policy.[6] In other words, the literature has left the mistaken impression that incomplete markets may not be enough to generate procyclicality in both government consumption and tax rates.

In this paper, we go back to basics and examine what the workhorse model of a small open economy, operating under incomplete markets, has to say about procyclical fiscal policy. We quickly (even in a static setup) uncover a key and novel theoretical result: while incomplete markets are a sufficient condition to generate procyclical government spending, the same is *not* true of tax rates. In fact, under incomplete markets, tax rates may be acyclical, countercyclical, or procyclical depending on how the ratio of private to public consumption comoves with real GDP over the business cycle. Intuitively, if this comovement is positive,

[6] Specifically, they argue that "[i]n Riascos and Vegh the government can commit to pay its debt, so it faces the same interest rate across states. Since the government always borrows at the international risk free rate, the model is not able to generate a negative correlation between output and tax rates" (p. 455).

then in good times the tax base (consumption) is increasing more than government spending, which will induce the government to reduce tax rates (procyclical tax policy). Conversely, if this comovement is negative, the tax base is increasing less than government spending in good times, which will lead the fiscal authority to increase tax rates (countercyclical tax policy). We trace back this endogenous comovement to preference parameters over private and public consumption, which allows us to perform various experiments that shed further light on the mechanism involved. For expositional purposes, we will refer to this channel as the (private) "consumption preference channel."

Having used a static model to illustrate this novel result on the cyclicality of tax rates in the simplest possible setup, we then build a standard DSGE model with incomplete markets to examine whether the intuition built with the static model holds for an infinite-horizon model. Further, to understand the precise role of the degree of market completeness, we also examine the cases of financial autarky and complete markets. To get rid of the unit root that characterizes the basic theoretical small open economy model, we incorporate an upward-supply of funds that renders the model stationary and thus amenable to computer solutions. Interestingly, the intertemporal nature of the DSGE model brings a second channel into the picture: since households can borrow to smooth consumption in response to, say, a positive shock, then consumption (and thus the tax base) will increase less than otherwise, which will reduce the fiscal authority's need to reduce taxes (i.e., tax policy will be, all else equal, less procyclical/more countercyclical). For expositional purposes, we will refer to this second channel as the (private) "consumption smoothing channel."

How do these two channels interact in the DSGE model? The steeper the upward-sloping supply of funds, the less consumption smoothing will take place, and, hence, the more similar will be the results to the static case (i.e., the cyclicality of tax policy will essentially depend on the consumption preference channel). Conversely, the flatter the upward-sloping supply of funds, the more important the consumption smoothing channel becomes and, hence, the less relevant the consumption preference channel. For the more empirically-relevant specification (based on estimations of the debt elasticity), the consumption preference channel clearly dominates and hence our insights from the static model go through. We thus conclude that the cyclicality of tax policy will be essentially determined by the preference for private consumption relative to public consumption.

We then examine the role of the degree of persistence of TFP shocks. The main motivation is the work of Aguiar and Gopinath (2007) who suggest that cycles in emerging markets are more volatile because shocks are more persistent. The question is then: how does more persistence affect optimal tax policy? Our analysis clearly indicates that more persistence is associated with more procyclical tax policy. The intuition is simple enough: the more persistent are TFP shocks, the more consumption will have to react. Hence, in response to a negative TFP shock, for example, consumption will fall by more than if the shock were not as persistent, which reduces the tax base and forces the fiscal authority to increase taxes more. This is thus another channel that would make developing countries more procyclical.

Up to this point, the analysis will have illustrated the effect of various kinds of financial frictions on fiscal cyclicality, without inquiring how these dynamics may explain the actual data. To this effect, we conduct a matching moments exercise by calibrating the model for non-OECD countries. We conclude that the model does a remarkably good job in matching our four targeted moments: the standard deviations of output and private consumption, and the correlations between government spending and output and tax rates and output. In addition, the model matches very well the correlation between GDP and the ratio of private to public consumption, even though this was not a targeted moment.

We then offer a formal quantification of the welfare costs of fiscal procyclicality. As stressed by Reinhart et al. (2004), fiscal procyclicality makes an already volatile business cycle in emerging markets even more pronounced (the "when it rains, it pours" phenomenon). To our knowledge, however, no paper has yet provided an estimate of this cost. By calibrating the model for OECD and non-OECD countries using different persistence and volatility of TFP shocks, we conclude that Lucas-type welfare costs of business cycles are ten times larger in non-OCED than in OECD countries. While more volatile business cycles in emerging markets may reflect factors other than fiscal procyclicality, our analysis makes clear that higher output volatility leads to more fiscal procyclicality, which will in turn increase output volatility and hence the resulting welfare costs.

Finally, we offer empirical evidence in favor of (a) the results of the model and (b) the financial frictions that we emphasize in the theoretical and calibration analysis. In terms of the results of the model, we show that non-OECD countries exhibit statistically significant procyclical fiscal policy both on the spending and taxation sides. In terms of the financial frictions, we proxy market incompleteness with capital control measures and show that these are significantly higher in non-OECD than OECD countries. Further, we estimate the debt elasticities used in the model with two different proxies for debt and conclude that the debt elasticities are significantly higher in non-OECD than OECD countries. Finally, we show that GDP volatility is significantly higher in non-OECD than OECD countries. In sum, the empirical evidence is consistent with the idea that countries that exhibit procyclical fiscal policy are characterized by deeper financial frictions (i.e., more market incompleteness and higher debt elasticities) and display more output volatility.

The paper proceeds as follows. Section 2 develops the static model, which isolates the consumption preference channel. Section 3 turns to the DSGE model and focuses on how the consumption smoothing channel interacts with the consumption preference channel. Section 4 presents empirical evidence that supports our main findings. Section 5 offers concluding remarks.

2 A static model

To illustrate our main point, consider a simple, static, open economy model of optimal fiscal policy.[7] We will consider two different asset market structures: (i) incomplete markets and (ii) complete markets. The purpose of this model is to examine, in the simplest possible framework, how market incompleteness affects the cyclical behavior of fiscal instruments (i.e., government spending and tax rates) in response to output fluctuations. The key punchlines will be: (i) under complete markets, spending and tax policy are acyclical; and (ii) while market incompleteness *always* generates procyclical government spending, the same is *not* true of tax rates. In fact, the cyclical behavior of tax rates will depend on the preference for private consumption relative to public consumption.

[7] The reader is referred to Appendix A for detailed derivations.

2.1 Setup

Consider a small open economy perfectly integrated with the rest of worlds' goods markets. There is a single tradable good. Output is exogenous and uncertain, and follows the binomial distribution:

$$y = \begin{cases} y_H = \bar{y} + \gamma, \text{ with probability } 1/2, \\ y_L = \bar{y} - \gamma, \text{ with probability } 1/2, \end{cases} \tag{1}$$

where $\bar{y}$ and γ are positive parameters, and H and L denote the high and low output states of nature, respectively. Since $E(y) = \bar{y}$ and $V(y) = \gamma^2$, an increase in γ represents a mean-preserving spread.

Following Baxter and King (1993), we assume that households' preferences are separable in private and public consumption:

$$U(c_i, g_i) = \begin{cases} \underset{i=H,L}{E}\left[\alpha \frac{c_i^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha)\frac{g_i^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}}\right], & \sigma_c \neq 1 \text{ and } \sigma_g \neq 1, \\ \underset{i=H,L}{E}\left[\alpha \ln(c_i) + (1-\alpha)\frac{g_i^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}}\right], & \sigma_c = 1 \text{ and } \sigma_g \neq 1, \\ \underset{i=H,L}{E}\left[\alpha \frac{c_i^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha)\ln(g_i)\right], & \sigma_c \neq 1 \text{ and } \sigma_g \neq 1, \end{cases} \tag{2}$$

where σ_c and σ_g denote the relative preference for private and public consumption, respectively, which will determine how the ratio of private to public consumption correlates with output. As will become clear below, the relative size of σ_c and σ_g will be crucial for our results.

The household's budget constraint in each state of nature is given by

$$y_i = (1 + \tau_i)c_i, \qquad i = L, H, \tag{3}$$

where τ_i is a consumption tax.[8]

The government's budget constraints are thus

[8] As shown in Appendix A, in this simple world the results would be identical if we assumed an output (endowment) tax.

$$g_i = \tau_i c_i, \qquad i = L, H. \tag{4}$$

Combining the household's constraints, given by (3), with the government's, given by (4), yields the economy's resource constraints:

$$y_i = c_i + g_i, \qquad i = L, H. \tag{5}$$

For the sake of tractability, we will consider two polar cases in terms of asset market completeness: financial autarky and complete markets. We begin with the extreme case of financial autarky (i.e. full absence of financial instruments). Before proceeding, notice that we can solve this problem as a social planner because the consumption tax does not distort intertemporally (i.e., the model is static) or intratemporally (i.e., there is no labor/leisure choice). Once we have solved for the social planner's optimal allocation, we can use the government's constraints, given by (4), to recover the optimal consumption tax rates.[9]

In the financial autarky case, the planner's problem consists in choosing $\{c_H, c_L, g_H, g_L\}$ to maximize households' utility, given by (2), subject to the economy's resource constraints. As shown in Appendix A, at an optimum, the marginal utilities of private and public consumption will be the same in each state of nature:

$$U_{c_i}(c_i, g_i) = U_{g_i}(c_i, g_i), \;\; i = H, L. \tag{6}$$

In the complete markets case, the economy may buy/sell contingent claims that promise to deliver one unit of output if states H and L occur for a price q_H and q_L, respectively.[10] Prices are assumed to be actuarially fair, which implies that

$$\frac{q_H}{q_L} = \frac{p}{1-p}.$$

[9] We should note that, for simplicity, we will assume that initial assets are zero. As shown in Appendix A, this assumption is not without loss of generality in the case of financial autarky (i.e., initial assets matter for the degree of procyclicality of tax policy, though not for spending policy). The role of initial assets will be explored formally in the context of the dynamic model of Section 3.

[10] The state contingent bonds are intra-period; that is, they are purchased at the beginning of the period, before the shock materializes, and the households receive the pay-off at the end of the period, once the shock materializes.

The economy's resource constraint is thus

$$q_H y_H + q_L y_L = q_H(c_H + g_H) + q_L(c_L + g_L). \tag{7}$$

The social planner chooses $\{c_H, c_L, g_H, g_L\}$ to maximize households' utility, given by (2), subject to constraint (7). In addition to condition (6), it is also the case that

$$\begin{aligned} U_{c_H}(c_H, g_H) &= U_{c_L}(c_L, g_L), \\ U_{g_H}(c_H, g_H) &= U_{g_L}(c_L, g_L). \end{aligned}$$

In other words, the marginal utilities of private and and public consumption are equalized across states of nature (which, by definition, implies full risk sharing under complete markets). Since the utility function is separable, these two optimality conditions imply that $c_H = c_L$ and $g_H = g_L$. The latter implies that government spending is the same across states of nature regardless of the relation between σ_c and σ_g. We now fully characterize the properties of fiscal policy across states of nature.

2.2 Cyclical properties of fiscal policy

Let θ_g and θ_τ capture, respectively, the cyclicality of government spending and tax rates:

$$\begin{aligned} \theta_g^X &\equiv \ln\left(\frac{g_H}{g_L}\right), \\ \theta_\tau^X &\equiv \ln\left(\frac{\tau_H}{\tau_L}\right). \end{aligned}$$

where $X \equiv CM$ in the case of complete markets and $X \equiv FA$ in the case of financial autarky. A positive (negative) value of θ_g; that is, $g_H > g_L$ ($g_H < g_L$), would indicate that government spending is procyclical (countercyclical). If $g_H = g_L$, then $\theta_g = 0$, which implies acyclical government spending. A positive (negative) value of θ_τ; that is, $\tau_H > \tau_L$ ($\tau_H < \tau_L$), would indicate that tax rates are countercyclical (procyclical). If $\tau_H = \tau_L$, then $\theta_\tau = 0$, implying acyclical tax policy.

We are now ready to fully characterize the cyclical properties of optimal fiscal policy in the static model under complete and incomplete markets (i.e., financial autarky). We will do so by establishing the following two propositions.

Proposition 1 *Government spending is acyclical under complete markets (i.e., $\theta_g^{CM} = 0$) and is procylical under financial autarky ($\theta_g^{FA} > 0$) regardless of the values of σ_c and σ_g.*

Proof. See Appendix A. ■

Proposition 2 *Tax rates are acyclical under complete markets (i.e., $\theta_\tau^{CM} = 0$). Under financial autarky, the cyclicality of tax rates depends on the relative values of σ_c and σ_g. Tax rates are acyclical ($\theta_\tau^{FA} = 0$) if $\sigma_c = \sigma_g$, countercyclical ($\theta_\tau^{FA} > 0$) if $\sigma_c < \sigma_g$, and procyclical ($\theta_\tau^{FA} < 0$) if $\sigma_c > \sigma_g$.*

Proof. See Appendix A. ■

The key and novel results of this section are thus captured by Propositions 1 and 2, which establish that market incompleteness (as captured in this instance by financial autarky) is a sufficient condition for government spending to be procyclical. In sharp contrast, market incompleteness is neither a necessary nor a sufficient condition for tax policy to be procyclical. In fact, acyclicality, procyclicality, and countercyclicality are all possible depending on the relative values of $\sigma_c > \sigma_g$. In sum, market incompleteness does not tell us anything, in principle, about the cycliclality of optimal tax policy.

The role of the relative values of σ_c and σ_g becomes clear when we realize that these parameters will determine how the ratio g/c moves over the business cycle. In order to understand precisely the role of these two parameters, we can use (**??**) and rewrite θ_τ as follows:

$$\theta_\tau \equiv \ln\left(\frac{\tau_H}{\tau_L}\right) = \ln\left(\frac{g_H/c_H}{g_L/c_L}\right). \tag{8}$$

Therefore, the tax rate cyclicality is tightly linked to the optimal ratio g/c across states of nature which, in turn, is determined by σ_c and σ_g:

- When $\sigma_c = \sigma_g$, then g/c is constant across states of nature (i.e., $g_H/c_H = g_L/c_L$), and hence $\tau_H = \tau_L$. Since private and public consumption are equally valued, the two types of consumption increase by the same proportion in the good state of nature. The higher tax base (i.e., the higher private consumption) thus enables the fiscal authority to leave the tax rate unchanged and still finance the higher government spending.

- When $\sigma_c < \sigma_g$, then $g_H/c_H > g_L/c_L$, and hence $\tau_H > \tau_L$. Since households have stronger preferences for public than private consumption, then private consumption increases proportionally less than public consumption in the good state of nature. Hence, if the fiscal authority kept the same tax rate, tax revenues would fall short. The fiscal authority needs to increase the tax rate to finance the higher public consumption ($\tau_H > \tau_L$), thus engaging in countercyclical tax policy

- When $\sigma_c > \sigma_g$, then $g_H/c_H < g_L/c_L$, and hence $\tau_H < \tau_L$. Since households' preferences are stronger for private than public consumption, then c increases proportionately more than g in the good state of nature. The relatively higher tax base allows the fiscal authority to reduce the tax rate and still finance the higher public consumption (i.e., $\tau_H < \tau_L$; procyclical tax rates), thus engaging in procyclical fiscal policy. We will refer to this channel as the (private) "consumption preference channel." In other words, the households' stronger preference for private than public consumption leads to procyclical tax rates.

It will thus be of interest to relate the degree of cyclicality of tax rates in the data to the dynamics of the ratio of public to private consumption over the business cycle. We will do so in the next section in the context of a fully-calibrated dynamic model.

Finally, the degree of asset market completeness will also matter for the extent to which higher income volatility affects the cyclicality of fiscal policy. The next proposition formalizes this.

Proposition 3 *Under financial autarky, the procyclicality of government spending increases with output volatility (i.e., $(d\theta_g^{FA}/d\gamma) > 0$). When tax rates are procyclical (i.e., $\sigma_c > \sigma_g$), then tax procyclicality increases with output volatility ((i.e., $(d\theta_\tau^{FA}/d\gamma) < 0$). When tax rates are countercyclical (i.e., $\sigma_c < \sigma_g$), then tax countercyclicality also increases with output volatility (i.e., $(d\theta_\tau^{FA}/d\gamma) > 0$). When tax rates are acyclical (i.e., $\sigma_c = \sigma_g$), then output* *volatility has no effect (i.e.,* $d\theta_{\tau}^{FA}/d\gamma) = 0$*). Under complete markets, output volatility does not affect the cyclicality of either government spending or tax rates.*

Proof. See Appendix A. ■

In sum, the simple model developed in this section has enabled us to characterize fiscal policy in a tractable way while relating it to the extreme cases of asset market completeness (complete markets and financial autarky). The natural question, to be explored next, is whether the key theoretical insights that we have derived in the context of this simple and static model remain valid in a much richer and more realistic modeling environment.

3 A DSGE model

This section further explores the cyclical implications of optimal (Ramsey) fiscal policy in both tax rates and public spending and their relationship with the degree of asset market completeness in open economies, but in a richer environment (i.e., the current DSGE models used for the analysis of business cycles in small open economies). Agents will now operate within an infinite horizon setting, production will be endogenous, and households will choose labor supply optimally. An important new feature is that households will now be able to smooth consumption (which we will refer to as the "consumption smoothing channel") by issuing risk-free debt in international markets, which will have a direct effect on the cyclicality of fiscal policy. While we will continue to explore the degree of asset completeness in debt markets, the presence of debt opens up the possibility to study the presence of a different, but related, type of financial friction: the existence of an upward sloping supply of external savings. We will also explore the extent to which the slope of this supply curve – a proxy for other kinds of financial frictions – affects the cyclicality of fiscal policy in response to productivity shocks. Lastly, we will explore how higher output volatility, driven by more persistent TFP shocks, influences the cyclicality of fiscal policy.

Despite the richness of the new environment, results will show that the main take-aways from the model continue to be those from the much simpler static one in the previous section. In other words, financial frictions will be positively correlated with fiscal procyclicality, though results will differ between government spending and tax rates insofar as tax policy cyclicality will also depend on the preference of private consumption relative to public consumption.

3.1 Setup

3.1.1 Households

Households maximize the expected present value of utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t), \tag{9}$$

where c_t, g_t, l_t, and β denote, respectively, private consumption, public consumption, labor supply, and the discount factor.

In what will be our benchmark case henceforth, households will have access to international markets by issuing one period non-state contingent bonds. This assumption will be later relaxed when we consider, as in the previous static framework, the polar cases of financial autarky and complete markets.

The representative household's budget constraint is given by:

$$d_t = R_{t-1} d_{t-1} + \theta_t c_t - y_t, \tag{10}$$

where d_t is the stock of private debt at the end of period t; R_t is the (gross) interest rate of debt contracted in period $t-1$ and repaid in period t; and θ_t is the (gross) tax rate on consumption (and equal to $1+\tau_t$).[11] Lastly, output y_t, is given

$$y_t = A_t l_t, \tag{11}$$

where A_t is a stochastic productivity factor.

[11] Labor income taxation would deliver similar, though not identical, results because the distortions introduced are not exactly the same as in the previous section.

3.1.2 Real interest rates

The gross international real interest rate faced by households, R_t, is assumed to be equal to the stochastic gross world real interest rate (R_t^*) and an endogenous risk premium, S_t, that depends on the stock of debt:

$$\begin{aligned} R_t &= R_t^* S_t, \\ S_t &= 1 + p\left(\widetilde{d}_t\right). \end{aligned} \tag{12}$$

We follow Schmitt-Grohé and Uribe (2003) and assume that $p(\cdot)$ is a country-specific interest rate premium, and $\widetilde{d}_t$ is the aggregate level of foreign debt which, in equilibrium, is equal to household's debt. The functional form that we use for $p(\cdot)$ also follows their work:

$$p(d) = \psi^c \left(e^{d-\overline{d}} - 1\right), \tag{13}$$

where ψ^c and $\overline{d}$ are parameters.

Note that ψ^c governs the elasticity of the spread to changes in private debt (i.e., the slope of the supply of external funds). In our benchmark case, we will follow Schmitt-Grohé and Uribe (2003) and consider small values of ψ^c so as to render the model stationary. As explained in more detail below, we will nonetheless consider also deviations from this level as an additional source of financial frictions (García-Cicco *et al.,* 2010; Chang and Fernández, 2013; and Fernández and Gulan, 2015).

