Good Jobs and Bad Jobs over the Business Cycle:  
Implications for Inclusive Monetary Policy*

ChaeWon Baek †

Job Market Paper

This draft: January 10, 2021

Click here for the most recent version

Abstract

The costs of business cycles are not evenly distributed in the economy. Nevertheless, monetary policies have relied only on “aggregate” labor market variables, ignoring this heterogeneity in the incidence of economic fluctuations. How can monetary policy be more inclusive and benefit people who are more vulnerable to economic fluctuations? To shed light on this question, which is at the core of policy discussions, I study heterogeneity (“types”) in labor market arrangements and implications of this heterogeneity for welfare and optimal monetary policy. I document that the experiences of regular and irregular workers over the business cycle differ considerably. For example, the share of irregular workers in employment rises during recessions, suggesting that firms actively adjust labor composition over the business cycle. I develop a tractable New Keynesian model with regular and irregular labor types that reflect the cyclical nature of labor composition. I find that workers, who are marginally attached to either the regular or the irregular labor market, face larger volatilities in their consumption and disutility from labor supply and hence suffer larger welfare losses over the business cycle. I find that optimal monetary policy rule should react to employment dynamics in specific segments of the labor market than the overall stance of the labor market. When a central bank follows that rule, it benefits not only people who are more vulnerable to economic fluctuations but generate higher economy-wide welfare.

Keywords: optimal monetary policy, distributional analysis, labor market composition, regular workers, irregular workers

*I am deeply indebted to Yuriy Gorodnichenko as well as David Romer and Benjamin Schoefer for their continuous support and invaluable guidance. I am very grateful to Pierre-Olivier Gourinchas and Byoungchan Lee for their valuable comments and helpful discussions. I also thank Woojin Kim, Todd Messer, Peter B. McCrory, Preston Mui, Claudia Sahm, Nick Sander, Jacob Weber, Vitaliia Yaremko, and Chaoran Yu, as well as seminar participants at the University of California, Berkeley. I also gratefully acknowledge financial support from the Institute for Research on Labor and Employment.

†Department of Economics, University of California, Berkeley. E-mail: 100cwbcw@berkeley.edu.
“With regard to the employment side of our mandate, our revised statement emphasizes that maximum employment is a broad-based and inclusive goal. This change reflects our appreciation for the benefits of a strong labor market, particularly for many in low- and moderate-income communities.”

1 Introduction

When the Federal Reserve Chair Jerome Powell unveiled the new framework for monetary policy this summer, he emphasized that maximum employment is a broad-based and inclusive goal for the Federal Reserve. He stressed that a strong labor market benefits particularly people from low- and moderate-income communities who are more vulnerable to economic fluctuations. The costs of business cycles are not evenly distributed in the economy. Some groups of people suffer more from economic fluctuations. Nevertheless, monetary policies have relied only on “aggregate” labor market variables, ignoring this heterogeneity in the incidence of business cycle costs. Can monetary policies be more inclusive? How should monetary policies be implemented to ease the burden of the vulnerable and yet to increase aggregate welfare? This paper seeks to answer these questions by shedding new light on heterogeneity (“types”) in labor market arrangements and the implications of this heterogeneity for welfare and optimal monetary policy.

To study the differential experience of different types, I first classify various types of labor arrangements into two broad types: permanent, full-time jobs (“regular types”) and temporary jobs, independent contractors, part-time jobs, and so on (“irregular types”). Regular types have higher productivity but difficult to hire/fire and irregular types have lower productivity but easier to hire/fire. Using microdata from the Current Population Survey (CPS) in the United States and full-time and part-time jobs as proxies for regular and irregular jobs, I document four stylized facts. First, the share of irregular jobs rises during recessions. Second, many workers move directly from regular to irregular jobs during economic downturns. These direct flows are not only highly countercyclical but also quantitatively large. Third, the share of irregular workers signifi-

1The latter group has received a number of names in the literature, e.g., marginally attached workers, temporary workers, informal jobs, part-time workers, etc. I will use “irregular workers” to highlight that this group is likely to be least protected from business cycle fluctuations.
2I validate the use of full-time and part-time jobs as proxies for regular and irregular jobs in Appendix A.1.
3Heterogeneous experiences of different types of labor over the business cycle are well documented in the literature.
cantly rises during recessions, because many regular workers directly move to irregular jobs, not because those out of the labor force enter the irregular labor market. Fourth, while the number of regular workers increases in response to a positive government spending shock, the number of irregular workers decreases. Similarly, the number of people who participate in the regular labor market significantly decreases in response to a contractionary monetary policy shock. The opposite is true for the number of those participating in the irregular labor market. This illustrates that regular and irregular jobs exhibit differential dynamics in response to shocks.

Informed by these facts from the data, I develop a tractable New Keynesian model following an approach of [Galí (2011, 2020)] and [Christiano, Trabandt, and Walentin (2020)]. who reformulate the standard New Keynesian framework to incorporate unemployment. In particular, my paper extends the modeling strategy of [Christiano, Trabandt, and Walentin (2020)]. Importantly, the approach of [Christiano, Trabandt, and Walentin (2020)] deviates from the assumption of perfect consumption insurance against individual workers’ labor market outcomes. This feature enables me to have a meaningful heterogeneity in consumption (and hence welfare) over the business cycle for individual workers who face different labor market risks.

Unlike [Christiano, Trabandt, and Walentin (2020)], my model features two types of labor. Therefore, it is possible to examine the differential dynamics between the two labor markets and different income risks that different types of workers face. Specifically, I introduce regular and irregular types of labor into [Christiano, Trabandt, and Walentin (2020)]. When firms decide to adjust the total labor input, firms either newly create or destruct each type of job or they transfer one type to the other via promotion and demotion. That is, they alter the composition of job types. Workers choose which labor market to participate in and what types of jobs to seek. Workers and firms make decisions over labor types every period, which leads to changes in the composition of labor types. I call this margin of labor adjustment “the composition margin” and examine its implications for welfare and monetary policy in a standard New Keynesian model with sticky prices.

After calibrating my model, I show that my framework can successfully replicate the empirical patterns. My model can generate differential dynamics of the regular and the irregular labor markets in response to aggregate shocks. For example, consistent with the results from the data,

Particularly in the context of the United States, there are empirical studies examining the distinct feature of involuntary part-time workers from full-time workers during the Great Recessions (see, for example, [Canon et al., 2014], [Cajner et al., 2014], [Warren, 2017], [Larran, 2018], [Mukoyama, Shintani, and Teramoto, 2019], [Borowczyk-Martins and Lale, 2019]).
firms decrease the number of regular workers but increase the number of irregular workers in response to a contractionary monetary policy shock. This, in turn, generates a large increase in the share of irregular workers. I demonstrate that aggregate (un)employment dynamics mask considerable heterogeneity in outcomes for different labor types. The changes in the relative demand for the two types make some portion of workers move frequently between the two labor markets, increasing these workers’ uncertainties in their labor market outcomes.

I use my model to explore heterogeneity in the costs of economic fluctuations borne by different workers. I show that workers, who are likely to move between the regular and irregular labor markets (contingent “regular” workers) or between the irregular labor market and not-in-the-labor-force (contingent “irregular” workers), pay substantially larger welfare costs over the business cycle. Workers who are marginally attached to either the regular or the irregular jobs, contingent regular and irregular workers, respectively, encounter higher uncertainties over their labor market status. Hence, they face the risks of larger fluctuations in their disutility from supplying labor and consumption, which varies according to their labor market status with the imperfect consumption insurance. In addition, the levels of consumption per each labor market status themselves vary over the business cycles, which makes those workers experience the largest consumption volatility. Contingent regular workers, in particular, pay the highest costs of economic fluctuations as they face the largest risks regarding their labor market status. Moreover, the share of contingent regular workers is substantially larger than the share of contingent irregular workers, with the changes of the composition of worker types.