3.1.3 Government

The government's flow budget constraint is given by

$$d_t^g = R_{t-1}^g d_{t-1}^g + g_t - (\theta_t - 1)c_t, \tag{14}$$

where d_t^g is the stock of public debt.

We will further assume that a similar debt elastic interest rate premium applies to government debt; that is, $R_t^g = R_t^* S_t^g$, where $S_t^g = 1 + p\left(d_t^g\right) = \psi^g \left(e^{d_t^g - \overline{d}^g} - 1\right)$ and ψ^g and

$\overline{d}^g$ are parameters.

3.1.4 Driving processes

There are two independent sources of uncertainty modeled as stochastic driving forces. The first one is for TFP:

$$\ln\left(A_t/\bar{A}\right) = \rho_A \ln\left(A_{t-1}/\bar{A}\right) + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID(0, \sigma_A^2), \tag{15}$$

where $\bar{A}$ is TFP in the steady state. The second one is for the world real interest rate:

$$\ln\left(R_t^*/\overline{R}^*\right) = \rho_R \ln\left(R_{t-1}^*/\overline{R}^*\right) + \varepsilon_t^R, \quad \varepsilon_t^A \sim NIID(0, \sigma_R^2), \tag{16}$$

where $\overline{R}^*$ is the non-stochastic real interest rate.

3.1.5 Ramsey problem

The Ramsey planner maximizes the welfare of the representative agent (9), subject to the private and public budget constraints, (10) and (14), and the implementability constraints from the household's problem:

$$\begin{aligned} \Gamma_t &= A_t, \\ \lambda_t &= \beta R_t E_t \lambda_{t+1}, \end{aligned}$$

where $\Gamma_t(c_t, g_t, l_t, \theta_t)$ is the marginal rate of substitution between consumption and leisure and $\lambda_t = h(c_t, g_t, l_t, \theta_t)$ is the shadow price of wealth (further details can be found in Appendix B).

3.2 Financial frictions

As in the static case, we will study the effects of varying the level of market completeness. This is captured schematically in Figure 3. In addition to the benchmark case (point B in Figure 3), we will explore the two extremes polar cases of financial autarky (point A) and complete markets (point C).

Following Uribe and Schmitt-Grohé (2003), in the case of complete asset markets, agents have access to a complete array of state-contingent claims:

$$E_t r_{t+1} b_{t+1} = b_t + y_t - \theta_t c_t, \tag{17}$$

where b_{t+1} is a random variable that captures the number of assets purchased in t to be delivered in each state of period $t+1$ and r_{t+1} denotes the period-t price of an asset that pays one unit of good in a particular state of period $t+1$ divided by the probability of occurrence of that state given information available in period t. A no-Ponzi-game constraint exists, given by $\lim E_t q_{t+j} b_{t+j} \geq 0$ as $j \to \infty$, for all dates and for all contingencies, where $q_t = r_1 \cdot r_2 \cdot ... \cdot r_t$, with $q_0 \equiv 1$.

On the other hand, the case of financial autarky assumes that neither households nor the government can buy or sell financial securities from/to the rest of the world. In other words, assets can only be exchanged within the country. Formally, then, the following general equilibrium condition must be added for this case:

$$d_t + d_t^g = 0 \quad \forall t. \tag{18}$$

Furthermore, in this case the real interest rate R is no longer an exogenous variable as in (12), (13), and (16), and instead adjusts endogenously to ensure that condition (18) is satisfied.

The second type of financial friction, illustrated schematically by the vertical axis in Figure 3 and captured in reduced form by (13), is a debt-elastic risk premium, with ψ^c and ψ^g denoting the two elasticities considered. Larger values of this elasticity, which imply a stronger response of the real interest rate paid by the country to the country's stock of debt, appear to be empirically necessary to bring DSGE models closer to emerging markets data (Garcia-Cicco et.al, 2010; Chang and Fernandez, 2013).

While in the limit (i.e., as ψ^c and ψ^g tend to ∞), the model with debt-elastic interest rates collapses to the financial autarky case (i.e., the vertical axis in Figure 3 rotates to the left eventually converging to the extreme left of the horizontal arrow), the two types of financial frictions are not completely isomorphic. The existence of debt-elastic real interest rates requires at least *some* level of asset market incompleteness, but higher levels of this elasticity do not necessarily imply more incompleteness. Fernández and Gulan (2015), for instance, provide microfoundations for the debt elasticity parameter in an environment where private debt is defaultable due to asymmetric information between domestic entrepreneurs and external lenders, holding the level of asset market incompleteness constant. This warrants a separate analysis of these two types of frictions.

A third and final dimension that we will study within this richer setup is the presence of varying degrees of persistence in the TFP process, A_t, as illustrated by the diagonal axis in Figure 3. Motivated by the observation that small emerging economies have more volatile GDP, Aguiar and Gopinath (2007) hypothesized that such phenomenon could be related to higher persistence in TFP which, in turn, captures larger frictions (whether of financial nature or not), such as frequent changes in regimes in fiscal, monetary, and trade policies. Our goal will be to study the extent to which a more persistent TFP process delivers fiscal procyclicality when the two kinds of financial frictions that we consider coexist.

3.3 Calibration

Following Baxter and King (1993), we continue to use separable preferences:

$$U(c,g,l) = \frac{(c)^{1-1/\sigma_c} - 1}{1 - 1/\sigma_c} + \frac{(g)^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \log(1-l)$$

The calibration of the various parameters in the DSGE model is summarized in Table 1. We mostly rely on previous studies of small open economies. Without loss of generality, the steady-state level of TFP ($\bar{A}$) is normalized to one. Following Schmitt-Grohé and Uribe (2003), the steady-state level of the international real interest rate ($\overline{R}^*$) is set to 4 percent on an annual basis. The steady-state ratio of total debt (public and private) to (quarterly) income is set to 1.34, using Lane and Milesi-Ferretti's (2007) data on net foreign assets for the non-OECD countries in Figures 1 and 2.

The persistence of the TFP process, governed by the AR(1) coefficient ρ_A is set to 0.95 in our benchmark case, taken from Neumeyer and Perri (2005). The volatility of the shock to this process will vary throughout our various experiments, including one where we calibrate it so as to match certain moments in the data, but in our benchmark case it will be set to 0.0129, also taken from Schmitt-Grohé and Uribe (2003). The values chosen for ρ_{R^*} and σ_{R^*}, which govern the persistence and volatility of the real interest rate process, come from Uribe and Yue (2006) and are set to 0.83 and 0.007, respectively.

As usual, the discount factor (β) is calibrated as the inverse of the gross international real interest rate. In our baseline case of incomplete markets, the parameters capturing the debt elasticity of the real interest rate in both private and public debt, ψ^c and ψ^g, will be set to 0.125, using the estimated value to be discussed later in the empirical section. However, in the various experiments that we will conduct, the values considered for these parameters will change so as to capture varying degrees of financial frictions, and also consider alternative values that aim at better matching some of the moments in the data.

Last, but not least, we need to calibrate the parameters governing the intertemporal elasticities of substitution for private (σ_c) and public consumption (σ_g). From the static model, we know that the relative values chosen for these parameters are crucial for determining the cyclicality of fiscal policy. We will therefore normalize σ_c to 1 and consider a range of values for σ_g from lower to higher than σ_c.[12]

3.4 Results

This section presents the quantitative results from the DSGE model. We study, separately, the effects of market incompleteness and varying debt elastic spreads on the optimal path of fiscal policy as it reacts to exogenous TFP disturbances. We do this through the analysis of second moments, simulations, and impulse response functions. In addition, we explore the fiscal consequences of higher income volatility resulting from more persistent TFP shocks. Lastly, we present an exercise that measures the performance of the DSGE model in matching some of the key moments in the data. All results come from a first-order Taylor approximation of the model around its non-stochastic steady state.

[12] It is worth pointing out that the calibration of σ_c and σ_g not only has relevance for the cyclical dynamics of fiscal policy, but also for the steady state of the model. A calibration where $\sigma_c > \sigma_g$ ($\sigma_c < \sigma_g$) delivers a ratio of private-to-public consumption, c/g, that is lower (higher) than one, and hence tax rates that are relatively higher (lower).

3.4.1 Second moments

We begin by exploring the two main second moments in our analysis: the contemporaneous correlations of government consumption (g) and tax rates (τ) with income (y). The key role played by the elasticities σ_g and σ_c in determining these moments and, thus, on the degree of fiscal procyclicality are illustrated in Figures 4 and 5 for, respectively, the cases when the debt elasticity (ψ^c and ψ^g) varies and for different degrees of market incompleteness.

When varying the debt elasticity in Figure 4, in addition to our benchmark calibration for $\psi^c = \psi^g = 0.125$, we consider four alternative values: two are lower than our calibrated benchmark ($\psi^c = \psi^g = 0.01$, $\psi^c = \psi^g = 0.001$), while the remaining two are higher ($\psi^c = \psi^g = 1, \psi^c = \psi^g = 2.8$). The lowest value considered (0.001) can be thought as the minimum level needed to render the model stationary in the process of debt and hence amenable to computing second moments, while the highest value comes from the estimated value for Argentina in Garcia-Cicco *et al.* (2010).

The main message from Figure 4 is essentially the same derived in the static model of Section 2 (recall Propositions 1 and 2): fiscal procyclicality in government spending does not depend on the relative values of σ_g and σ_c, but the same is *not* true of tax rates, for which the relative values of these parameters are crucial in determining the sign of the correlation. Indeed, as shown in the top panel, for the entire range considered for σ_g (recall that σ_c is fixed at 1), between 0.5 and 1.5, the correlation between g and y lies above 0.95 for all values of ψ^c and ψ^g considered. In contrast, as shown in the bottom panel, the sign of the correlation between τ and y changes drastically from -1 to 1 as σ_g increases. For example, in the extreme case where $\psi^c = \psi^g = 2.8$, and the economy is basically in financial autarky as it is too costly to issue debt (a case that will be explored in greater detail next), the results are identical to the static model: tax rates behave in a procyclical manner for σ_g less than σ_c (which is fixed at 1) while they behave countercyclically when the opposite occurs. Put differently, the consumption preference channel identified in the static model dominates the consumption smoothing channel.

An additional, and novel, feature of the correlation between τ and y that follows from Figure 4 is that the change in the sign of such correlation depends also on ψ^c and ψ^g. Indeed, as we move away from the polar case of $\psi^c = \psi^g = 2.8$ and the elasticity falls, so does the cut-off value of σ_g for which taxes change from being procyclical to countercyclical. The intuition for this result stems from the fact that agents in this dynamic model can issue debt in order to smooth out the effect of shocks on their consumption path, a mechanism that, by construction, was absent in the static case previously considered. For a given σ_g, as agents are confronted with a negative TFP shock, the cheaper it is for them to issue debt (i.e. ψ^c and ψ^g fall), the more the effect of the TFP shocks on consumption will diminish and so will the effect on the tax base, reducing the need for taxes to increase in a procyclical manner. The latter will occur only for cases when the degree of intertemporal substitution of government consumption falls (i.e. σ_g low).

Figure 5 reports the correlations between public consumption and output (top panel) and tax rates and output (bottom panel) for the three cases of market incompleteness considered (financial autarky, incomplete markets, and complete markets). The less complete markets are, the higher the correlation between output and public consumption, regardless of the value of σ_g: public consumption is uncorrelated with output under complete markets and perfectly correlated under financial autarky. As in the static framework, results cannot be extrapolated for the correlation between tax rates and output (bottom panel) where the sign of such correlation depends on the value of σ_g, more so as markets are less complete. In the extreme case of financial autarky, such correlation turns around completely from -1 to 1 as σ_g crosses 1 (note how similar this case is to the one in Figure 4 for $\psi^c = \psi^g = 2.8$).

We report other second moments of the model in Table 2. In addition to the two moments already analyzed – correlation of tax rates and government spending with output – the table shows the standard deviation of these three variables and consumption and the correlation of the public spending ratio with output. The table reports the moments for the three cases of market completeness considered and the various possible calibrations of σ_g considered.

The case of market completeness is trivial as all moments (except income's variability) are zero, regardless of the calibration of σ_g as consumption, public spending and, hence, tax rates, are perfectly smoothed thanks to complete markets.[13] For the case of financial

[13] Note that, for the case of complete markets, the table presents covariances instead of correlations, as the

autarky, as expected from the discussion above, the moments vary considerably depending on the calibration of σ_g. Low (high) values of this elasticity relative to σ_c render private consumption more (less) volatile, while public consumption becomes less (more) volatile. This, in turn, implies that the ratio c/g continues to commove positively (negatively) with income only if $\sigma_g \leq 1$ ($\sigma_g > 1$). The case of incomplete asset markets is an intermediate case, though closer to the case of financial autarky given the relatively high spread elasticity considered ($\psi^c = \psi^g = 0.125$).

A final look at the dynamics of fiscal variables is explored in the simulation presented in Figure 6, which illustrates the deviations from the steady state for the key variables in the model from randomly drawing TFP shocks during 40 quarters. The particular pattern of the draws is such that roughly in the first half of the period a sequence of positive TFP shocks takes place while the opposite is true towards the end of the sample (Panel A). The path of government spending (Panel C) shows an increase in the period of positive TFP shocks and a fall towards the end of the sample, indicating a procyclical behavior. Private consumption comove positively with public consumption (Panel B). As expected, the path of tax rates (Panel D) critically depends on whether σ_g is above or below $\sigma_c = 1$: (i) when $\sigma_g > \sigma_c$, the path of tax rates is procyclical (i.e., tax rates co-move negatively with GDP); (ii) when $\sigma_g < \sigma_c$, the path of tax rates is countercyclical. (i.e., tax rates comove positively with GDP); and (iii) when $\sigma_g = \sigma_c = 1$, the path of tax rates is acyclical. Such simulations were conducted for the extreme case where $\psi^c = \psi^g = 2.8$.

3.4.2 Impulse responses

A complementary analysis of fiscal policy is conducted by means of impulse response functions (IRFs) following a fall of TFP by one percentage point relative to the steady state. Figures 7A, 7B, and 8 show the IRFs under different degrees of asset market completeness (Figures 7A and 7B) and varying debt elasticities of spreads (Figure 8). The figures present the dynamics of the key variables in the model: productivity, private consumption, public consumption, output, tax rates, private debt, tax rates, real interest rates, and private and

latter cannot be computed.

public debt.[14] Units are in percentage deviations from steady state levels.

The columns of Figures 7A and 7B illustrate the three cases considered in terms of asset market completeness: the left-most column is the case of financial autarky, the middle column is the benchmark case (i.e., one non-state contingent bond) and the right-most column is the case of complete markets. Focusing first on the top row of the figures that deals with the case of $\sigma_g = 0.5$, the key result is that, again, going from complete markets to financial autarky increases the procyclicality of both public expenditures as well as tax rates. In response to a fall of TFP by 1 percent below steady state, under financial autarky tax rates increase and public consumption falls (Figure 7A). On the other hand, under complete markets (right column), neither public nor private consumption and, thus, tax rates react as debt increases automatically given that debt is no longer a state variable. In the case of incomplete markets (middle column), debt is now a state variable and hence does not react following the shock. However, next period debt increases in order to smooth the effect on consumption of the shock (Figure 7B). Note that under the benchmark calibration, issuing debt is relatively expensive (high ψ^c and ψ^g) and the shock is quite persistent (high ρ_A) so that the amount of debt issued following the shock does not help much in smoothing its effect, delivering a result that is quantitatively similar to that of financial autarky (Figure 7A). Note also that, under financial autarky, the government borrowing needed to pay for public consumption, in addition to higher tax rates, pushes the domestic real interest rate up (Figure 7B).[15]

The second and third rows of Figures 7A and 7B illustrate the effects of different values of σ_g, the only parameter that varies as we move across rows, with the second row showing results for $\sigma_g = 1$, and the third for $\sigma_g = 2$. In the financial autarky case, left column (when $\sigma_g = \sigma_c = 1$), the fall of public and private consumption is identical and, in turn, the

[14] Notice that in the financial autarky case, there is a single (domestic) real interest rate, whereas in the incomplete markets case, the real interest rate faced by the private and public sector are not the same.

[15] The considerable increase of the (now endogenous) interest rate in the financial autarky model following a negative TFP shock is explained by the outward shift in the deamand for debt by the government. This effect is stronger the lower is the elasticity of substitution of public consumption. Being debt a state variable, on impact what clears this market is the spike in the interest rate. Such large volatility in the interest rate can be curbed to more realistic values of interest rates by increasing ϕ, the parameter that governs the cost of adjusting debt. We have verified this claim in additional simulations -not reported but available upon request-.

same as output. This implies that tax rates do not need to change and become acyclical. The same is essentially true in the incomplete markets case due to debt being relatively expensive. When σ_g is twice as large as σ_c, the Ramsey planner can actually use tax rates in a countercyclical way because the fall in public consumption is larger than that in private consumption. Quantitatively similar results are observed under incomplete markets. Lastly, as expected, increases in σ_g do not have any relevance in the complete markets case as neither of the two types of consumption react.

The effects of increasing the spread elasticity are illustrated in the first row of Figure 8, comparing the benchmark case (middle column) with one where ψ^c and ψ^g increase to 2.8 (left column) and another where ψ^c and ψ^g fall to 0.001 (right column). As it becomes more costly to issue debt, a negative TFP shock has larger negative effects on private and public consumption. For relatively low values of σ_g, as is the case of the first row in Figure 8, tax rates become procyclical because the effects on public consumption are smaller than on private consumption. The opposite occurs, however, when σ_g increases, as in the second and third rows, to the point that taxes become countercyclical (i.e., they fall as the recession takes place). Public consumption always responds procyclically by falling relative to its steady state.

3.4.3 Output volatility

As mentioned above, higher persistence in the TFP residual is viewed as a potential source of the higher output volatility observed in EMEs (Aguiar and Gopinath, 2007). Motivated by this hypothesis, we now study the extent to which changes in the persistence of this shock affect the implications of financial frictions on fiscal procyclicality analyzed above.

Figure 9 illustrates the IRFs following a similar fall in TFP to the one considered thus far – 1 percentage point below the steady state – under the three alternative cases of market completeness that we have studied, focusing on the case where the persistence of the TFP process, captured by the AR(1) coefficient, falls to 0.42 as in Mendoza (1991). The results are shown in the plots on the lower row in the figure and, for comparison, the benchmark cases are reported on top. Such an increase implies that the half life of a TFP shock falls from close to seven quarters to less than a quarter, the volatility of the Solow residual decreases from 1.7 to 1.3 percent, and the standard deviation of income falls from 2.9 to 2.5 percent in our benchmark case.

Overall, the lower persistence reduces the tendency for financial imperfections to increase the procyclicality of fiscal policy. The mechanism is simple and intuitive: as the shock becomes less persistent, agents react less vigorously in terms of private consumption, calling for a less pronounced fiscal response, i.e. a reduction in public consumption and an increase in tax rates. The exception is, of course, the case of complete markets (right column) where the shock does not have any effect except a larger increase in debt. This contrasts with the case of financial autarky (left column) where the increase in taxes is considerably less persistent following the shock[16]. For the case of market incompleteness, results on public spending and tax rates change drastically relative to the benchmark case as the persistence of the shock diminishes (centered subplot in middle column). The strong procyclical tax rates largely vanishes. However, when we interact the fall in persistence with a higher spread elasticity (bottom plot, middle column), the procyclicality in tax rates is restored and tax rates increase in the wake of the shock.

The role of persistence in TFP is further illustrated in Figure 10, which focuses on the two key correlations between public consumption and tax rates with output, and reports their values as the AR(1) coefficient in the TFP process increases from 0.4 to near 1 (from below). Results are qualitatively robust for the correlation of government expenditures and income (top panel) where all cases considered display a positive correlation regardless of the persistence in the shock.

Regarding the correlation between tax rates and income (bottom panel), for the intermediate case of incomplete markets, we see that, as persistence decreases, the degree of procyclicality falls. For relatively low levels of debt elasticity ($\psi = 0.125$), the lack of persistence of the shock is enough for consumption not to drop as much, which leads to countercyclical tax rates (i.e., the correlation turns positive). Results are also affected for the extreme case of financial autarky where lower persistence in the shock turns tax rates countercyclical. As mentioned above in the context of Figure 9 (see previous footnote), this

[16] In fact, in the Financial Autarky case, tax rates fall on impact. This is linked to the increase in the interest rate which provides income to the government who is a net creditor in steady state.

is linked to the increase in the interest rate which provides income to the government who is a net creditor in steady state, thereby implying less need to resort to higher tax rates to support public consumption.