Can the monetary authority achieve higher aggregate welfare by stabilizing the income fluctuations of these contingent workers? I find that an alternative interest rate (Taylor) rule, which stabilizes the labor-type composition based on the size of each labor market, not only improves the welfare of contingent workers but also achieves higher economy-wide welfare. This alternative monetary policy rule stabilizing the composition of labor types can minimize the number of contingent workers and the movements of them between the two labor markets. As a result, it can stabilize the consumption volatility they experience over the business cycle. This alternative specification of a Taylor rule achieves higher aggregate welfare than the conventional specification with the overall unemployment gaps. These results suggest that the central bank can achieve higher aggregate welfare by targeting more vulnerable groups, who move across the two labor
markets in response to the business cycle, rather than by focusing on the overall stance of the labor market.

**Related Literature**

My paper studies optimal monetary policy when incorporating the heterogeneity in labor market arrangements. In this regard, the most closely related paper is Gali (2020). He studies monetary policy implications of introducing insider-outsider labor markets into a New Keynesian model embedding a theory of involuntary unemployment of Gali (2011). In contrast to Gali (2020), my approach features imperfect consumption insurance, and therefore the level of consumption is different according to labor market status. This distinct feature enables me to explore heterogeneity in the costs of economic fluctuations borne by different workers according to labor market status, and therefore to examine if monetary policy can be more inclusive.

With regular and irregular work arrangements, my paper examines differential labor market dynamics between the two labor markets. In this regard, my work is closely related to the literature studying labor market dynamics with two types of labor (see, for example, Blanchard and Summers, 1986; Alonso-Borrego, Fernández-Villaverde, and Galdón-Sánchez, 2005; Mukoyama, Shintani, and Teramoto, 2019). Among them, the most closely related paper is Mukoyama, Shintani, and Teramoto (2019). They introduce the full-time and the part-time labor markets into monetary DSGE models and generate differential labor market dynamics between the two labor markets. However, I explore different mechanisms than theirs. While Mukoyama, Shintani, and Teramoto (2019) focus on the on-the-job search of part-timers for full-time jobs, my model focuses more on the firm side. In my model, firms’ ability to transfer one type of labor to the other via promotion and/or demotion is the key to generate the opposite responses of the two labor market variables. Moreover, in contrast to Mukoyama, Shintani, and Teramoto (2019), my work explores the welfare and policy implications of incorporating the two types of labor.

Lastly, my paper contributes to explore heterogeneity in the costs of economic fluctuations borne by different workers. To study the welfare implications of incorporating more than one type of labor, I calculate the welfare costs of eliminating business cycles in a fashion similar to Lucas (1987). Similar to the results in the earlier literature (see, for example, Mukoyama and Şahin, 2006; Krusell et al., 2009), I show that there is substantial heterogeneity in the cost of business cycles
among workers with different characteristics. In particular, my model introduces a new group of workers who pay substantially larger costs of economic fluctuations: contingent regular workers who frequently move between the regular and the irregular labor markets and switch their job types. Because these workers face larger uncertainties regarding their labor market status, they experience larger fluctuations in their labor income over the business cycle.

The rest of the paper is organized as follows. Section 2 documents the importance of compositional changes of labor types using the micro data from the Current Population Survey in the United States. Section 3 lays out the model with two types of labor and multiple ways for firms to adjust labor composition. Section 4 explains calibration of the model. Section 5 examines the labor market dynamics from the model. Section 6 investigates welfare implications of the findings in Section 5 and discuss monetary policy implications. The paper concludes in Section 7.

2 Evidence: the Importance of the Composition Margin over the Business Cycle

---

*Note:* This figure summarizes the findings from the CPS data in the United States. This illustrates large flows across the two labor markets. The red arrows represent cyclically and quantitatively important flows, while the dashed blue arrows denote cyclically and quantitatively less meaningful flows. This figure illustrates that there are large and strongly (counter-)cyclical flows from the full-time labor market to the part-time labor market, which derives the changes in the composition of workers over the business cycle. For the flows between labor markets and not-in-the-labor-force (NLF), flows between NLF and part-time labor markets are more important in terms of cyclicality and magnitudes. See Appendix A.3, A.4, and A.5 for other flows.
This section documents basic macroeconomic facts (see Figure 1 for a graphical summary) about regular and irregular labor over the business cycle using micro data from the Current Population Survey (CPS) in the United States between January 1976 and December 2019. To that end, I group labor market outcomes into five states: (i) employed full-time ($e^{FT}$), (ii) employed part-time ($e^{PT}$), (iii) unemployed full-time ($u^{FT}$), (iv) unemployed part-time ($u^{PT}$), and (v) not in the labor force ($n$), following the CPS distinction of full-time and part-time status. The CPS distinguishes full-time workers, who work over 35 hours or more per week, and part-time workers, who work less than 35 hours per week. For the unemployed, the full-time versus part-time status is determined by the sort of jobs they are mainly looking for. If they mainly seek full-time (part-time) jobs, they are classified as unemployed in full-time, $u^{FT}$ (unemployed in part-time, $u^{PT}$). I then examine if the composition of the two types significantly changes in response to aggregate shocks, and if the two labor markets exhibit differential dynamics to those shocks.

I use full-time and part-time workers as proxies for broader notions of regular and irregular types in the model I develop in Section 3. These proxies give me sufficiently long-time series at the business cycle frequency. Moreover, full-time and part-time status has been consistently defined compared to other classifications of work arrangements. (e.g. contingent workers, contract workers, etc.) Because full-time and part-time status does not represent all the regular and irregular jobs, I validate the use of full-time and part-time classification by comparing the relative shares of full-time and part-time workers with those of permanent and temporary workers in other countries where data on both full-time and part-time workers and permanent and temporary workers are available at the annual frequency (see Appendix A.1). I show that the relative shares of the full-time and part-time workers are comparable to those of permanent and temporary workers in other countries where data on both full-time and part-time workers and permanent and temporary workers at the annual frequency are available: Germany, Italy, Greece, and the United Kingdom. Moreover, I show that the evolution of the share of involuntary part-timers is in line with the evolution of the share of temporary workers in these countries.

\footnote{For the summary statistics, refer to Table A1 in Appendix A. See also A for the details about the construction of the series.}
Fact 1. Regular jobs are procyclical and irregular jobs are countercyclical.

Figure 2: The Size of Full-time and Part-time Workers out of Population (age > 15) and The Relative Employment Share of the Part-time Workers.

Panel (a): Full-time and Part-time Workers

Panel (b): The Relative Employment Share of Part-timers

Note: In Panel (a), the red dash-dot line is the size of full-time workers out of population of age greater than fifteen in percentage with the y-axis on the left, and the blue solid line is the size of part-time workers in percentage with the y-axis on the right. In Panel (b), the black solid line denotes the relative employment share of part-time workers, that is, \(e_{PT}/(e_{FT} + e_{PT})\). Recession periods from NBER classifications are denoted as grey shaded areas. Source: CPS microdata from January 1976 to December 2019.

First, I show that the relative size of full-time employment to part-time employment in the United States changes over the business cycle. Panel (a) of Figure 2 plots the time series of full-time and part-time employment \((e_{FT} \text{ and } e_{PT})\) as a share of population of age over fifteen, and Panel (b) plots the relative share of the part-time workers out of total employment. While the full-time employment is procyclical (the red dash-dot line), the part-time employment is countercyclical (the blue solid line). Countercyclicality of part-time employment particularly stands out during longer recessions of the early 1980s and the Great Recession. Panel (b) shows that the relative employment share of part-time workers increases during recessions. This is consistent with one of the findings in Katz and Krueger (2017) that weak labor market conditions lead to an increase in irregular jobs.\(^5\)

The fact that the relative size of the two labor markets changes over the business cycle implies that the composition of workers significantly varies over the business cycle.

\(^5\)Their “non-traditional” work corresponds to irregular jobs in my paper.
Fact 2: Flows from the regular employment to the irregular employment are counter-cyclical.