3.4.4 Moment matching

The analysis presented thus far has illustrated the effect of various kinds of financial frictions on fiscal cyclicality, without asking how these dynamics account for those observed in the data. Matching the data, however, is important to discipline the calibration of the model. In the static model, for example, we showed how the behavior of the ratio c/g was relevant for pinning down the relative values of σ_g and σ_c which, in turn, determines the degree of procyclicality of tax rates.

This subsection addresses the issue of data matching by calibrating some of the key parameters in the DSGE model to match the most salient moments in the data. The four parameters to be calibrated are: (i) the volatility and persistence of TFP shocks (σ_A and ρ_A, respectively), (ii) the intertemporal elasticity of substitution for government consumption (σ_g), and (iii) the debt elasticity of the spread ($\psi^c = \psi^g = \psi$). The targeted moments in the data are the standard deviations of income and private consumption (σ_y and σ_c, respectively) and the correlations of taxes and public spending with real GDP ($\rho_{\tau,y}$ and $\rho_{g,y}$, respectively}.

The calibration procedure consists of the following steps. First, a grid is defined over all the possible combinations of values that the four parameters to be calibrated can take.[17] Second, the model is simulated for each point of the grid (based on random TFP and world real interest rate shocks) and the four targeted moments are computed using the HP-filtered series. Third, a quadratic loss function – defined as the sum of the (percentage) squared deviations of simulated (HP-filtered) moments from their empirical counterparts – is minimized over a four-dimensional grid defined for each of the parameters. Finally, we select the combination of values of $\{\sigma_A, \rho_A, \sigma_g, \psi\}$ for which the loss function takes its minimum value. We use the data for non-OECD countries (see Figures 1 and 2) and restrict the sample to balanced panel observations (within country) of more than ten annual consecutive

[17] The rest of the parameters are taken from Table 1.

observations.

The results of this exercise are reported in Table 3. The first column shows the second moments in the data. The second column reports the results from the simulated moments. The model performs rather well, considering how parsimonious it is. It captures the higher volatility of private consumption relative to that of output, as well as the positive correlation of government spending with output (procyclical government spending) and the negative correlation of tax rates with output (procyclical tax policy). Remarkably, the model matches the comovement between the ratio c/g and output, a moment that the exercise was not targeted to match. As shown in Table 3, the good performance of the model is based on a relative inelastic government consumption intertermporal elasticity ($\sigma_g = 0.25$), relatively high debt elasticity of the spread ($\psi = 1$), and persistent TFP shocks ($\rho_A = 0.95$).

The model, however, is too parsimonious to account for the high volatility of government spending, which is about one tenth of that in the data. It also accounts for only a fifth of the observed volatility in taxes. Clearly, the true data generating process exhibits other sources of volatility in the fiscal instruments, in addition to the Ramsey prescriptions captured by our model. Likewise, the model accounts for only a fraction of the correlation between c and y, signalling perhaps that additional shocks are needed that can increase the comovement between these two variables.

3.4.5 Welfare costs

Finally, we compute the welfare costs associated with the business cycles for OECD and non-OECD countries implied by our model. To this effect, all parameters are held constant and equal across the two groups of countries except for those characterizing the TFP process (i.e., persistence and volatility).[18] These values are pinned down by matching the standard deviation of consumption and income across the two groups of countries. The costs of business cycles are then computed following Lucas (1987). In other words, the cost is given by the percentage fall in steady state consumption that makes households indifferent between living in one model or the other. Welfare costs are computed using a second-order

[18] Specifically, the parameters used are those reported in Table 1, except for σ_c and σ_g (set to 0.5), the debt elasticity ψ (set to 2.8), ρ_A (set to 0.7 for OECD countries and 0.9 for non-OECD), and σ_A (set to 0.01 for OECD countries and 0.025 for non-OECD).

approximation of the welfare function.

Table 4 reports the costs of business cycles. First notice that, by construction, we hit exactly the standard deviations of income and consumption by using a standard deviation of TFP (A) of 7.9 for non-OECD countries and 1.8 for OECD countries. Given these calibrations, welfare costs of business cycles are 10 times larger in non-OECD countries (0.131 percent) than in OECD countries (0.013 percent).

Naturally, while highly suggestive, these numbers should be taken with a grain of salt because, except for volatility and persistence of output, we have kept all other parameters equal across both groups of countries. In any event, this sizable difference in the welfare costs of business cycles reflects the large difference in output volatility, which in turn implies a more procyclical fiscal policy in developing countries (both on the spending and tax sides), as illustrated in subsubsection 3.4.3. While larger output volatility will certainly reflect other factors (such as policy regime changes that may increase output persistence), part of these welfare costs may be viewed as a quantification of Kaminsky *et al.*'s "when-it-rains-it-pours" phenomenon; that is, the idea that procyclical fiscal policy reinforces the underlying business cycle in developing countries by making booms larger and recessions deeper.

4 Empirical evidence

This section complements our previous theoretical analysis by providing empirical evidence supporting the links between financial frictions and fiscal procyclicality in expenditures and tax rates. Specifically, we now test whether several plausible empirical proxies for the two types of financial frictions considered in our theoretical framework differ between industrial/OECD and non-OECD countries.

4.1 Data and estimation

As a proxy for market incompleteness, we use the dataset on restrictions on capital inflows and outflows from Fernández *et al.* (2016), which quantifies *de jure* restrictions on cross border flows across 32 types of transactions in 10 different assets (equity, bonds, FDI, and so forth), over the period 1995-2015. We use the following four specific indices: (i) overall assets inflow restrictions index (kai); (ii) overall assets outflow restrictions index (kao); (iii) bond inflow restrictions (boi); and (iv) bond outflow restrictions (boi). The value of these four measures is an average of several restrictions indices within the corresponding category (e.g., assets outflows). Each index varies between 0 (no restrictions) and 1 (restrictions on all assets).

Our proxy for the spread elasticity is derived from direct estimation of the function used in the model:

$$S_{i,t} = \phi_i + \psi \left[\exp \left(\frac{D_{i,t}}{Y_{i,t}} - \overline{\left(\frac{D_i}{Y_i} \right)} \right) - 1 \right] + \varepsilon_{i,t},$$

where i and t are, respectively, country and time indices, $S_{i,t}$ is the country spread, ϕ_i is a constant, ψ is the estimated elasticity, $D_{i,t}$ is debt, $Y_{i,t}$ is output, and $\varepsilon_{i,t}$ is a mean zero $i.i.d$ disturbance. The above equation is estimated using panel fixed effects for two different samples of countries: OECD and non-OECD.

Two measures of debt are used: (i) total public debt from the World Bank's WDI; and (minus) net foreign assets from Lane and Milesi-Ferretti (2015). $S_{i,t}$ is proxied with the EMBIG for EMEs; the 10-year T-bill spread with respect to German T-bills for EU countries; and for the remaining developed countries we use a UIP condition between the domestic 10-year T-bills and the U.S. 10-year T-bills.

Lastly – and following Frankel *et al.* (2013) – we update the two measures of fiscal procyclicality for each country plotted in Figures 1 and 2. The first, $\rho(Y, G)$, is the correlation between the cyclical components of real GDP and real government consumption expenditure (using the HP filter). The second, $\rho\left(Y, \tau^{ind}\right)$, is the correlation between the cyclical component of real GDP and a tax index that averages the corporate tax rate, value added tax rate, and labor income tax rate (also using the HP filter).

4.2 Results

The key take-away from the results, summarized in Table 5, is that as discussed in the Introduction, non-OECD countries, characterized by procyclical fiscal policies, unlike OECD ones where government spending and tax policy are countercyclical or acyclical, are also characterized by deeper financial frictions as per the two proxies that we quantify, and display more macroeconomic volatility.

The top panel reports the average cyclicality measures across the two groups of countries. The average correlation of government spending and the business cycle in non-OECD and OECD countries are 0.27 and -0.12, respectively, and one can reject the null hypothesis of equality between the two groups of countries. A similar result is found for the tax rate in the sense that non-OECD countries behave in a more procyclical manner.

The second panel reports the results in term of market incompleteness. As shown, all four non-OECD indices proxing the inability of countries to participate in capital markets are higher than those of OECD countries by an order of magnitude, and the hypothesis of similar degrees of completeness is statistically rejected.

The third panel presents our estimated debt elasticities of spreads for the two groups of countries using two proxies of debt. In both cases, the elasticities are considerably higher for the non-OECD group of countries and the null hypothesis of equality is easily rejected. For the case of total public debt, the estimate for the non-OECD countries is $\psi = 0.125$ and significant at the one percent level, whereas the point estimate elasticity for OECD countries is 0.002 and not significant at the ten percent level. Using (negative) NFA as proxies of debt delivers similar qualitative results.

Lastly, average GDP volatility in non-OECD countries, measured with the standard deviation of the filtered real GDP process, 3.28, is more than twice that of OECD countries, 1.47. This further corroborates previous work that documented the relatively higher business cycle volatility in less advanced countries. This basic empirical evidence complements our previous theoretical analysis by providing evidence in favor of the connections between financial frictions and fiscal procyclicality in expenditures and tax rates.

5 Concluding remarks

This paper has examined the effects of incomplete markets on optimal fiscal policy (both on the spending and tax rate sides). Unlike other papers in the literature, we have gone to basics and use the standard, off-the shelve model with uncertainty and a risk-free bond.

We have shown that the literature had missed an important insight, which had been lost in more complex (and hence more obscure) models. Specifically, and contrary to popular belief, market incompleteness is not a sufficient condition to generate procyclicality of fiscal policy on both the spending and the revenue sides. In fact, while market incompleteness is indeed sufficient to generate procyclical government spending, the same is *not* true of tax rates. Tax rates may be countercyclical, acyclical, or procyclical depending on the relative preference for private versus public consumption.

Specifically, if the ratio of private to public consumption comoves positively with the cycle, then optimal tax policy is procyclical. If the comovement is zero, then tax policy is acyclical. If the comovement is negative, then tax policy is countercyclical. We isolate this channel in a static model and refer it to as the (private) "consumption preference channel." When we turn to a DSGE model with market incompleteness, a second channel comes into the picture: the (private) "consumption smoothing channel." The ability to smooth consumption implies that in response to, for example, a positive shock, consumption increases less than otherwise, which implies that the tax rate is reduced by less (thus making tax procyclicality less likely, all else equal).

When we add an upward-sloping supply of funds to the model and calibrate it, we find that the consumption preference channel prevails. Hence, for realistic parameterizations of the model, optimal tax policy will be procyclical when the ratio of private to public consumption comoves positively with the cycle, which is the case for emerging markets. We thus conclude that the simplest model of incomplete markets is capable of explaining the stylized facts.

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Appendix A: Static model (online; not for publication)

5.1 Financial autarky case

Consider a static model with a stochastic endowment and consumption tax. Output (endowment) can take two different values with probability p and $1-p$:

$$y = \begin{cases} y_H = \bar{y} + \gamma, \text{ with probability } p, \\ y_L = \bar{y} - \gamma, \text{ with probability } 1-p. \end{cases} \tag{19}$$

For simplicity, p is assumed to be equal to $1/2$. It can be easily checked that

$$\begin{aligned} E(y) &= \bar{y}, \\ V(y) &= \gamma^2. \end{aligned}$$

Preferences are given by

$$U(c_i, g_i) = \begin{cases} \underset{i=H,L}{E}\left[\alpha \frac{c_i^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha)\frac{g_i^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}}\right], & \sigma_c \neq 1 \text{ and } \sigma_g \neq 1, \\ \underset{i=H,L}{E}\left[\alpha \ln(c_i) + (1-\alpha)\frac{g_i^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}}\right], & \sigma_c = 1 \text{ and } \sigma_g \neq 1, \\ \underset{i=H,L}{E}\left[\alpha \frac{c_i^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha)\ln(g_i)\right], & \sigma_c \neq 1 \text{ and } \sigma_g \neq 1. \end{cases} \tag{20}$$

We can solve this problem as a social planner because the consumption tax does not distort intertemporally (i.e., the model is static) or intratemporally (i.e., there is no labor/leisure choice). The economy's resource constraints take the form

$$y_i = c_i + g_i, \qquad i = L, H. \tag{21}$$

The planner's choice variables are $\{c_H, c_L, g_H, g_L\}$. The Lagrangian is given by

$$\mathcal{L} = p\left[\alpha\frac{c_H^{1-\frac{1}{\sigma_c}}-1}{1-\frac{1}{\sigma_c}} + (1-\alpha)\frac{g_H^{1-\frac{1}{\sigma_g}}-1}{1-\frac{1}{\sigma_g}}\right] + (1-p)\left[\alpha\frac{c_L^{1-\frac{1}{\sigma_c}}-1}{1-\frac{1}{\sigma_c}} + (1-\alpha)\frac{g_L^{1-\frac{1}{\sigma_g}}-1}{1-\frac{1}{\sigma_g}}\right]$$
$$+\lambda_H(y_H - c_H - g_H) + \lambda_L(y_H - c_H - g_H).$$

Once we have solved the planner's problem, the government constraint in each state of nature can be used to find out the corresponding tax rates:

$$g_i = \tau_i c_i, \qquad i = L, H. \tag{22}$$

The first-order conditions for$\{c_H, c_L, g_H, g_L\}$ are given by, respectively,

$$\alpha c_H^{-\frac{1}{\sigma_c}} = \lambda_H, \tag{23}$$
$$(1-\alpha)\, g_H^{-\frac{1}{\sigma_g}} = \lambda_H, \tag{24}$$
$$\alpha c_L^{-\frac{1}{\sigma_c}} = \lambda_L, \tag{25}$$
$$(1-\alpha)\, g_L^{-\frac{1}{\sigma_g}} = \lambda_L. \tag{26}$$

These first-order conditions imply that the marginal utilities of private and public consumption are equalized in each state of nature (i.e., $U_{c^H} = U_{g^H}$ and $U_{c^L} = U_{g^L}$) but not across states of nature (because there is no full insurance).

Specifically, combining first-order conditions (23) and (24), we obtain:

$$c_H = g_H^{\frac{\sigma_c}{\sigma_g}}\left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c}. \tag{27}$$

By the same token, from (25) and (26):

$$c_L = g_L^{\frac{\sigma_c}{\sigma_g}}\left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c}. \tag{28}$$

Combining (21), (27), and (28), it follows that

$$\begin{aligned} y_H &= g_H^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} + g_H, \\ y_L &= g_L^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} + g_L. \end{aligned}$$

Define

$$\begin{aligned} \phi(g_i) &\equiv g_i^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} + g_i, \qquad i = L, H, \\ \phi'(g_i) &= 1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} \frac{\sigma_c}{\sigma_g} g_i^{\frac{\sigma_c}{\sigma_g}-1} > 0. \qquad i = L, H. \end{aligned}$$

Then,

$$\begin{aligned} y_i &= \phi(g_i), \qquad i = L, H, \\ g_i &= \Gamma(y_i) > 0, \qquad i = L, H, \\ \Gamma'(y_i) &> 0, \qquad i = L, H, \end{aligned} \tag{29}$$

where $\Gamma \equiv \phi^{-1}$ and

$$\frac{dg_i}{dy_i} = \frac{1}{\phi'} > 0. \qquad i = L, H.$$

Define the cyclicality of government spending as

$$\theta_g \equiv \log\left(\frac{g_H}{g_L}\right) = \log(g_H) - \log(g_L).$$

Using (29), this expression can be rewritten as

$$\theta_g = \log(\Gamma(y_H)) - \log(\Gamma(y_L)) > 0,$$

because $\Gamma'(y_i) > 0$ and $y_H > y_L$, which shows that government spending is procyclical regardless of the values of σ_c and σ_g (Proposition 1).

Taking into account (1), we now show that θ_g is increasing in γ (Proposition 3):

$$\frac{d\theta_g}{d\gamma} = \frac{1}{\Gamma(y_H)}\Gamma'(y_H) + \frac{1}{\Gamma(y_L)}\Gamma'(y_L) > 0.$$

To obtain a reduced form for τ_i, combine (22) with (27) and (28) to obtain

$$\tau_i = \left(\frac{1-\alpha}{\alpha}\right)^{\sigma_c} g_i^{1-\frac{\sigma_c}{\sigma_g}}, \qquad i = L, H. \tag{30}$$

Now define the cyclicality of the tax rate as

$$\theta_\tau \equiv \log\left(\frac{\tau_H}{\tau_L}\right)$$

which, using (30), can be rewritten as

$$\theta_\tau = \log\left(\frac{g_H^{1-\frac{\sigma_c}{\sigma_g}}}{g_L^{1-\frac{\sigma_c}{\sigma_g}}}\right) = \left(1-\frac{\sigma_c}{\sigma_g}\right)\log\left(\frac{g_H}{g_L}\right) = \left(1-\frac{\sigma_c}{\sigma_g}\right)\theta_g \gtrless 0. \tag{31}$$

As stated in Proposition 2, it follows that

$$\theta_\tau = \begin{cases} + & \text{(countercyclical)}, \quad \sigma_c < \sigma_g, \\ 0 & \text{(acyclical)}, \quad \sigma_c = \sigma_g, \\ - & \text{(procyclical)}, \quad \sigma_c > \sigma_g. \end{cases}$$

What happens in response to a mean-preserving spread in output? Differentiating (31) with respect to τ,

$$\frac{d\theta_\tau}{d\gamma} = \left(1-\frac{\sigma_c}{\sigma_g}\right)\frac{d\theta_g}{d\gamma},$$

where $(d\theta_g/d\gamma) > 0$. Hence,

$$\frac{d\theta_\tau}{d\gamma} = \begin{cases} + & \sigma_c < \sigma_g, \\ 0 & \sigma_c = \sigma_g, \\ - & \sigma_c > \sigma_g. \end{cases}$$

As stated in Proposition 3, if τ is countercyclical (i.e., positive), it will become more countercyclical. If it is zero, it remains zero of course. If it is negative (procyclical), it becomes more negative (i.e., more procyclical). In other words, a mean-preserving spread always amplifies the cyclicality of tax rates.

5.2 Complete markets

While the model is static, we assume that households have access to contingent claims that can insure them against the outcomes in each state of nature (i.e., high and low output). The state-contingent bonds are intra-period; that is, they are purchased at the beginning of the period (i.e., before the shock materializes), and the households receive the pay-off at the end of the period (i.e., after the shock takes place).

As in the case of financial autarky, we can solve the planner's problem (with the planner having access to complete markets abroad) since, in the absence of any distortions, the government will be able to implement the first-best policy.

The planner chooses $\{c_H, c_L, g_H, g_L\}$ to maximize

$$pU(c_H, g_H) + (1-p)U(c_L, g_L),$$

subject to

$$q_H y_H + q_L y_L = q_H(c_H + g_H) + q_L(c_L + g_L),$$

where $U(c_i, g_i)$ is given by (20).

The Lagrangian is given by

$$
\begin{aligned}
\mathcal{L} &= pU(c_H, g_H) + (1-p)U(c_L, g_L) \\
&\quad + \lambda \left[q_H y_H + q_L y_L - q_H(c_H + g_H) - q_L(c_L + g_L) \right].
\end{aligned}
$$

The first-order conditions are given by

$$
\begin{aligned}
pU_{c_H}(c_H, g_H) &= \lambda q_H, & (32) \\
pU_{g_H}(c_H, g_H) &= \lambda q_H, & (33) \\
(1-p)U_{c_L}(c_L, g_L) &= \lambda q_L, & (34) \\
(1-p)U_{g_L}(c_L, g_L) &= \lambda q_L. & (35)
\end{aligned}
$$

Notice that, as in the financial autarky case, the marginal utilities of private and public consumption are equalized in each state of the world; that is, $U_{c_H} = U_{g_H}$ and $U_{c_L} = U_{g_L}$.

Combining first-order conditions across states of the world (i.e., (32) and (33) on the one hand, and (34) and (35) on the other), we obtain

$$
\begin{aligned}
\frac{pU_{c_H}(c_H, g_H)}{q_H} &= \frac{(1-p)U_{c_L}(c_L, g_L)}{q_L}, \\
\frac{pU_{g_H}(c_H, g_H)}{q_H} &= \frac{(1-p)U_{g_L}(c_L, g_L)}{q_L}.
\end{aligned}
$$

Assuming actuarially fair insurance (i.e., $q_H/q_L = p/(1-p)$), we can rewrite these optimality conditions as

$$
\begin{aligned}
U_{c_H}(c_H, g_H) &= U_{c_L}(c_L, g_L), & (36) \\
U_{g_H}(c_H, g_H) &= U_{g_L}(c_L, g_L). & (37)
\end{aligned}
$$

Marginal utilities of private and public consumption are equalized across states of nature (implying full risk sharing). Since the utility function, given by (20), is separable, conditions (36) and (37) imply, respectively, that $c_H = c_L$ and $g_H = g_L$. The latter implies that government spending is acyclical in the sense defined above (Proposition 1). In other words, under complete markets, government spending is acyclical regardless of the relation between σ_c and σ_g.

Further, given (22), $c_H = c_L$ and $g_H = g_L$ imply that $\tau_H = \tau_L$. Hence, tax policy is also acyclical for any values of σ_c and σ_g (Proposition 2).

In either case (spending or tax rates), fiscal policy is acyclical regardless of the variance of the distribution (Proposition 3).

5.3 Income tax

Suppose that we have an income tax in the form of an endowment tax (to keep it symmetrical with the consumption tax case). In other words, the consumer's budget constraints are given by:

$$c_i = (1 - \xi_i)y_i, \qquad i = L, H,$$

where ξ_i is the endowment tax.

Other than this, the model is exactly the same as in the case analyzed above. As in the consumption tax case, this endowment tax is non-distortionary. We can thus solve the planner's problem as we did for the consumption tax case. All results for both financial autarky and complete markets go through since they do not depend on what is the government's source of tax income as long as it is non-distortionary.

Given the planner's optimal choices (which will be the same as in the consumption tax case), the income tax will follow from the government budget constraint:

$$g_i = (1 - \xi_i)y_i, \qquad i = L, H.$$

We conclude that Propositions 1-3 would also hold for the case of an income (endowment) tax.

5.4 Financial autarky with initial assets

Suppose that the government has initial assets, given by a.[19] Then the consumer's and government's constraints are given by

$$\begin{aligned} y_i &= (1+\tau_i)c_i, \qquad i = L, H, \\ g_i &= a + \tau_i c_i, \qquad i = L, H. \end{aligned} \tag{38}$$

Initial assets are, of course, the same across states of nature. Combining these two constraints, we get the economy's resource constraints:

$$y_i + a = c_i + g_i, \qquad i = L, H. \tag{39}$$

The Lagrangian becomes:

$$\begin{aligned} \mathcal{L} &= p\left[\alpha \frac{c_H^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha)\frac{g_H^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}}\right] + (1-p)\left[\alpha \frac{c_L^{1-\frac{1}{\sigma_c}} - 1}{1-\frac{1}{\sigma_c}} + (1-\alpha)\frac{g_L^{1-\frac{1}{\sigma_g}} - 1}{1-\frac{1}{\sigma_g}}\right] \\ &\quad + \lambda_H(y_H + a - c_H - g_H) + \lambda_L(y_L + a - c_L - g_L). \end{aligned}$$

Of course, the first-order conditions do not change due to the presence of a, and thus conditions (27) and (28) continue to hold.

Using the economy's resource constraint, given by (39), it follows that

$$y_i + a = g_i^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} + g_i. \qquad i = L, H.$$

Proceeding as before, define

[19] We could assume that one or both sectors (private and public) have initial assets, as long as net assets are not zero (otherwise we would be back to our original formulation with $a = 0$).

$$\begin{aligned}
\phi(g_i) &\equiv g_i^{\frac{\sigma_c}{\sigma_g}} \left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} + g_i - a, \qquad i = L, H. \\
\phi^{'}(g_i) &= 1 + \left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} \frac{\sigma_c}{\sigma_g} g_i^{\frac{\sigma_c}{\sigma_g}-1} > 0, \qquad i = L, H.
\end{aligned}$$

As before, it is easy to conclude that government spending is always procyclical and increasing in γ. The presence of initial assets does not change anything.

In contrast, initial assets will matter for the cyclicality of tax rates. To obtain a reduced form for τ_i, recall (27), (28), and (38):

$$\begin{aligned}
g_i - a &= \tau_i c_i, \qquad i = L, H, \\
\alpha c_i^{-\frac{1}{\sigma_c}} &= (1-\alpha)\, g_i^{-\frac{1}{\sigma_g}}, \qquad i = L, H.
\end{aligned}$$

Rewrite the latter as:

$$\left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} g_i^{\frac{\sigma_c}{\sigma_g}} = c_i, \qquad i = L, H.$$

Substitute the latter into (38) to obtain:

$$\tau_i = \frac{g_i - a}{\left(\frac{\alpha}{1-\alpha}\right)^{\sigma_c} g_i^{\frac{\sigma_c}{\sigma_g}}} \qquad i = L, H.$$

As before, define the procyclicality of tax rates as

$$\theta_\tau \equiv \log\left(\frac{\tau_H}{\tau_L}\right) = \log\left(\frac{(g_H - a)\, g_L^{\frac{\sigma_c}{\sigma_g}}}{g_H^{\frac{\sigma_c}{\sigma_g}}\, (g_L - a)}\right).$$

Notice that since this expression cannot be expressed as a function of the ratio g_H/g_L, we cannot establish the clean results stated in Proposition 2. Of course, if $a = 0$, then we recover our original case.

6 Appendix B (online, not for publication)

This appendix formally sets up and solves the three variations of the main model used in the text. First, we consider a small open economy operating under financial autarky (i.e., no borrowing from/lending to the rest of the world). Second, we deal with a standard small open economy that has access to a world risk-free bond. Third, we consider a small open economy that is operating under complete markets.

6.1

Financial Autarky Model

6.1.1 Households' Problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

subject to:

$$d_t = (1 + r_{t-1}) d_{t-1} - y_t + (1 + \tau_t) c_t + \Phi(d_t) - \Pi_t$$

$$y_t = A_t l_t$$

$$\ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID\left(0; \sigma_A^2\right)$$

and a no-Ponzi condition.

Note that we assume households face convex portfolio transactions costs, $\Phi(d_t)$. These administrative services (for either assets or liabilities) are provided by a government agency at zero cost. Profits (Π) are transferred to households in a lump-sum way so as to get rid of any wealth effects associated with these portfolio adjustment/transaction costs.

Notice also that here the portfolio costs are internalized by both the household and Ramsey planner. In the SOE case (see below), only the Ramsey planner internalizes the upward sloping supply of funds.

6.1.2 Lagrangian

$$\pounds = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t \left[d_t - (1 + r_{t-1})\, d_{t-1} + A_t l_t - (1 + \tau_t)\, c_t - \Phi\left(d_t\right) + \Pi_t \right] \right\}$$

F.O.C.:

$$[c_t] : \beta^t U_c(c_t, g_t, l_t) - \beta^t \lambda_t \left(1 + \tau_t\right) = 0$$

$$[l_t] : \beta^t U_l(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0$$

$$[d_t] : \beta^t \lambda_t \left[1 - \Phi'(d_t)\right] - (1 + r_t)\, \beta^{t+1} E_t \lambda_{t+1} = 0$$

Simplifying:

$$U_c(c_t, g_t, l_t) = \lambda_t \left(1 + \tau_t\right)$$

$$U_l(c_t, g_t, l_t) = -\lambda_t A_t$$

$$\lambda_t \left[1 - \Phi'(d_t)\right] = (1 + r_t)\, \beta E_t \lambda_{t+1}$$

Government's flow budget constraint

$$d_t^g = (1 + r_{t-1})\, d_{t-1}^g - \tau_t c_t + g_t - \Phi\left(d_t\right) + \Pi_t$$

Aggregate Constraints

Financial autarky implies:

$$d_t + d_t^g = 0 \quad \forall t$$

Combining the household´s and government flow budget constraints with the restriction above (in t and $t-1$):

$$\begin{aligned} (1 + r_{t-1})\, d_{t-1} - y_t + (1 + \tau_t)\, c_t + (1 + r_{t-1})\, d_{t-1}^g - \tau_t c_t + g_t &= 0 \\ -y_t + (1 + \tau_t)\, c_t - \tau_t c_t + g_t &= 0 \\ c_t + g_t &= y_t \end{aligned}$$

Implementability Conditions

$$A_t = -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)}(1+\tau_t) \equiv \Gamma_t(c_t, g_t, l_t, \tau_t)$$

$$\frac{U_c(c_t, g_t, l_t)}{1+\tau_t} = \lambda_t \equiv \lambda_t(c_t, g_t, l_t, \tau_t) = \frac{1+r_t}{1-\Phi'(d_t)}\beta E_t \lambda_{t+1}(c_{t+1}, g_{t+1}, l_{t+1}, \tau_{t+1})$$

Ramsey Problem:

$$\pounds = E_0 \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) \\ +\beta^t \mu_{1,t} [\Gamma_t - A_t] \\ +\beta^t \mu_{2,t} \left[\lambda_t(c_t, g_t, l_t, \tau_t) - \frac{1+r_t}{1-\Phi'(d_t)}\beta E_t \lambda_{t+1}(c_{t+1}, g_{t+1}, l_{t+1}, \tau_{t+1})\right] \\ +\beta^t \mu_{3,t} [d_t - (1+r_{t-1}) d_{t-1} + A_t l_t - (1+\tau_t) c_t] \\ +\beta^t \mu_{4,t} [A_t l_t - c_t - g_t] \end{array} \right\}$$

F.O.C

$$[c_t] : U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\lambda_{c_t} - \mu_{3,t}(1+\tau_t) - \mu_{4,t} = 0$$

$$[l_t] : U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\lambda_{l_t} + \mu_{3,t}A_t + \mu_{4,t}A_t = 0$$

$$[g_t] : U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\lambda_{g_t} - \mu_{4,t} = 0$$

$$[\tau_t] : \mu_{1,t}\Gamma_{\tau_t} + \mu_{2,t}\lambda_{\tau_t} - \mu_{3,t}c_t = 0$$

$$[d_t] : \mu_{2,t}\left[-\frac{1+r_t}{[1-\Phi'(d_t)]^2}\Phi'(d_t)\,\beta E_t \lambda_{t+1}(c_{t+1}, g_{t+1}, l_{t+1}, \tau_{t+1})\right] + \mu_{3,t} - \beta(1+r_t) E_t \mu_{3,t+1} = 0$$

$$[r_t] : -\frac{1}{1-\Phi'(d_t)}\mu_{2,t}\beta E_t \lambda_{t+1} - \beta d_t E_t \mu_{3,t+1} = 0$$

$$[\mu_{1,t}] : \Gamma_t - A_t = 0$$

$$[\mu_{2,t}] : \lambda_t - \frac{1+r_t}{1-\Phi'(d_t)}\beta E_t \lambda_{t+1} = 0$$

$$[\mu_{3,t}] : d_t - (1+r_{t-1}) d_{t-1} + A_t l_t - (1+\tau_t) c_t = 0$$

$$[\mu_{4,t}] : A_t l_t - c_t - g_t = 0$$

Parametrization

- Portfolio transactions costs:

$$\Phi(d_t) = \frac{\phi}{2}\left(d_t - \overline{d}\right)^2$$
$$\Phi'(d_t) = \phi\left(d_t - \overline{d}\right);\ \Phi'(d_t) = \phi;$$

- Preferences:

$$U(c_t, g_t, l_t) = \frac{c_t^{1-1/\sigma} - 1}{1 - 1/\sigma} + \frac{g_t^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + \ln(1 - l_t)$$

$$U_{c_t} = c_t^{-1/\sigma};\quad U_{g_t} = g_t^{-1/\sigma_g};\quad U_{l_t} = -\frac{1}{1 - l_t}$$

- Derivatives of Γ_t and λ_t:

$$\Gamma_t = -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)}(1 + \tau_t) = \frac{1 + \tau_t}{(1 - l_t)c_t^{-1/\sigma}}$$

$$\Gamma_{c_t} = \frac{1 + \tau_t}{\sigma(1 - l_t)c_t^{1-1/\sigma}};\quad \Gamma_{l_t} = \frac{1 + \tau_t}{(1 - l_t)^2 c_t^{-1/\sigma}}; \Gamma_{g_t} = 0;\quad \Gamma_{\tau_t} = \frac{1}{(1 - l_t)c_t^{-1/\sigma}};$$

$$\lambda_t = \frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} = \frac{c_t^{-1/\sigma}}{1 + \tau_t}$$

$$\lambda_{c_t} = -\frac{c_t^{-1/\sigma-1}}{\sigma(1 + \tau_t)};\quad \lambda_{l_t} = 0; \lambda_{g_t} = 0;\quad \lambda_{\tau_t} = -\frac{c_t^{-1/\sigma}}{(1 + \tau_t)^2};$$

Dynamic System

Model is a system of 12 endogenous and 1 exogenous variables:

$$\{c_t, \tau_t, l_t, g_t, d_t, r_t, y_t, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t\} \text{ and} \{A_t\},\ \text{respectively.}$$

that are determined by 13 equations:

$$
\begin{aligned}
U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\lambda_{c_t} - \mu_{3,t}\left(1+\tau_t\right) - \mu_{4,t} &= 0 \\
[1] \quad c_t^{-1/\sigma} + \mu_{1,t}\frac{1+\tau_t}{\sigma(1-l_t)c_t^{1-1/\sigma}} - \mu_{2,t}\frac{c_t^{-1/\sigma-1}}{\sigma\left(1+\tau_t\right)} - \mu_{3,t}\left(1+\tau_t\right) - \mu_{4,t} &= 0
\end{aligned}
$$

$$
\begin{aligned}
U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\lambda_{l_t} + \mu_{3,t}A_t + \mu_{4,t}A_t &= 0 \\
[2] \qquad -\frac{1}{1-l_t} + \mu_{1,t}\frac{1+\tau_t}{(1-l_t)^2 c_t^{-1/\sigma}} + \mu_{3,t}A_t + \mu_{4,t}A_t &= 0
\end{aligned}
$$

$$
\begin{aligned}
U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\lambda_{g_t} - \mu_{4,t} &= 0 \\
[3] \qquad g_t^{-1/\sigma_g} - \mu_{4,t} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mu_{1,t}\Gamma_{\tau_t} + \mu_{2,t}\lambda_{\tau_t} - \mu_{3,t}c_t &= 0 \\
[4] \qquad \mu_{1,t}\frac{1}{(1-l_t)c_t^{-1/\sigma}} - \mu_{2,t}\frac{c_t^{-1/\sigma}}{\left(1+\tau_t\right)^2} - \mu_{3,t}c_t &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mu_{2,t}\left[-\frac{1+r_t}{\left[1-\Phi'(d_t)\right]^2}\Phi'(d_t)\,\beta E_t\lambda_{t+1}(c_{t+1},g_{t+1},l_{t+1},\tau_{t+1})\right] + \mu_{3,t} - \beta\left(1+r_t\right)E_t\mu_{3,t+1} &= 0 \\
[5] \qquad \mu_{2,t}\left[-\frac{1+r_t}{\left[1-\phi\left(d_t-\overline{d}\right)\right]^2}\phi\beta E_t\lambda_{t+1}(c_{t+1},g_{t+1},l_{t+1},\tau_{t+1})\right] + \mu_{3,t} - \beta\left(1+r_t\right)E_t\mu_{3,t+1} &= 0
\end{aligned}
$$

$$
\begin{aligned}
-\frac{1}{1-\Phi'(d_t)}\mu_{2,t}\beta E_t\lambda_{t+1} - \beta d_t E_t\mu_{3,t+1} &= 0 \\
[6] \qquad -\frac{1}{1-\phi\left(d_t-\overline{d}\right)}\mu_{2,t}\beta E_t\lambda_{t+1} - \beta d_t E_t\mu_{3,t+1} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_t - A_t &= 0 \\
[7] \qquad \frac{1+\tau_t}{(1-l_t)c_t^{-1/\sigma}} - A_t &= 0
\end{aligned}
$$

$$\lambda_t - \frac{1+r_t}{1-\Phi'(d_t)}\beta E_t \lambda_{t+1} = 0$$

$$[8] \qquad \lambda_t - \frac{1+r_t}{1-\phi\left(d_t-\overline{d}\right)}\beta E_t \lambda_{t+1} = 0$$

$$[9] \qquad d_t - (1+r_{t-1})\,d_{t-1} + A_t l_t - (1+\tau_t)\,c_t = 0$$

$$[10] \qquad A_t l_t - c_t - g_t = 0$$

$$[11] \qquad y_t = A_t l_t$$

$$[12] \qquad \lambda_t = \frac{c_t^{-1/\sigma}}{1+\tau_t}$$

$$[13] \qquad \ln A_t = (1-\rho_A)\ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID\left(0;\sigma_A^2\right)$$

with associated 8 parameters

$$\{\sigma, \sigma_g, \beta, \rho_A, \sigma_A^2, \bar{A}, \phi, \overline{d}\}$$

Steady State

The steady state is a system of 13 equations with 21 unknowns, adding 13 SS-variables

$$\{c, \tau, l, g, d, r, y, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, A\}$$

and 8 parameters $\{\sigma, \sigma_g, \beta, \rho_A, \sigma_A^2, \bar{A}, \phi, \overline{d}\}$

$$c_t^{-1/\sigma} + \mu_{1,t}\frac{1+\tau_t}{\sigma(1-l_t)c_t^{1-1/\sigma}} - \mu_{2,t}\frac{c_t^{-1/\sigma-1}}{\sigma\left(1+\tau_t\right)} - \mu_{3,t}\left(1+\tau_t\right) - \mu_{4,t} = 0$$

$$[1] \qquad c^{-1/\sigma} + \mu_1\frac{1+\tau}{\sigma(1-l)c^{1-1/\sigma}} - \mu_2\frac{c^{-1/\sigma-1}}{\sigma\left(1+\tau\right)} - \mu_3\left(1+\tau\right) - \mu_4 = 0$$

$$-\frac{1}{1-l_t} + \mu_{1,t}\frac{1+\tau_t}{(1-l_t)^2 c_t^{-1/\sigma}} + \mu_{3,t}A_t + \mu_{4,t}A_t = 0$$

$$[2] \qquad -\frac{1}{1-l} + \mu_1\frac{1+\tau}{(1-l)^2 c^{-1/\sigma}} + \mu_3 A + \mu_4 A = 0$$

$$g_t^{-1/\sigma_g} - \mu_{4,t} = 0$$

$$[3] \qquad g^{-1/\sigma_g} - \mu_4 = 0$$

$$\mu_{1,t}\frac{1}{(1-l_t)c_t^{-1/\sigma}} - \mu_{2,t}\frac{c_t^{-1/\sigma}}{(1+\tau_t)^2} - \mu_{3,t}c_t = 0$$

$$[4] \qquad \mu_1\frac{1}{(1-l)c^{-1/\sigma}} - \mu_2\frac{c^{-1/\sigma}}{(1+\tau)^2} - \mu_3 c = 0$$

$$\mu_{2,t}\left[-\frac{1+r_t}{\left[1-\phi\left(d_t-\overline{d}\right)\right]^2}\phi\beta E_t\lambda_{t+1}(c_{t+1},g_{t+1},l_{t+1},\tau_{t+1})\right] + \mu_{3,t} - \beta\left(1+r_t\right)E_t\mu_{3,t+1} = 0$$

$$[5] \qquad \mu_2\left[-\frac{1+r}{\left[1-\phi\left(d-\overline{d}\right)\right]^2}\phi\beta\lambda\right] + \mu_3 - \beta\left(1+r\right)\mu_3 = 0$$

$$-\frac{1}{1-\phi\left(d_t-\overline{d}\right)}\mu_{2,t}\beta E_t\lambda_{t+1} - \beta d_t E_t\mu_{3,t+1} = 0$$

$$[6] \qquad -\frac{1}{1-\phi\left(d-\overline{d}\right)}\mu_2\lambda - d\mu_3 = 0$$

$$\frac{1+\tau_t}{(1-l_t)c_t^{-1/\sigma}} - A_t = 0$$

$$[7] \qquad \frac{1+\tau}{(1-l)c^{-1/\sigma}} - A = 0$$

$$\lambda_t - \frac{1+r_t}{1-\phi\left(d_t-\overline{d}\right)}\beta E_t\lambda_{t+1} = 0$$

$$[8] \qquad 1 - \frac{1+r}{1-\phi\left(d-\overline{d}\right)}\beta = 0$$

$$[9] \qquad d - \left(1+r\right)d + Al - \left(1+\tau\right)c = 0$$

$$[10] \qquad Al - c - g = 0$$

$$[11] \qquad y = Al$$

$$[12] \qquad \frac{c^{-1/\sigma}}{1+\tau} = \lambda$$

$$[13] \qquad A = \bar{A}$$

After calibrating some of these parameters (see text for details), we solve this system numerically.