Figure 3: Flows between Full-Time Employment and Part-Time Employment

Note: This figure shows the gross flows between full-time employment, $e^{FT}$ with the blue solid line in percentage (y-axis on the left) and part-time employment, $e^{PT}$ with the red dash-dot line in percentage (y-axis on the right). Recession periods from the NBER classifications are denoted as grey shaded areas. All the flows are margin error adjusted, deNUNfied following [Elsby, Hobijn, and Sahin (2015)](their suggested method of correcting for classification errors), corrected for temporal aggregation bias following [Shimer (2012)] and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1976 to December 2019.

The changes in the relative size of the two labor markets stem from the flows of workers between them. The transition from part-time employment to full-time employment, $f^{ePT,eFT}$ (red dash-dot line), does not exhibit any cyclicality. The transition from full-time employment to part-time employment, $f^{eFT,ePT}$ (blue solid line), is clearly countercyclical. It sharply rises during recessions and gradually decreases during booms. Moreover, the magnitude of the flow, $f^{eFT,ePT}$ is significantly large. For example, in recessions, the number of workers who move from full-time employment to part-time employment is more than twice the number of workers who move from the employed to the unemployed.

---

6See Appendix A for the construction of the series.

7This countercyclical flow is not entirely driven by firms’ adjustment of intensive margins. To show this, I compare $f^{eFT,ePT}$ for hourly-paid workers with those who are not hourly-paid and non-respondents. I also compare this flow for those who work for the same employer and do the same job and for those who either do not work for the same employer or do different jobs than before. See Appendix A.2.

8For the other flows, $f^{eFT,uFT}$, $f^{ePT,uPT}$, $f^{eFT,uPT}$, $f^{ePT,uFT}$, $f^{eFT,uT}$, $f^{ePT,uT}$, $f^{eFT,nT}$, $f^{ePT,nT}$, and $f^{ePT,nT}$, see Appendix A.3 and A.5.
Fact 3: Flows from regular to irregular employment explain most of the changes in the composition of workers.

Figure 4: Flow Decomposition of Full-Time and Part-Time Employment Rates

Note: In Panel (a) (Panel (b)), the red line with circles plots the cumulative changes of the full-time employment rates (the part-time employment rates); the blue solid line is the cumulative contributions of the changes of the flows from full-time employment to part-time employment to explain the full-time employment rate changes (to explain the part-time employment rate changes); The black dash line is the cumulative contributions of the changes of the $E$ to $U$ and $U$ to $E$ transitions within the full-time labor market (within the part-time labor market), and the green dash-dot line is the contributions of the changes of the flows between part-time employment and full-time unemployment rate. Recession periods according to NBER classifications are denoted as grey shaded area. All the flows are margin error adjusted, desNUNfied following Elsby, Hobijn, and Sahin (2015) (their suggested method of correcting for classification errors), and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1994 to December 2019.

Following the method developed by Elsby et al. (2019), I calculate the contributions of the flows across the two labor markets to explain the evolution of full-time employment and part-time employment. Panel (a) of Figure 4 shows the contribution of each flow to explain full-time employment rate changes. The red line with circles shows the cumulative changes of full-time employments from January 1994 to December 2019. This shows that the stock of full-time employment significantly drops during recessions and gradually rises during expansions. As expected, the $E$
to $U$ and $U$ to $E$ transitions within the full-time labor market, $\Delta p_{e^{FT},u^{FT}}$ and $\Delta p_{u^{FT},e^{FT}}$, explain most of the changes of full-time employment, $\Delta e^{FT}$. We can observe this from the black dash line, which closely tracks the changes in full-time employment rates denoted as the red line with circles. On the other hand, the contribution of the flows from the full-time employment rate to the part-time employment rate, $\Delta p_{e^{FT},e^{PT}}$, is plotted with the blue solid line. While these are not as big as the contribution shown in the black dash line, which represents the $E$ to $U$ and $U$ to $E$ transitions within the full-time labor market, the flows of $\Delta p_{e^{FT},e^{PT}}$ still explain significant portions of the changes in full-time employment. For example, the full-time employment rate could have dropped by four percentage points during Great Recessions, if only the $E$ to $U$ and $U$ to $E$ transitions within the full-time labor market had changed. However, the transition rates from full-time to part-time employment have risen sharply during the Great Recession. This has accelerated the drop in full-time employment rates by two percentage points more.