6.2 SOE with Incomplete Asset Markets Model

Households' Problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

Subject to:

$$d_t = (1 + r_{t-1})\, d_{t-1} - y_t + (1 + \tau_t)\, c_t$$

$$y_t = A_t l_t$$

$$r_t = r_t^* + p(\tilde{d}_t)$$

$$\ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID\left(0; \sigma_A^2\right)$$

$$\ln r_t^* = (1 - \rho_r) \ln \bar{r}^* + \rho_R \ln r_{t-1} + \varepsilon_t^r, \quad \varepsilon_t^r \sim NIID\left(0; \sigma_r^2\right)$$

and a no-Ponzi condition.

Note that, in equilibrium, $\tilde{d}_t = d_t$. This is not internalized by the household, though it is by the Ramsey planner (below).

Lagrangian:

$$£ = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t \left[d_t - (1 + r_{t-1})\, d_{t-1} + A_t l_t - (1 + \tau_t)\, c_t \right] \right\}$$

F.O.C.:

$$[c_t] : \beta^t U_c(c_t, g_t, l_t) - \beta^t \lambda_t \,(1 + \tau_t) = 0$$

$$[l_t] : \beta^t U_l(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0$$

$$[d_t] : \beta^t \lambda_t - (1 + r_t)\, \beta^{t+1} E_t \lambda_{t+1} = 0$$

Simplifying:

$$U_c(c_t, g_t, l_t) = \lambda_t\, (1 + \tau_t)$$

$$U_l(c_t, g_t, l_t) = -\lambda_t A_t$$

$$\lambda_t = (1 + r_t)\, \beta E_t \lambda_{t+1}$$

Government's flow budget constraint:

$$d_t^g = \left(1 + r_{t-1}^g\right) d_{t-1}^g - \tau_t c_t + g_t$$

$$r_t^g = r_t^* + p(d_t^g)$$

Implementability Conditions

$$A_t = -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)}\, (1 + \tau_t) \equiv \Gamma_t(c_t, g_t, l_t, \tau_t)$$

$$\frac{U_c(c_t, g_t, l_t)}{1 + \tau_t} = \lambda_t \equiv \lambda_t(c_t, g_t, l_t, \tau_t) = (1 + r_t)\, \beta E_t \lambda_{t+1}(c_{t+1}, g_{t+1}, l_{t+1}, \tau_{t+1})$$

Ramsey problem

$$\pounds = E_0 \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t) \\ +\beta^t \mu_{1,t} \left[\Gamma_t - A_t\right] \\ +\beta^t \mu_{2,t} \left[\lambda_t - (1 + r_t^* + p(d_t))\, \beta E_t \lambda_{t+1}\right] \\ +\beta^t \mu_{3,t} \left[d_t - \left(1 + r_{t-1}^* + p(d_{t-1})\right) d_{t-1} + A_t l_t - (1 + \tau_t)\, c_t\right] \\ +\beta^t \mu_{4,t} \left[d_t^g - \left(1 + r_{t-1}^* + p(d_{t-1}^g)\right) d_{t-1}^g + \tau_t c_t - g_t\right] \end{array} \right\}$$

F.O.C

$$[c_t] : U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\lambda_{c_t} - \mu_{3,t}(1+\tau_t) + \mu_{4,t}\tau_t = 0$$

$$[l_t] : U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\lambda_{l_t} + \mu_{3,t}A_t = 0$$

$$[g_t] : U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\lambda_{g_t} - \mu_{4,t} = 0$$

$$[\tau_t] : \mu_{1,t}\Gamma_{\tau_t} + \mu_{2,t}\lambda_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t = 0$$

$$[d_t] : -\mu_{2,t}p'(d_t)\beta E_t\lambda_{t+1} + \mu_{3,t} - \beta\left(p'(d_t)d_t + 1 + r_t^* + p(d_t)\right)E_t\mu_{3,t+1} = 0$$

$$[d_t^g] : \mu_{4,t} - \beta\left(p'(d_t^g)d_t^g + 1 + r_t^* + p(d_t^g)\right)E_t\mu_{4,t+1} = 0$$

$$\left[\mu_{1,t}\right] : \Gamma_t - A_t = 0$$

$$\left[\mu_{2,t}\right] : \lambda_t - (1 + r_t^* + p(d_t))\beta E_t\lambda_{t+1} = 0$$

$$\left[\mu_{3,t}\right] : d_t - \left(1 + r_{t-1}^* + p(d_{t-1})\right)d_{t-1} + A_tl_t - (1+\tau_t)c_t = 0$$

$$\left[\mu_{4,t}\right] : d_t^g - \left(1 + r_{t-1}^* + p(d_{t-1}^g)\right)d_{t-1}^g + \tau_tc_t - g_t = 0$$

Parametrization

- Preferences:

$$U(c_t, g_t, l_t) = \frac{c_t^{1-1/\sigma} - 1}{1-1/\sigma} + \frac{g_t^{1-1/\sigma_g} - 1}{1-1/\sigma_g} + \ln(1-l_t)$$

$$U_{c_t} = c_t^{-1/\sigma}; \quad U_{g_t} = g_t^{-1/\sigma_g}; \quad U_{l_t} = -\frac{1}{1-l_t}$$

- Derivatives of Γ_t and λ_t:

$$\Gamma_t = -\frac{U_l(c_t, g_t, l_t)}{U_c(c_t, g_t, l_t)}(1+\tau_t) = \frac{1+\tau_t}{(1-l_t)c_t^{-1/\sigma}}$$

$$\Gamma_{c_t} = \frac{1+\tau_t}{\sigma(1-l_t)c_t^{1-1/\sigma}};\quad \Gamma_{l_t} = \frac{1+\tau_t}{(1-l_t)^2 c_t^{-1/\sigma}}; \Gamma_{g_t} = 0;\quad \Gamma_{\tau_t} = \frac{1}{(1-l_t)c_t^{-1/\sigma}};$$

$$\lambda_t = \frac{U_c(c_t, g_t, l_t)}{1+\tau_t} = \frac{c_t^{-1/\sigma}}{1+\tau_t}$$

$$\lambda_{c_t} = -\frac{c_t^{-1/\sigma-1}}{\sigma(1+\tau_t)};\quad \lambda_{l_t} = 0; \lambda_{g_t} = 0;\quad \lambda_{\tau_t} = -\frac{c_t^{-1/\sigma}}{(1+\tau_t)^2};$$

- Debt-elastic specification, following Uribe and Schmitt-Grohé (2003), for the case of the household is:

$$p(d_t) = \psi\left[\exp\left(d_t - \bar{d}\right) - 1\right]$$

$$p'(d_t) = \psi \exp\left(d_t - \bar{d}\right)$$

and for the case of the government:

$$p(d_t^g) = \psi_g\left[\exp\left(d_t^g - \bar{d}\right) - 1\right]$$

$$p'(d_t^g) = \psi_g \exp\left(d_t^g - \bar{d}\right)$$

Dynamic System

Model is a system of 13 endogenous and 2 exogenous variables

$$\{c_t, \tau_t, l_t, g_t, d_t, d_t^g, r_t, y_t, \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t\} \text{ and} \{A_t, r_t^*\}, \text{ respectively.}$$

that are determined by 15 equations:

$$U_{c_t} + \mu_{1,t}\Gamma_{c_t} + \mu_{2,t}\lambda_{c_t} - \mu_{3,t}(1+\tau_t) + \mu_{4,t}\tau_t = 0$$

$$[1]\quad c_t^{-1/\sigma} + \mu_{1,t}\frac{1+\tau_t}{\sigma(1-l_t)c_t^{1-1/\sigma}} + \mu_{2,t}\frac{c_t^{-1/\sigma-1}}{\sigma(1+\tau_t)} - \mu_{3,t}(1+\tau_t) + \mu_{4,t}\tau_t = 0$$

$$
\begin{aligned}
-U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{2,t}\lambda_{l_t} + \mu_{3,t}A_t &= 0 \\
[2] \quad \frac{1}{1-l_t} + \mu_{1,t}\frac{1+\tau_t}{(1-l_t)^2 c_t^{-1/\sigma}} + \mu_{3,t}A_t &= 0
\end{aligned}
$$

$$
\begin{aligned}
U_{g_t} + \mu_{1,t}\Gamma_{g_t} + \mu_{2,t}\lambda_{g_t} - \mu_{4,t} &= 0 \\
[3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\mu_{1,t}\Gamma_{\tau_t} + \mu_{2,t}\lambda_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0 \\
[4] \quad \mu_{1,t}\frac{1}{(1-l_t)c_t^{-1/\sigma}} - \mu_{2,t}\frac{c_t^{-1/\sigma}}{(1+\tau_t)^2} - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0
\end{aligned}
$$

$$
\begin{aligned}
-\mu_{2,t}p'(d_t)\beta E_t\lambda_{t+1} + \mu_{3,t} - \beta\left(p'(d_t)d_t + 1 + r_t^* + p(d_t)\right)E_t\mu_{3,t+1} &= \\
[5] \quad -\mu_{2,t}\psi\exp\left(d_t - \bar{d}\right)\beta E_t\lambda_{t+1} + \mu_{3,t} - \beta\left(\psi\exp\left(d_t - \bar{d}\right)d_t + 1 + r_t^* + \psi\left[\exp\left(d_t - \bar{d}\right) - 1\right]\right)E_t\mu_{3,t+1} &=
\end{aligned}
$$

$$
\begin{aligned}
\mu_{4,t} - \beta\left(p'(d_t^g)d_t^g + 1 + r_t^* + p(d_t^g)\right)E_t\mu_{4,t+1} &= 0 \\
[6] \quad \mu_{4,t} - \beta\left(\psi_g\exp\left(d_t^g - \bar{d}^g\right)d_t^g + 1 + r_t^* + \psi_g\left[\exp\left(d_t^g - \bar{d}^g\right) - 1\right]\right)E_t\mu_{4,t+1} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_t - A_t &= 0 \\
[7] \quad \frac{1+\tau_t}{(1-l_t)c_t^{-1/\sigma}} - A_t &= 0
\end{aligned}
$$

$$[8] \quad \lambda_t - \left(1 + r_t^* + \psi\left[\exp\left(d_t - \bar{d}\right) - 1\right]\right)\beta E_t\lambda_{t+1} = 0$$

$$[9] \quad d_t - \left(1 + r_{t-1}^* + \psi\left[\exp\left(d_{t-1} - \bar{d}\right) - 1\right]\right)d_{t-1} + A_t l_t - (1+\tau_t)c_t = 0$$

$$[10] \quad d_t^g - \left(1 + r_{t-1}^* + \psi_g\left[\exp\left(d_{t-1}^g - \bar{d}^g\right) - 1\right]\right)d_{t-1}^g + \tau_t c_t - g_t = 0$$

$$[11] \quad y_t = A_t l_t$$

$$[12] \quad \lambda_t = \frac{c_t^{-1/\sigma}}{1+\tau_t}$$

$$[13]\quad r_t = r_t^* + \psi\left[\exp\left(d_t - \bar{d}\right) - 1\right]$$

$$[14]\quad \ln A_t = (1-\rho_A)\ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID\left(0;\sigma_A^2\right)$$

$$[15]\quad \ln r_t^* = (1-\rho_r)\ln \bar{r}^* + \rho_r \ln r_{t-1} + \varepsilon_t^r, \quad \varepsilon_t^r \sim NIID\left(0;\sigma_r^2\right)$$

with associated 13 parameters

$$\{\sigma, \sigma_g, \beta, \psi, \bar{d}, \psi_g, \bar{d}^g, \rho_A, \sigma_A^2, \rho_r, \sigma_r^2, \bar{A}, \bar{r}^*\}$$

Steady State

The steady state is a system of 15 equations with 28 unknowns, adding 15 SS-variables

$$\{c, \tau, l, g, d, d^g, r, y, \lambda, \mu_1, \mu_2, \mu_3, \mu_4, A, r^*\}$$

and 13 parameters $\{\sigma, \sigma_g, \beta, \psi, \bar{d}, \psi_g, \bar{d}^g, \rho_A, \sigma_A^2, \rho_r, \sigma_r^2, \bar{A}, \bar{r}^*\}$

$$[1]\quad c^{-1/\sigma} + \mu_1 \frac{1+\tau}{\sigma(1-l)c^{1-1/\sigma}} + \mu_2 \frac{c^{-1/\sigma-1}}{\sigma(1+\tau)} - \mu_3(1+\tau) + \mu_4\tau = 0$$

$$[2]\quad -\frac{1}{1-l} + \mu_1 \frac{1+\tau}{(1-l)^2 c^{-1/\sigma}} + \mu_3 A = 0$$

$$[3]\quad g^{-1/\sigma_g} - \mu_4 = 0$$

$$[4]\quad \mu_1 \frac{1}{(1-l)c^{-1/\sigma}} - \mu_2 \frac{c^{-1/\sigma}}{(1+\tau)^2} - \mu_3 c + \mu_4 c = 0$$

$$[5]\quad -\mu_2\psi\exp\left(d-\bar{d}\right)\beta\lambda + \mu_3 - \beta\left(\psi\exp\left(d-\bar{d}\right)d + 1 + r^* + \psi\left[\exp\left(d-\bar{d}\right)-1\right]\right)\mu_3 = 0$$

$$[6]\quad \mu_4 - \beta\left(\psi_g\exp\left(d^g-\bar{d}^g\right)d^g + 1 + r^* + \psi_g\left[\exp\left(d^g-\bar{d}^g\right)-1\right]\right)\mu_4 = 0$$

$$[7]\quad \frac{1+\tau}{(1-l)c^{-1/\sigma}} - A = 0$$

$$[8]\quad \lambda - \left(1 + r^* + \psi\left[\exp\left(d-\bar{d}\right)-1\right]\right)\beta\lambda = 0$$

$$[9]\quad d - \left(1 + r^* + \psi\left[\exp\left(d-\bar{d}\right)-1\right]\right)d + Al - (1+\tau)c = 0$$

$$[10]\ \ d^g - \left(1 + r^* + \psi_g \left[\exp\left(d^g - \bar{d}^g\right) - 1\right]\right) d^g + \tau c - g = 0$$

$$[11]\ \ y = Al$$

$$[12]\ \ r = r^* + \psi \left[\exp\left(d - \bar{d}\right) - 1\right]$$

$$[13]\ \ \frac{c^{-1/\sigma}}{1+\tau} = \lambda$$

$$[14]\ \ \ln A = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A$$

$$[15]\ \ \ln r^* = (1 - \rho_r) \ln \bar{r}^* + \rho_r \ln r$$

Which can be reduced to the following 8 equations:

$$[1]\ \ c^{-1/\sigma} + \mu_1 \frac{1+\tau}{\sigma(1-l)c^{1-1/\sigma}} + \mu_2 \frac{c^{-1/\sigma-1}}{\sigma\left(1+\tau\right)} - \mu_3 \left(1+\tau\right) + \mu_4 \tau = 0$$

$$[2]\ \ -\frac{1}{1-l} + \mu_1 \frac{1+\tau}{(1-l)^2 c^{-1/\sigma}} + \mu_3 A = 0$$

$$[3]\ \ g^{-1/\sigma_g} - \mu_4 = 0$$

$$[4]\ \ \mu_1 \frac{1}{(1-l)c^{-1/\sigma}} - \mu_2 \frac{c^{-1/\sigma}}{\left(1+\tau\right)^2} - \mu_3 c + \mu_4 c = 0$$

$$[5]\ \ -\mu_2 \psi \beta \frac{c^{-1/\sigma}}{1+\tau} + \mu_3 - \beta \left(\psi \bar{d} + 1 + r^*\right) \mu_3 = 0$$

$$[6]\ \ \frac{1+\tau}{(1-l)c^{-1/\sigma}} - A = 0$$

$$[7]\ \ \bar{d} - \left(1 + r^*\right) \bar{d} + Al - \left(1+\tau\right) c = 0$$

$$[8]\ \ d^g - \left(1 + r^*\right) d^g + \tau c - g = 0$$

on the following 9 unknowns

$$\{c, \tau, l, g, \bar{d}, \mu_1, \mu_2, \mu_3, \mu_4\}$$

with the following equation added to close the system

$$(\bar{d} + d^g)/Al = DtoY$$

where $DtoY$ is calibrated.

Note that such system can be further simplified setting $d^g = 0$. To see this note that

$$\begin{aligned}
\mu_4 - \beta\left(\psi_g \exp\left(d^g - \bar{d}^g\right) d^g + 1 + r^* + \psi_g \left[\exp\left(d^g - \bar{d}^g\right) - 1\right]\right)\mu_4 &= 0 \\
\beta\left(\psi_g \exp\left(d^g - \bar{d}^g\right) d^g + 1 + r^* + \psi_g \left[\exp\left(d^g - \bar{d}^g\right) - 1\right]\right) &= 1 \\
\psi_g \exp\left(d^g - \bar{d}^g\right) d^g + 1 + r^* + \psi_g \left[\exp\left(d^g - \bar{d}^g\right) - 1\right] &= 1/\beta \\
\psi_g \exp\left(d^g - \bar{d}^g\right) d^g + \psi_g \left[\exp\left(d^g - \bar{d}^g\right) - 1\right] &= 0 \\
\psi_g \exp\left(d^g - \bar{d}^g\right) d^g + \psi_g \exp\left(d^g - \bar{d}^g\right) &= \psi_g \\
\exp\left(d^g - \bar{d}^g\right) d^g + \exp\left(d^g - \bar{d}^g\right) &= 1 \\
\exp\left(d^g - \bar{d}^g\right)\left(d^g - 1\right) &= 1
\end{aligned}$$

where one solution (of possibly many) is one where $\bar{d}^g = 0$ and hence $d^g = 0$. In other words, one can assume that $\bar{d}^g = 0$ and then it follows endogenously that $d^g = 0$.

After introducing this further simplification and calibrating some of the parameters in this model (see text for details), we solve this system numerically.

6.3 SOE with Complete Asset Markets Model

Households' Problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, g_t, l_t),$$

Subject to:

$$E_t d_{t+1} r_{t+1} = d_t + A_t l_t - (1 + \tau_t) c_t$$

$$y_t = A_t l_t$$

$$\ln A_t = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon_t^A, \quad \varepsilon_t^A \sim NIID\left(0; \sigma_A^2\right)$$

and a no-Ponzi condition.

Lagrangian:

$$\pounds = E_0 \sum_{t=1}^{\infty} \beta^t U(c_t, g_t, l_t) + \beta^t \lambda_t [d_t + A_t l_t - (1+\tau_t) c_t - d_{t+1} r_{t+1}]$$

F.O.C.:

$$[c_t] : \beta^t U_{c_t}(c_t, g_t, l_t) - \beta^t \lambda_t (1+\tau_t) = 0$$

$$[l_t] : \beta^t U_{l_t}(c_t, g_t, l_t) + \beta^t \lambda_t A_t = 0$$

$$[d_{t+1}] : -\beta^t \lambda_t E_t r_{t+1} + \beta^{t+1} E_t \lambda_{t+1} = 0$$

Simplifying:

$$U_{c_t}(c_t, g_t, l_t) = \lambda_t (1+\tau_t)$$

$$U_{l_t}(c_t, g_t, l_t) = -\lambda_t A_t$$

$$\lambda_t E_t r_{t+1} = \beta E_t \lambda_{t+1}$$

Governments' flow budget constraint:

$$E_t d^g_{t+1} r_{t+1} = d^g_t - g_t + \tau_t c_t$$

Implementability conditions

$$A_t = -\frac{U_{l_t}(c_t, g_t, l_t)}{U_{c_t}(c_t, g_t, l_t)}(1+\tau_t) \equiv \Gamma_t(c_t, g_t, l_t, \tau_t)$$

$$\frac{U_{c_t}(c_t, g_t, l_t)}{1+\tau_t} = \lambda_t = \lambda_t(c_t, g_t, l_t, \tau_t)$$

Following Uribe and Schmitt-Grohé (2003),

$$\frac{U_{c_t}}{1+\tau_t} = \Psi_{CAM}$$

This condition is incorporated into the model through s and s^g. Where,

$$s_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} d_{t+1}$$

$$s_t^g = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} d_{t+1}^g$$

This definition came from the following condition of the Households' Problem:

$$[d_{t+1}] : -\lambda_t E_t r_{t+1} + \beta E_t \lambda_{t+1} = 0$$

$->$

$$E_t r_{t+1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}$$

We multiply both sides by debt,

$$E_t r_{t+1} d_{t+1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} d_{t+1}$$

$$E_t r_{t+1} d_{t+1}^g = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} d_{t+1}^g$$

Let's define,

$$s_t \equiv E_t r_{t+1} d_{t+1}$$

$$s_t^g \equiv E_t r_{t+1} d_{t+1}^g$$

Ramsey Problem

$$\pounds = E_0 \left\{ \begin{array}{c} \sum_{t=1}^{\infty} \beta^t U(c_t, g_t, l_t) \\ +\beta^t \mu_{1,t} [\Gamma_t - A_t] \\ +\beta^t \mu_{3,t} [d_t + A_t l_t - (1 + \tau_t) c_t - s_t] \\ +\beta^t \mu_{4,t} [d_t^g - g_t + \tau_t c_t - s_t^g] \end{array} \right\}$$

F.O.C.:

$$[c_t] : U_{c_t} + \mu_{1,t} \Gamma_{c_t} - \mu_{3,t} (1 + \tau_t) + \mu_{4,t} \tau_t = 0$$

$$[l_t] : U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{3,t}A_t = 0$$

$$[g_t] : U_{g_t} + \mu_{1,t}\Gamma_{g_t} - \mu_{4,t} = 0$$

$$[\tau_t] : \mu_{1,t}\Gamma_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t = 0$$

$$[d_{t+1}] : -\mu_{3,t}E_t(\lambda_{t+1}/\lambda_t) + E_t\mu_{3,t+1} = 0$$

$$[d^g_{t+1}] : -\mu_{4,t}E_t(\lambda_{t+1}/\lambda_t) + E_t\mu_{4,t+1} = 0$$

$$[\mu_{1,t}] : \Gamma_t - A_t = 0$$

$$[\mu_{3,t}] : d_t + A_t l_t - (1+\tau_t)c_t - s_t = 0$$

$$[\mu_{4,t}] : d^g_t - g_t + \tau_t c_t - s^g_t = 0$$

Parametrization

- Preferences:

$$U(c_t, g_t, l_t) = \frac{c_t^{1-1/\sigma} - 1}{1 - 1/\sigma} + \frac{g_t^{1-1/\sigma_g} - 1}{1 - 1/\sigma_g} + log(1 - l_t)$$

$$U_{c_t} = c_t^{-1/\sigma};\ U_{g_t} = g_t^{-1/\sigma_g};\ U_{l_t} = -\frac{1}{1 - l_t}$$

- Derivatives of Γ_t and λ_t:

$$\Gamma_t = -\frac{U_{l_t}(c_t, g_t, l_t)}{U_{c_t}(c_t, g_t, l_t)}(1+\tau_t) = \frac{1+\tau_t}{(1-l_t)c^{-1/\sigma}}$$

$$\Gamma_{c_t} = \frac{1+\tau_t}{\sigma(1-l_t)c_t^{1-1/\sigma}};\ \Gamma_{l_t} = \frac{1+\tau_t}{(1-l_t)^2 c_t^{-1/\sigma}}$$

$$\Gamma_{g_t} = 0;\ \Gamma_{\tau_t} = \frac{1}{(1-l_t)c_t^{-1/\sigma}}$$

Dynamic System

Model is a system of 13 endogenous and 1 exogenous variables

$$\{c_t, l_t, g_t, \tau_t, d_t, d_t^g, s_t, s_t^g, \mu_{1,t}, \mu_{3,t}, \mu_{4,t}, \lambda_t, y_t\} \text{ and } A_t, \text{ respectively.}$$

that are determined by 14 equations:

$$\begin{aligned} U_{c_t} + \mu_{1,t}\Gamma_{c_t} - \mu_{3,t}(1+\tau_t) + \mu_{4,t}\tau_t &= 0 \\ [1] \quad c_t^{-1/\sigma} + \mu_{1,t}\frac{1+\tau_t}{\sigma(1-l_t)c_t^{1-1/\sigma}} - \mu_{3,t}(1+\tau_t) + \mu_{4,t}\tau_t &= 0 \end{aligned}$$

$$\begin{aligned} U_{l_t} + \mu_{1,t}\Gamma_{l_t} + \mu_{3,t}A_t &= 0 \\ [2] \quad -\frac{1}{1-l_t} + \mu_{1,t}\frac{1+\tau_t}{(1-l_t)^2 c_t^{-1/\sigma}} + \mu_{3,t}A_t &= 0 \end{aligned}$$

$$\begin{aligned} U_{g_t} + \mu_{1,t}\Gamma_{g_t} - \mu_{4,t} &= 0 \\ [3] \quad g_t^{-1/\sigma_g} - \mu_{4,t} &= 0 \end{aligned}$$

$$\begin{aligned} \mu_{1,t}\Gamma_{\tau_t} - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0 \\ [4] \quad \mu_{1,t}\frac{1}{(1-l_t)c_t^{-1/\sigma}} - \mu_{3,t}c_t + \mu_{4,t}c_t &= 0 \end{aligned}$$

$$[5] \quad -\mu_{3,t}E_t(\lambda_{t+1}/\lambda_t) + E_t\mu_{3,t+1} = 0$$

$$[6] \quad -\mu_{4,t}E_t(\lambda_{t+1}/\lambda_t) + E_t\mu_{4,t+1} = 0$$

$$\begin{aligned} \Gamma_t - A_t &= 0 \\ [7] \quad \frac{1+\tau_t}{(1-l_t)c^{-1/\sigma}} - A_t &= 0 \end{aligned}$$

$$[8] \quad d_t + A_t l_t - (1+\tau_t)c_t - s_t = 0$$

$$[9] \quad d_t^g - g_t + \tau_t c_t - s_t^g = 0$$

$$[10]\quad s_{t+1} = \beta E_t(\lambda_{t+1}/\lambda_t) d_{t+1}$$

$$[11]\quad s^g_{t+1} = \beta E_t(\lambda_{t+1}/\lambda_t) d^g_{t+1}$$

$$[12]\quad \lambda_t - \Psi_{CAM} = 0$$

$$[13]\quad y_t - A_t l_t = 0$$

$$[14]\quad \ln A_t = (1-\rho_A)\ln \bar{A} + \rho_A \ln A_{t-1} + \varepsilon^A_t, \quad \varepsilon^A_t \sim NIID\left(0; \sigma^2_A\right)$$

with associated 6 parameters

$$\{\sigma, \sigma_g, \beta, \rho_A, \sigma^2_A, \bar{A},\}$$

Steady State System

The steady state is a system of 14 equations with 19 unknowns, adding 14 SS-variables

$$\{c, l, g, \tau, d, d^g, \mu_1, \mu_3, \mu_4, A, y, \lambda, s, s^g\}$$

and 5 parameters $\{\sigma, \sigma_g, \beta, \rho_A, \bar{A},\}$

$$c^{-1/\sigma} + \mu_1 \frac{1+\tau}{\sigma(1-l)c^{1-1/\sigma}} - \mu_3(1+\tau) + \mu_4\tau = 0$$

$$-\frac{1}{1-l} + \mu_1 \frac{1+\tau}{(1-l)^2 c^{-1/\sigma}} + \mu_3 A = 0$$

$$g^{-1/\sigma_g} - \mu_4 = 0$$

$$\mu_1 \frac{1}{(1-l)c^{-1/\sigma}} - \mu_3 c + \mu_4 c = 0$$

$$\frac{1+\tau}{(1-l)c^{-1/\sigma}} - A = 0$$

$$d + Al - (1+\tau)c - d = 0$$

$$d^g - g + \tau c - d^g = 0$$

$$\frac{d^g + d}{Al} - dy = 0$$

$$\lambda_t - \Psi_{CAM} = 0$$

$$s = d$$

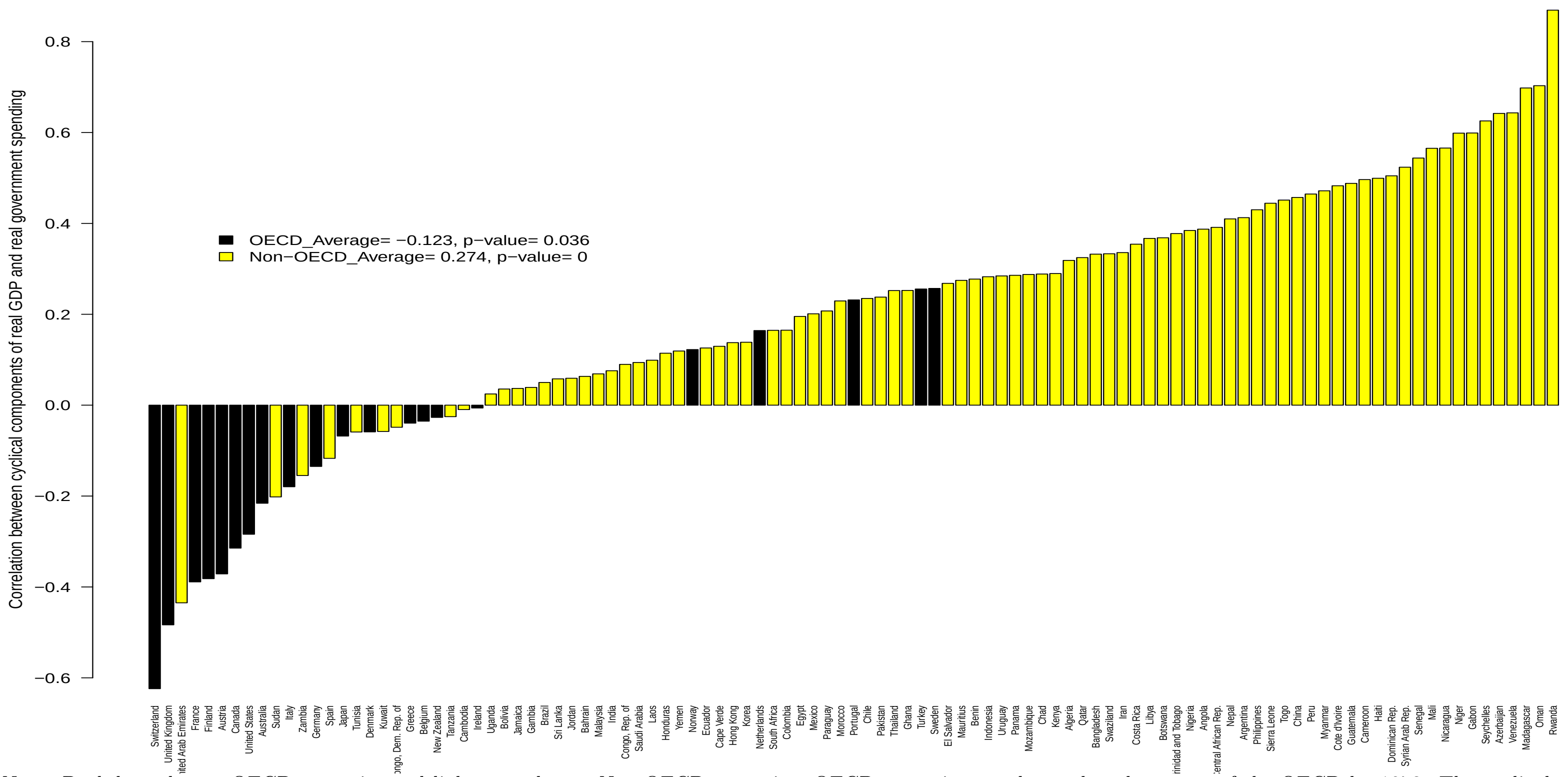
$$s^g = d^g$$

$$y = Al$$

$$\ln A = (1 - \rho_A) \ln \bar{A} + \rho_A \ln A$$

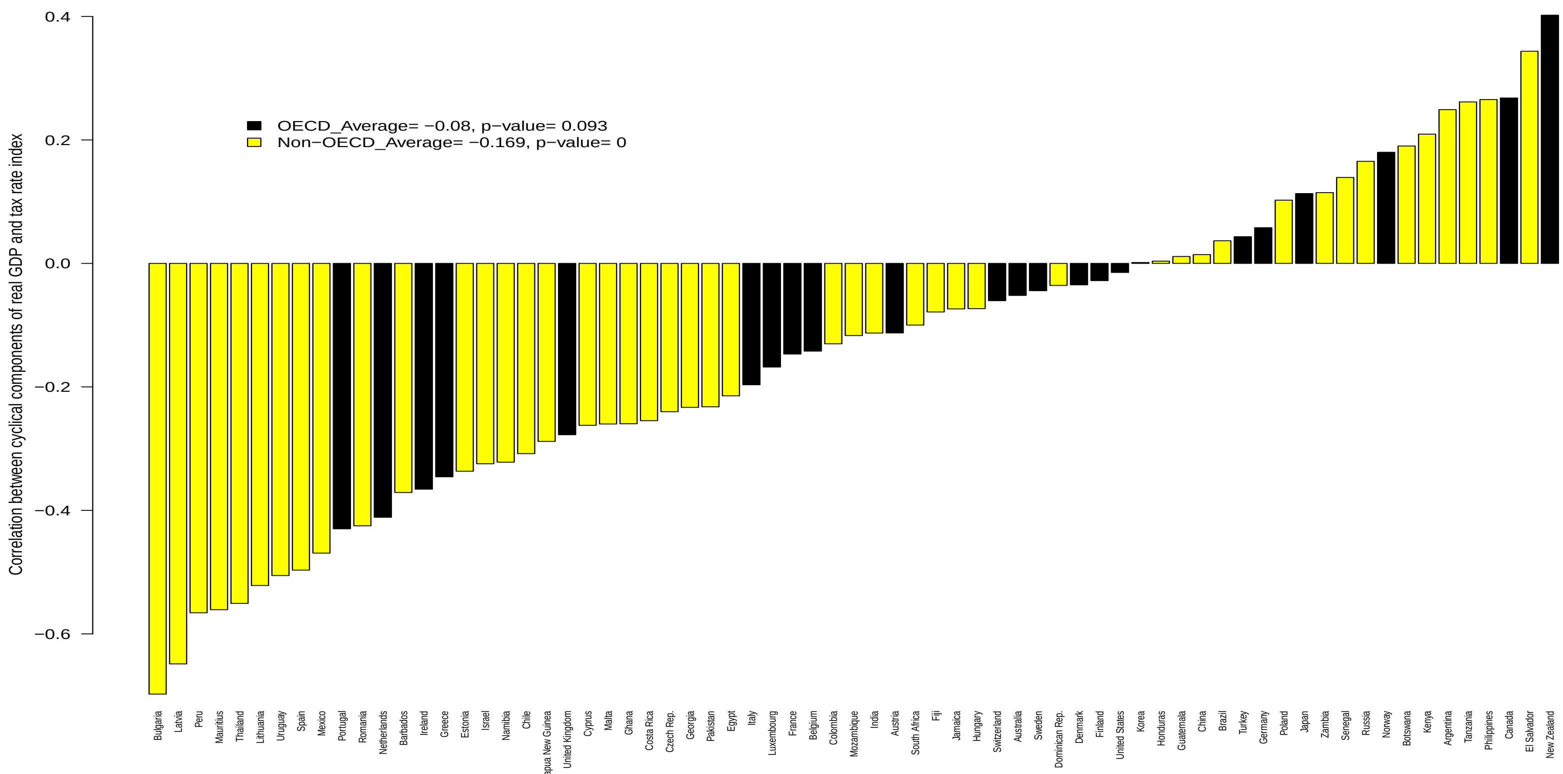
After calibrating some of these parameters (see text for details), we solve this system numerically.

Figure 1: Country correlations between the cyclical components of Real Government Expenditure and real GDP



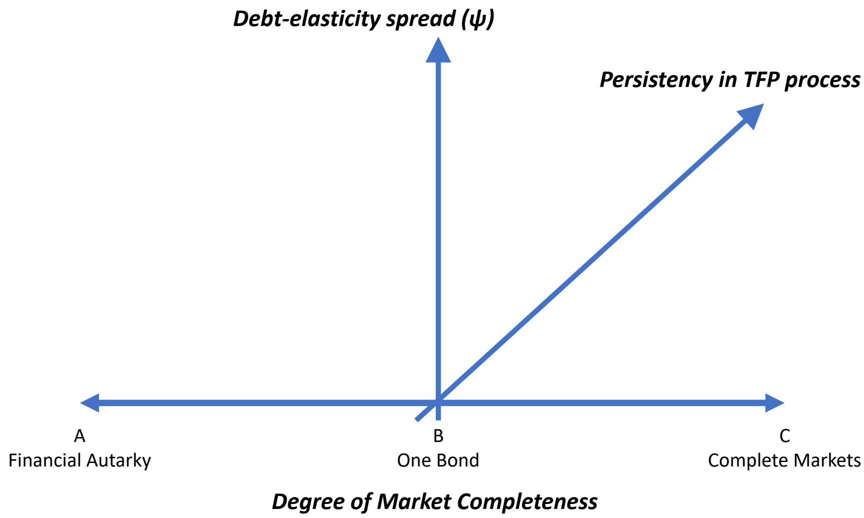
Note: Dark bars denote OECD countries and light ones denote Non-OECD countries. OECD countries are those who where part of the OECD by 1973. The cyclical components have been estimated using the Hodrick-Prescott filter. Real government expenditure is defined as central government expenditure and net lending deflated by the GDP deflator. A positive (negative) correlation indicates procyclical (countercyclical) fiscal policy. Sample is based on data availability, it includes 105 countries for period 1960-2017. The statistics presented in the legend correspond to the average of the correlations by country for each interest group. The p-values result from testing the hypothesis that the mean correlation is equal to zero within each group. Sources of raw data are Frankel et. al (2013) and Vegh and Vuletin (2015).

Figure 2: Country correlations between the percentage changes of Tax Index and Real GDP



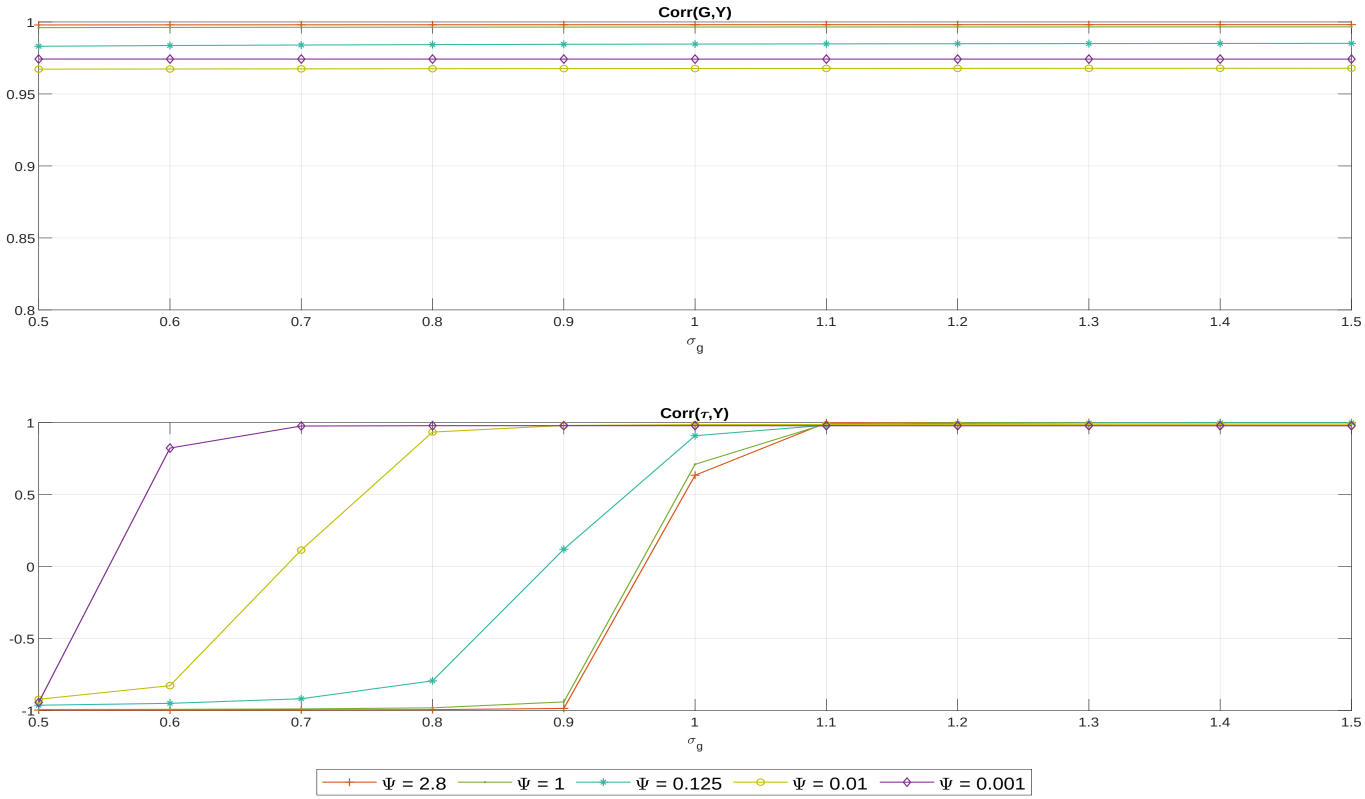
Note: Dark bars denote OECD countries and light ones denote Non-OECD countries. OECD countries are those who where part of the OECD by 1973. The percentages changes of tax index correspond to a weighted mix of the percentage changes in personal income, corporate income, and value-added tax rates. The weights used to build it capture the average importance of each tax in each country as a proportion of total tax revenues. A negative (positive) correlation indicates procyclical (countercyclical) fiscal policy. Sample is based on data availability, it includes 69 countries for period 1961-2013. The statistics presented in the legend correspond to the average of the correlations by country for each interest group. The p-values result from testing the hypothesis that the mean correlation is equal to zero within each group. Sources of raw data are Frankel et. al (2013) and Vegh and Vuletin (2015).