The importance of the flows across the two labor markets stand out even more in the flow decomposition of the part-time employed. Panel (b) of Figure 4, the red line with circles shows the cumulative changes in the part-time employments, $\Delta e^{PT}$ from January 1994 to December 2019. Consistent with Figure 2, the stock of part-time employment sharply rose during the Great Recession. What contributes the most to the changes in part-time employment rate, $\Delta e^{PT}$, is the flow from the full-time employment rate to the part-time employment rate, $\Delta p_{e^{FT},e^{PT}}$ (blue solid line). This time series closely tracks $\Delta e^{PT}$, suggesting that most of the changes in part-time employment are explained by higher transitions from full-time to part-time employment. Other important sources explaining the evolution of the part-time employment rates are the flows between part-time and full-time unemployment, $\Delta p_{e^{PT},u^{FT}}$ and $\Delta p_{u^{FT},e^{PT}}$ (the green dash-dot line). For example, during the Great Recession, the part-time employment rate could have risen by two percent, if only the transition probabilities from the full-time employment to the part-time employment, $\Delta p_{e^{FT},e^{PT}}$ had risen. However, the part-time employment rate has risen by one and half percent, because more part-time workers exit to full-time unemployment (looking for full-time jobs), and fewer full-time unemployed workers enter part-time employment. Due to these changes, the increases in the part-time employment rate during the Great Recession were attenuated. In contrast to these cross-market flows, flows within the part-time labor market, $\Delta p_{e^{PT},e^{PT}}$ and $\Delta p_{u^{PT},e^{PT}}$, do not explain the evolution of part-time employment, as is clear from the black dash line.
In summary, the flows of workers across the two labor markets rather than the flows between not-in-the-labor-force and each labor market explain the changes in the composition of worker types. This further illustrates that a significant portion of the workforce may experience fluctuations in their labor income from switching job-types over the business cycle. In other words, switching job types could be an important source of income risks for a large number of workers over the business cycle.

**Fact 4: The two labor markets behave differentially over the business cycle.**

To study the behavior of labor markets, I now estimate impulse responses of each labor market variables to observable structural shocks: Romer and Romer (2004) monetary policy shocks; uti-

\[ y_{t+h} - y_{t-1} = \alpha_h + \beta_h x_t + \psi_h (L) z_{t-1} + \epsilon_{t+h}, \quad \text{for} \quad h = 0, 1, 2, \ldots , \tag{1} \]

where \( y \) denotes an endogenous dependent variable of interest, \( x_t \) denotes a structural shock series, and \( z_{t-1} \) is a vector of control variables. \( \{ \beta_h \} \) gives the responses of \( x \) at time \( t+h \) to the shock at time \( t \) for \( h = 0, 1, 2, \ldots \).

Figure 5 presents the estimated impulse response functions of the share of part-time workers out of the total employment. In response to a one standard deviation of positive government spending shocks, the share of part-time workers out of the total number of workers significantly decreases by 0.35 percentage points at the peak. Similarly, in response to a one standard deviation of an expansionary monetary policy shock, the share of part-timers drops by 0.2 percentage points at the peak. Lastly, the share decreases by 0.15 percentage points in response to one standard deviation of utilization-rate adjusted TFP shock.

Moreover, Figure 6–8 show that each labor market variables exhibit differential responses to structural shocks. Figure 6 shows the estimated responses of total labor market variables and each labor market’s variables to government spending shocks, Figure 7 shows those to monetary policy shocks, and Figure 8 shows those to utilization-rate-adjusted total factor productivity shocks, using Equation (1).