Figure 3: Scheme of Frictions Considered



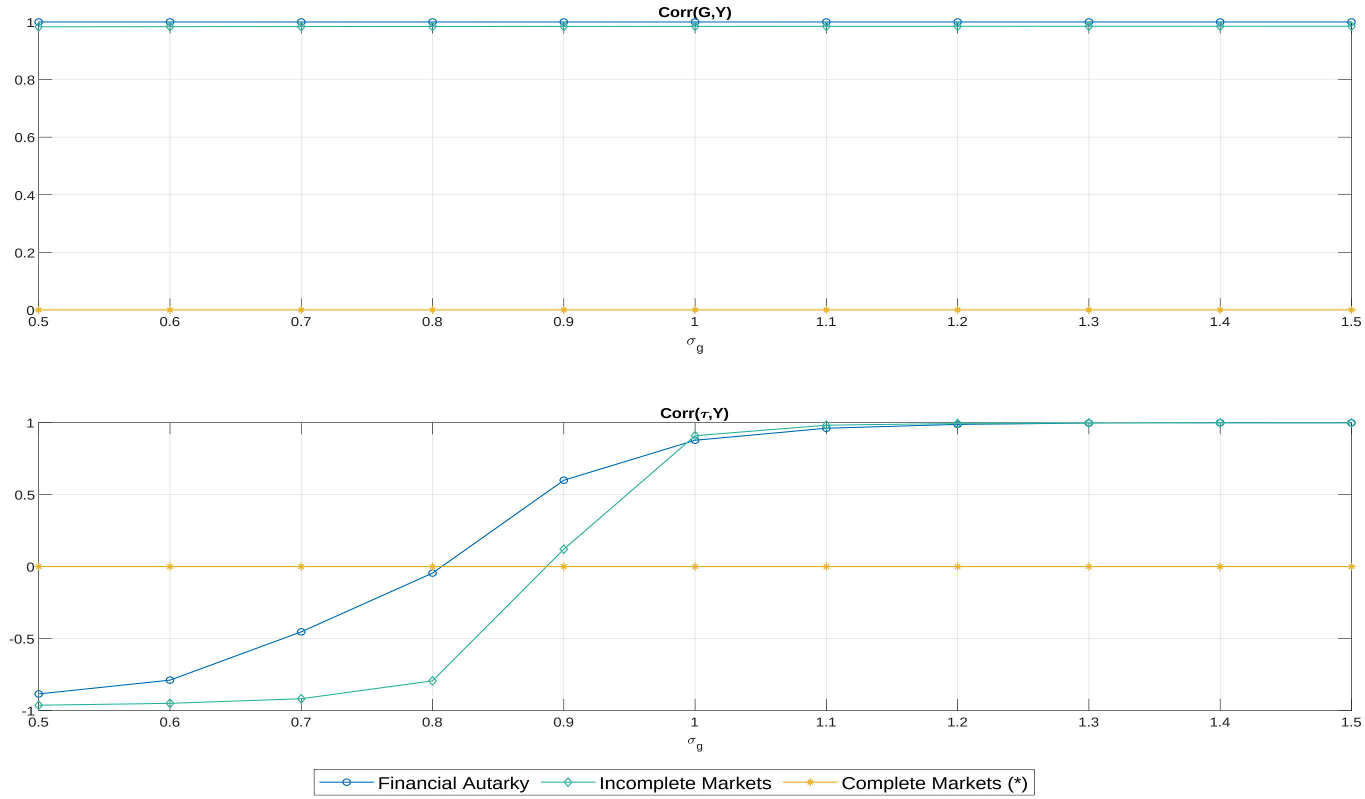
Note: The figure presents, in a schematic format, the dimensions of the three frictions considered in the model and their interactions. The horizontal axis depicts the various degrees of completeness in financial markets considered, with financial autarky and complete markets as two opposite extremes, and an intermediate case with a one period, non-state contingent bond. In the latter case, a varying debt elasticity of spreads interacts as the second friction considered, depicted by the vertical axis. The third friction, depicted by the diagonal axis, captures varying degrees of persistence (and, hence, volatility) of the TFP process.

Figure 4: Fiscal Procyclicality and the Intertemporal Substitution of Government Spending - Incomplete Asset Markets



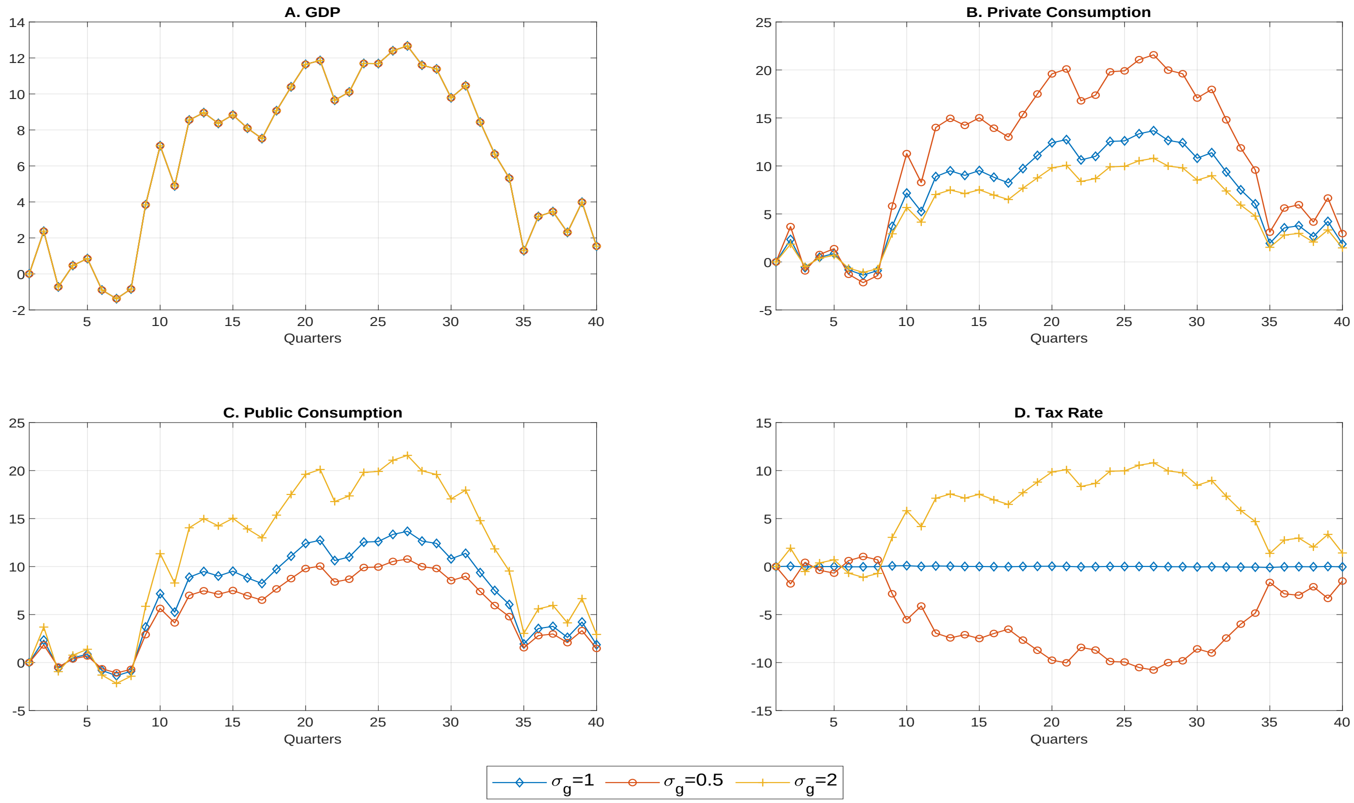
Note: The figure depicts the correlation between government consumption and output (first row) and between tax rates and output (second row) for various levels of the intertemporal elasticity of substitution of governement spending (σ_g). Correlations are computed from (HP-filtered) simulated data through 100.000 quarters randomly drawing TFP shocks only. Correlations shown are derived from varying the value of σ_g and the debt elasticity of the interest rate (ψ) in the framework of the Incomplete Asset Markets' model.

Figure 5: Fiscal Procyclicality and the Intertemporal Substitution of Government Spending - Market Completeness



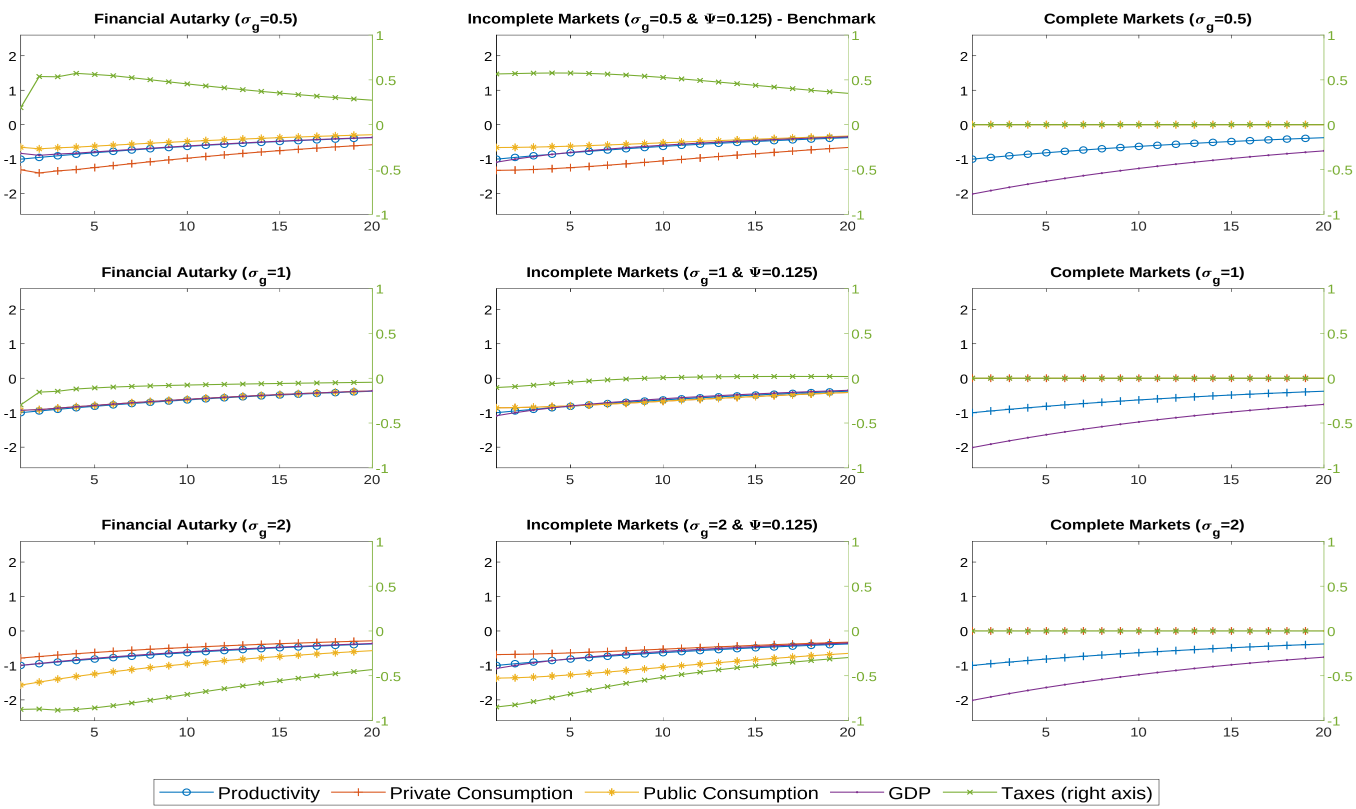
Note: The figure depicts the correlation between government consumption and output (first row) and between tax rates and output (second row) for various levels of the intertemporal elasticity of substitution of government spending (σ_g). Correlations are computed from (HP-filtered) simulated data through 100.000 quarters randomly drawing TFP shocks only. This figure presents the results under the three alternative cases of market completeness considered. (*) Given that under complete markets government expenditure and consumption are totally smoothed, it is not possible to calculate the correlations between government expenditure and output and between tax rates and output. For this reason, in the complete markets case the plot presents the covariance term.

Figure 6: Cyclical Dynamics and the Intertemporal Substitution of Government Spending: A Simulation



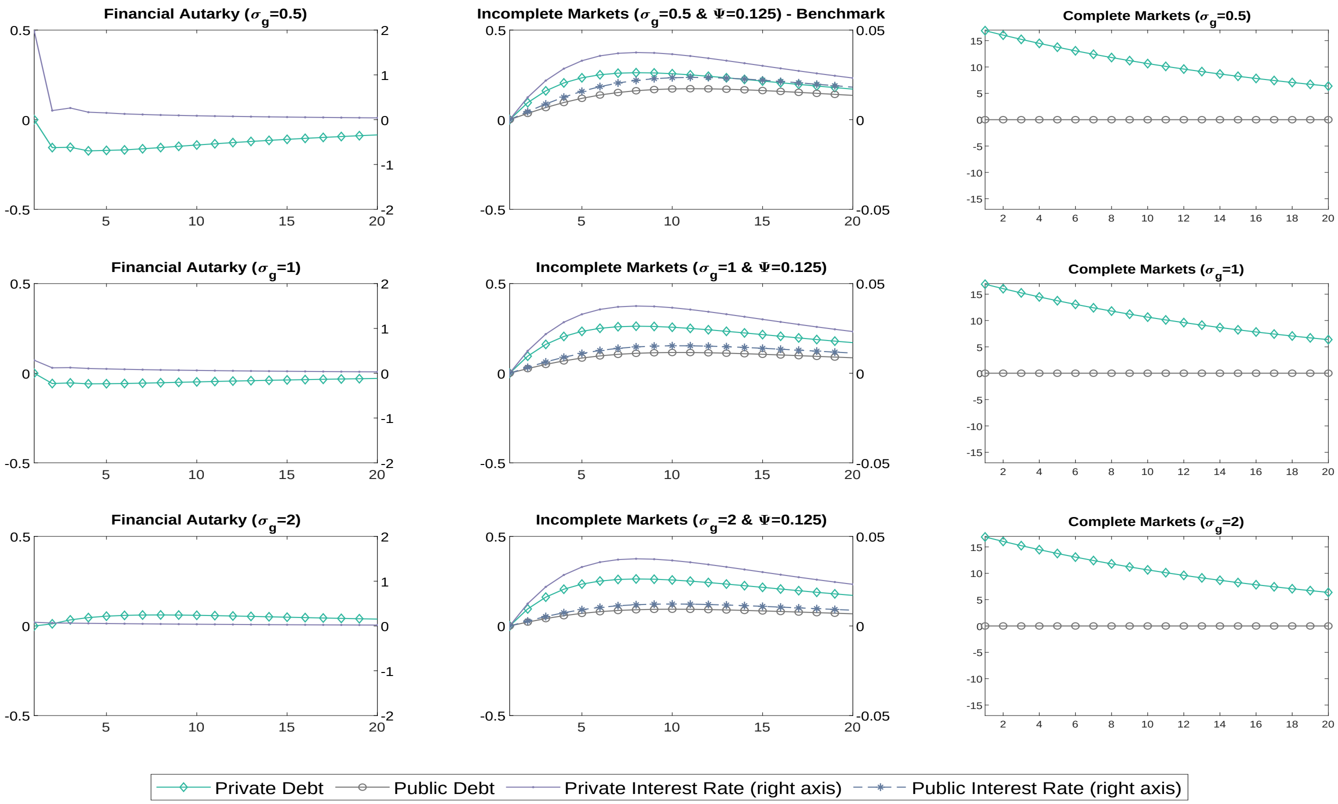
Note: The figure depicts the deviations from steady state for GDP, Private Consumption, Public Consumption, and Tax Rates from randomly drawing TFP shocks during 40 quarters. For visual purposes, in the simulation the parameter governing the debt elasticity of interest rates (ψ) was set at a higher value than in our benchmark calibration (2.8). Each of the three times series considered per variable is associated to an alternative value for the intertemporal elasticity of substitution of government spending (σ_g).

Figure 7A: Varying Degree of Market Completeness: Impulse Response Functions



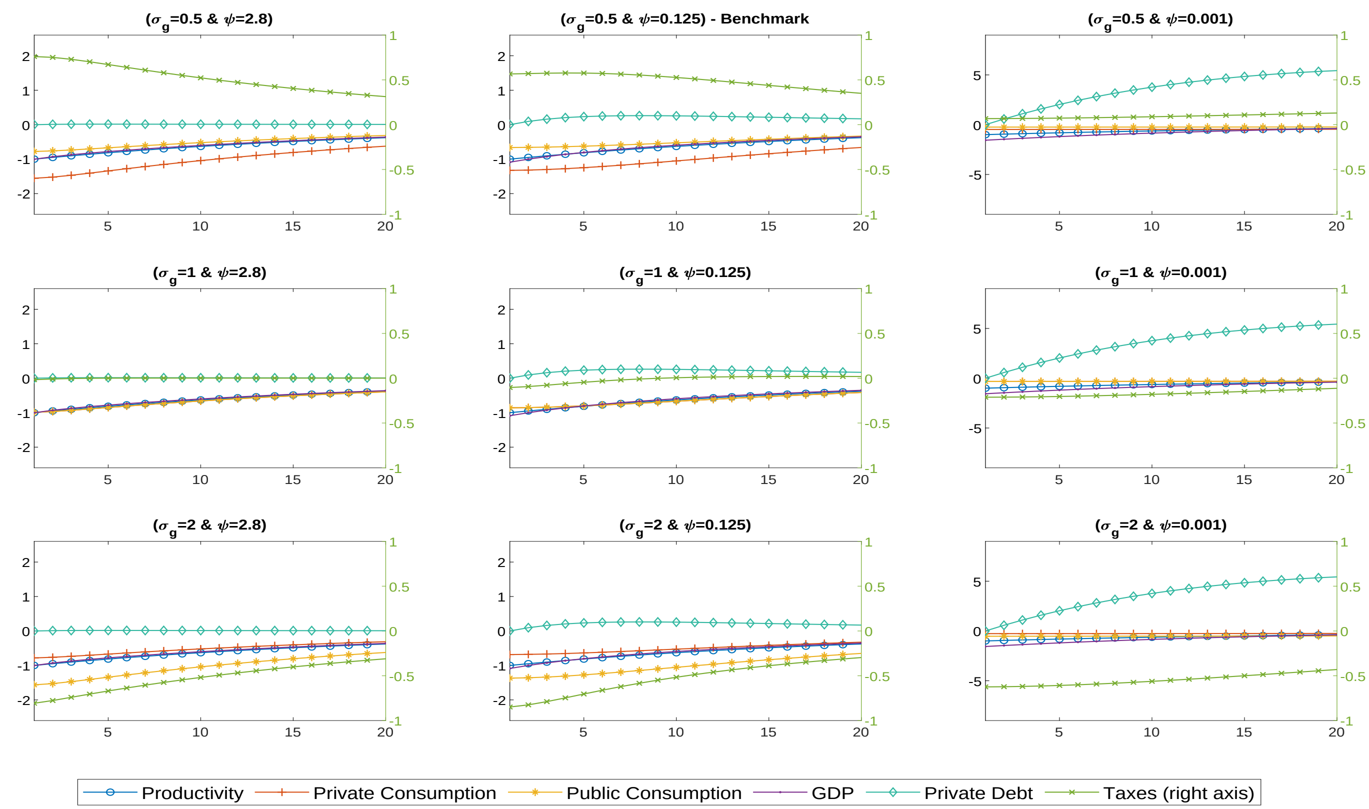
Note: The figure displays the IRFs of Private Consumption, Public Consumption, GDP, and Taxes following a fall of TFP by one percentage point relative to steady state. The responses are expressed in percentage deviation from steady state levels. The Figure documents the IRF through 20 quarters, including the initial one ($t = 1$) where the shock occurs. The three columns depict the three alternative cases of market completeness considered in the analysis. The difference across rows is the value for the intertemporal elasticity of substitution of government consumption (σ_g).

Figure 7B: Varying Degree of Market Completeness: Impulse Response Functions



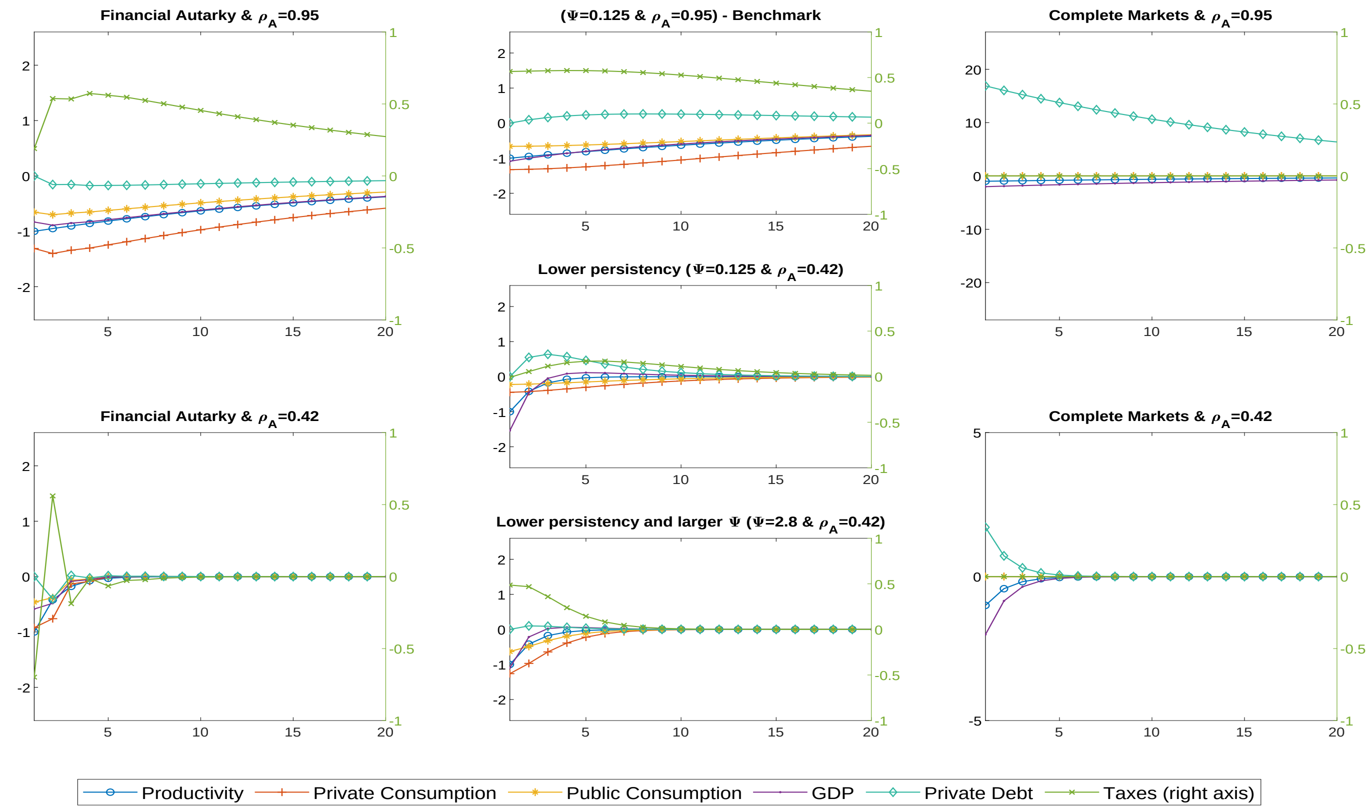
Note: The figure displays the IRFs of Private debt, Public debt, Private interest rate, and Public interest rate following a fall of TFP by one percentage point relative to steady state. The responses are expressed in linear deviation from the steady states levels. The Figure documents the IRF through 20 quarters, including the initial one ($t = 1$) where the shock occurs. The three columns depict the three alternative cases of market completeness considered in the analysis. The difference across rows is the value for the intertemporal elasticity of substitution of government consumption (σ_g).

Figure 8: Varying Debt Elasticity of Spreads under Incomplete Markets: Impulse Response Functions



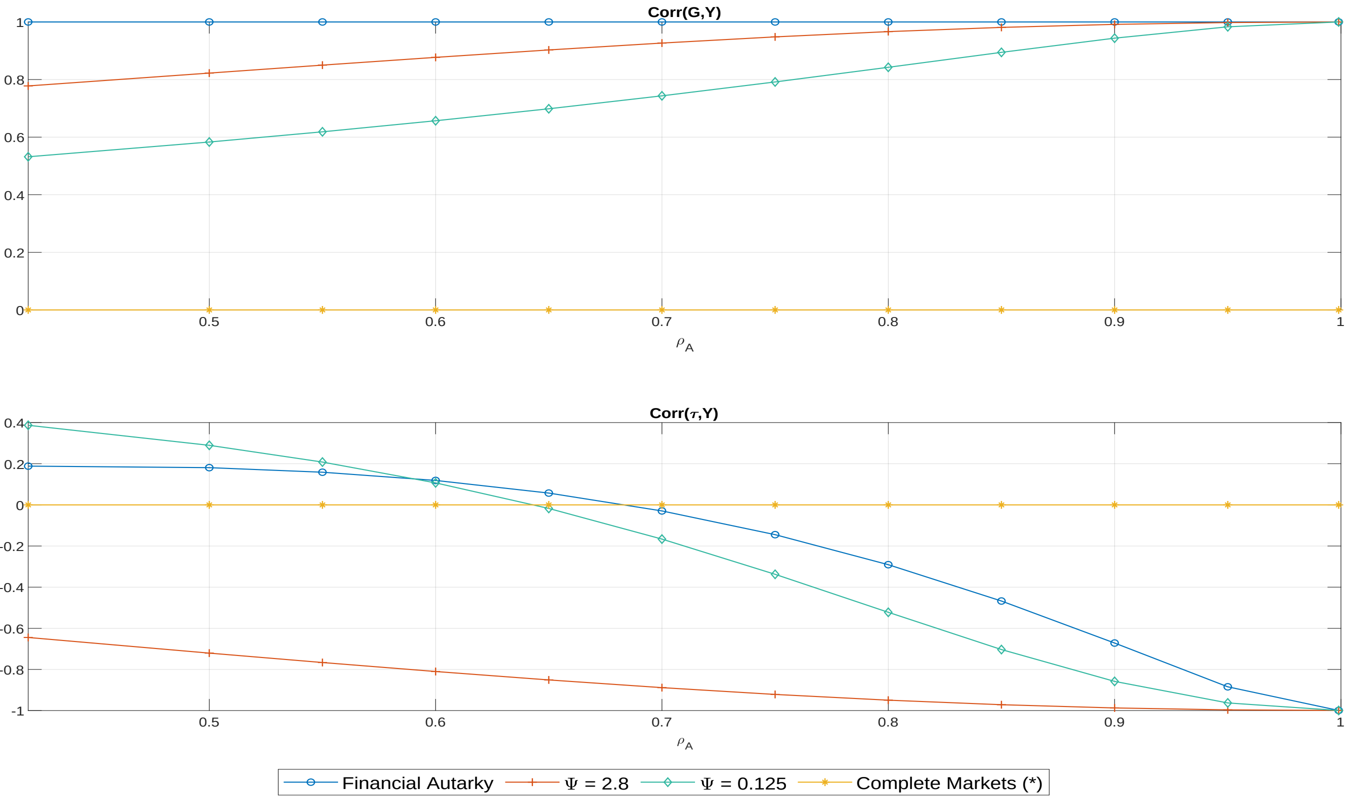
Note: The figure displays the IRFs of Private Consumption, Public Consumption, GDP, Private Debt, and Taxes following a fall of TFP by one percentage point relative to steady state. The responses are expressed in percentage deviation from steady state levels, except for the private debt which is expressed in linear deviation. The Figure documents the IRF through 20 quarters, including the initial one ($t = 1$) where the shock occurs. The three columns depict three alternative values of the debt elasticity of the interest rate (ψ). The difference across rows is the value for the intertemporal elasticity of substitution of government consumption (σ_g). All IRFs were computed under the case on incomplete markets, assuming a one period non-state contingent bond.

Figure 9: Varying TFP Persistence: Impulse Response Functions



Note: The figure displays the IRFs of Private Consumption, Public Consumption, GDP, Private Debt, and Taxes following a fall of TFP by one percentage point relative to steady state. The responses are expressed in percentage deviation from steady state levels, except for the private debt which is expressed in linear deviation. The Figure documents the IRF through 20 quarters, including the initial one ($t = 1$) where the shock occurs. The three columns depict the three alternative cases of market completeness considered in the analysis. The difference across rows in left column and right column is the value for the the persistency of the TFP process (ρ_A). In the middle column, under the framework of the Incomplete Asset Markets' model, both the ρ_A and the debt elasticity of the interest rate (ψ) varies.

Figure 10: Fiscal Procyclicality and the Persistence of TFP



Note: The figure depicts the correlation between government consumption and output (first row) and between tax rates and output (second row) for various levels of persistency in the TFP process, as governed by the AR(1) coefficient of its law of motion - depicted in the horizontal axis. Correlations are computed from (HP-filtered) simulated data through 100.000 quarters randomly drawing TFP shocks only. Results are reported under the three alternative cases of market incompleteness considered in the analysis, though for the case of incomplete markets two values of the debt elasticity of the interest rate (ψ) are considered. (*) Given that under complete markets government expenditure and consumption are totally smoothed, it is not possible to calculate the correlations between government expenditure and output and between tax rates and output. For this reason, in the complete markets case the plot presents the covariance term. For this exercise, σ_g and σ_c are set at 0.5 and 1, respectively.

Table 1: Calibration

Parameter	Description	Value	Source
$\bar{A}$	TFP in SS	1	Assumed
$\bar{R}^*$	Real Interest Rate	$1.04^{\frac{1}{4}}$	Schmitt-Grohé-Uribe (2003)
$(\bar{d}+\bar{d}^g)/y$	Foreign Debt Share	1.34	NFA data (see below)
ρ_A	TFP AR(1)	varies	Neumeyer-Perri (2005)/MSM
σ_A	SD TFP Shock	varies	Schmitt-Grohé-Uribe (2003)/MSM
ρ_{R^*}	Interest Rate AR(1)	0.83	Uribe and Yue (2006)
σ_{R^*}	SD Interest Rate Shock	0.007	Uribe and Yue (2006)
β	Discount factor	$1/1.04^{\frac{1}{4}}$	Endogenous $\beta\bar{R}^*=1$
$\Psi=\Psi^g$	Debt-elasticity	varies	Garcia-Cicco et al. (2010)/Own Estimates
σ_c	IES Private Consumption	1	Assumed
σ_g	IES Public Consumption	varies	MSM
ϕ	Debt Adjustment Cost parameter in the Financial Autarky Model	0.1	Assumed

Note: This table summarizes the calibration of the parameters in the DSGE model. These values mostly rely on previous studies of small open economies noted in the column "Source". The persistency of the TFP process (ρ_A), the volatility of the TFP shocks (σ_A), the debt elasticity of the interest rate in both public and private debt, (ψ) and (ψ^g), and the intertemporal elasticity of substitution of public consumption (σ_g) will vary through experiments considered in the analysis. Nevertheless, in the benchmark case these parameters are set at 0.95, 0.0129, 0.125, and 0.5, respectively. Results from matching second moments (MSM) are presented in tables 3.3 and C.4.

Table 2: Varying Degree of Market Completeness: Second Moments

(1) $\sigma_c = 1; \sigma_g = 0.5$			
Moments	A. Fin. Autarky	B. Incomplete Markets	C. Complete Markets (*)
Std(y)	0.015	0.018	0.034
Std(c)	0.024	0.023	0.000
Std(g)	0.012	0.012	0.000
Std(τ)	0.010	0.010	0.000
Corr(τ, y)	-0.884	-0.963	0.000
Corr(g,y)	1.000	0.983	0.000
Corr(c,y)	1.000	0.983	0.000
Corr((c/g),y)	1.000	0.983	0.000

(2) $\sigma_c = 1; \sigma_g = 1$			
Moments	A. Fin. Autarky	B. Incomplete Markets	C. Complete Markets (*)
Std(y)	0.016	0.018	0.034
Std(c)	0.016	0.015	0.000
Std(g)	0.016	0.015	0.000
Std(τ)	0.004	0.002	0.000
Corr(τ, y)	0.877	0.909	0.000
Corr(g,y)	1.000	0.985	0.000
Corr(c,y)	1.000	0.985	0.000
Corr((c/g),y)	-0.377	0.993	0.000

(3) $\sigma_c = 1; \sigma_g = 2$			
Moments	A. Fin. Autarky	B. Incomplete Markets	C. Complete Markets (*)
Std(y)	0.017	0.018	0.034
Std(c)	0.013	0.012	0.000
Std(g)	0.026	0.024	0.000
Std(τ)	0.016	0.015	0.000
Corr(τ, y)	0.987	0.997	0.000
Corr(g,y)	1.000	0.985	0.000
Corr(c,y)	1.000	0.985	0.000
Corr((c/g),y)	-1.000	-0.985	0.000

Note: This table presents second moments of key variables in the model. These moments are computed from (HP-filtered) simulated data through 100.000 quarters randomly drawing TFP shocks only. The three columns depict the three alternative cases of market completeness considered in the analysis. The difference across panels is the value for the intertemporal elasticity of substitution of government consumption (σ_g). (*) Given that under complete markets government expenditure and consumption are totally smoothed, it is not possible to calculate the correlations between government expenditure and output and between tax rates and output. For this reason, in the complete markets case the plot presents the covariance term.

Table 3: Matching Second Moments

Targeted moments	$\sigma_y; \sigma_c; \rho_{\tau,y}; \rho_{g,y}$	
Calibration obtained via MSM	$\sigma_g = 0.25, \sigma_A = 0.005,$ $\psi = 1, \rho_A = 0.95$	
Moments	Data	Model
Std(y)	0.017	0.010
Std(c)	0.026	0.018
Std(g)	0.056	0.005
Std(τ)	0.054	0.014
Corr(τ, y)	-0.104	-0.058
Corr(g,y)	0.142	0.143
Corr(c,y)	0.629	0.143
Corr((c/g),y)	0.126	0.143

Note: The calibration procedure that support the results presented in this table is constituted by the following steps: (1) a grid is defined with all the possible combinations of values that the parameters to be calibrated can take. These parameters are the intertemporal elasticity of substitution of government consumption (σ_g), the volatility of the TFP shocks (σ_A), the debt elasticity of the interest rate (ψ), and the persistency of the TFP process (ρ_A), (2) on each point of the grid the dynamics of the model are simulated and the target moments (the volatility of output (σ_y), the volatility of consumption (σ_c), the correlation between taxes and output ($\rho_{\tau,y}$), and the correlation between government spending and output ($\rho_{g,y}$)) are calculated from the Hodrick-Prescott (HP) filtered series. Both the simulation of the model and the calculation of the moments are made considering simultaneous random TFP shocks and world interest rate shocks, (3) a quadratic loss function between the estimated theoretical moments and the moments in the data is computed, and finally, (4) the combination of values of σ_g, σ_A, ψ, and ρ_A for which the function takes the lowest value is selected.

Table 4: Costs of Business Cycles

	Moments	Non-OECD	OECD
Data	Std(Y)	3.3	1.5
	Std(C)	4.8	1.5
	Std(C)/Std(Y)	1.5	1.0
DSGE Model	Std(Y)	3.3	1.3
	Std(C)	4.9	1.5
	Std(C)/Std(Y)	1.5	1.1
	Std(A)	7.9	1.8
	Lucas-type cost of Business Cycles (%)	0.131	0.013

Note: Empirical moments come from the pool of countries in the OECD and non-OECD groups in Table 5. When calibrating the DSGE model for this exercise, all parameters are held constant and equal across the two groups of countries, except for those in the TFP process (persistence and volatility of TFP shock). Values for the parameters are pinned down by matching the S.D of consumption and income across the two groups. Costs of business cycles are computed following Lucas (1987): they capture the percentage fall of steady consumption that makes households indifferent with an environment of shocks. Costs are computed with a second order approximation of the welfare function.

Table 5: Empirical Results

(1) Procyclicality Measure							
	Non-OECD		OECD				
Procyclicality Measure	Average	S.D	Average	S.D	T Stat	C.V	Test Result
G	0.27	0.23	-0.12	0.25	6.58	1.31	Reject H_0
Tax Index	-0.17	0.27	-0.08	0.21	1.48	1.3	Reject H_0
(2) Market Incompleteness							
	Non-OECD		OECD				
Capital Controls Measure	Average	S.D	Average	S.D	T Stat	C.V	Test Result
kai	0.41	0.3	0.1	0.09	7.42	1.29	Reject H_0
boi	0.39	0.37	0.03	0.1	7.04	1.29	Reject H_0
kao	0.44	0.36	0.11	0.12	6.48	1.29	Reject H_0
boo	0.51	0.4	0.15	0.17	5.73	1.29	Reject H_0
(3) Debt Elasticity - Panel							
	Non-OECD		OECD				
Reg	Coeff		Coeff		Wald Stat	C.V	Test Result
1 (Total Public Debt)	0.125***		0.002		61.18	3.84	Reject H_0
2 (NFA)	0.062***		0.026*		16.40	3.84	Reject H_0
(4) GDP Volatility							
	Non-OECD		OECD				
Business Cycle Measure	Average	S.D	Average	S.D	T Stat	C.V	Test Result
σ_Y	3.28	3.07	1.47	0.46	5.64	1.29	Reject H_0

Note 1 (Panel 1): The statistics displayed in this panel are based on the full sample of countries that we have for each procyclicality measure. For G and TaxIndex the number of countries that we have is, respectively, 105 and 69. The last column represents the result of the (one-tail) hypothesis test that the means for both groups of countries are equal for each procyclicality measure. Note 2 (Panel 2): We used the capital controls indexes from the updated version of Fernandez et al (2016), that has a set of 87 countries for which there is information on measures of procyclicality. The four specific indexes that we use to capture the degree of controls across capital flows are: overall assets inflow restrictions index (kai); bond inflow restrictions (boi); overall assets outflow restrictions index (kao); and bond outflow restrictions (boo). The indexes for each country are normalized from zero to one, such that higher degree of capital controls implies and index closer to one. The last column represents the result of the (one-tail) hypothesis test that the means for both groups of countries, Non-OECD and OECD, are equal for each capital controls index. Note 3 (Panel 3): ***,** and * are significant at, respectively, 1%, 5% and 10% levels of significance. OECD countries are those who where part of the OECD by 1973. Non-OECD countries are the rest for which we have data on their spread index (EMBIG or their UIP spread in the case of countries considered developed by the BofA Merril Lynch Bond Index Guide but that are not within our OECD group). The number of countries for each regression is 16 OECD and 20 Non-OECD in regression 1, and 17 OECD and 29 Non- OECD in regression 2. The last column represents the result of the (one-tail) hypothesis test that the coefficients for both groups of countries are equal in each regression. Note 4 (Panel 4): The statistics displayed in this panel are based on a sample of 123 countries. (σ_Y) corresponds to the volatility of the cyclical component of the HP-filtered data for the yearly real GDP of those countries. The last column represents the result of the (one-tail) hypothesis test that the means of (σ_Y) for both groups of countries are equal.