Across all figures from Figure 6 to Figure 8, the employment rates in each labor market move to the opposite directions in response to structural shocks. For example, while the full-time employment rates significantly increase in response to positive demand and supply shocks, the part-time employment rates decrease. For example, in response to a one standard deviation of positive government spending shocks, the total employment rate and full-time employment rate out of the total population increase by one percent, the part-time employment rate decreases by a similar amount.
Figure 6: The Responses of Each Labor Market Variables to Government Spending Shocks

Note: This figure shows the responses of total labor market variables and each labor market’s variables: employment rates, the size of labor forces, and unemployment rates to positive government spending shocks. The estimates for the total labor market variables’ responses are denoted as black solid lines with black dotted lines as 90 percent confidence bands and with grey shaded area as 68 percent confidence bands. The estimates for the full-time labor market variables’ responses are denoted as red solid lines with red dotted lines as 90 percent confidence bands and with the pink shaded area as 68 percent confidence bands. Those for the part-time labor market variables’ responses are denoted as blue solid lines with blue dotted lines as 90 percent confidence bands and with the sky-blue shaded area as 68 percent confidence bands. Sources: All the labor market variables are calculated from the CPS from March 1976 to December 2013. Government spending shocks from the second quarter of 1976 to the fourth quarter of 2013 are taken from Ramey (2016).

In contrast, the unemployment rates in general move in the same direction. For example, in response to positive demand shocks, both full-time and part-time unemployment rates significantly decrease. The magnitudes of the responses are different. The unemployment rate in the full-time labor market responds to the most, while the part-time unemployment rate moves only modestly to the shocks. The responses of the total unemployment rate are in between the two, while closer to the responses of the full-time unemployment rates. For instance, the unemployment rate in the full-time labor market decreases by 0.49 percentage points in response to a one standard deviation of positive government spending shock at the peak, but the part-time unemployment rate decreases by 0.25 percentage points at the peak. The total unemployment rate decreases by 0.36 percentage points at the peak.
Figure 7: The Responses of Each Labor Market Variables to Expansionary Monetary Policy Shocks

Note: This figure shows the responses of total labor market variables and each labor market’s variables: employment rates, the size of labor forces, and unemployment rates to a one standard deviation of expansionary monetary policy shocks. The estimates for the total labor market variables’ responses are denoted as black solid lines with black dotted lines as 90 percent confidence bands and with grey shaded area as 68 percent confidence bands. The estimates for the full-time labor market variables’ responses are denoted as red solid lines with red dotted lines as 90 percent confidence bands and with the pink shaded area as 68 percent confidence bands. Those for the part-time labor market variables’ responses are denoted as blue solid lines with blue dotted lines as 90 percent confidence bands and with the sky-blue shaded area as 68 percent confidence bands. Sources: All the labor market variables are calculated from the CPS from March 1976 to December 1996. I use quarterly-aggregated Romer and Romer (2004) monetary policy shocks from March 1976 to December 1996.

These differences in the magnitudes of the unemployment rates’ responses in each labor market stem from the differential responses of the labor force sizes in each labor market. First, the total labor force participation rates respond only modestly, which is consistent with the modest procyclicality of the total labor force participation rates documented in the literature (see, for example, Elsby, Hobijn, and Sahin, 2015). This, however, masks heterogeneous responses of each labor market’s labor force. For example, in response to one standard deviation of positive government spending shocks, the total labor force participation rate increases by 0.55 percent. In contrast to this, the full-time labor force significantly rises by 0.65 percent at the peak and the part-time labor force significantly decreases by 0.78 percent at the peak. The magnitudes of the changes in the full-time labor force are, however, smaller compared to the magnitudes of the changes in the full-time employment rates, while the changes in the part-time labor force are greater than the changes in the part-time employment rates. This generates different magnitudes of each labor market’s unemployment rate changes but in the same direction.
To summarize, this section documents the differential experiences of the full-time and the part-time labor markets over the business cycle. (Figure 1 provides a graphic summary.) While full-time employment is procyclical, part-time employment is strongly countercyclical. This is largely due to the countercyclical flows from full-time employment to part-time employment. Flow decompositions show that these countercyclical flows explain a large portion of the falls of the full-time employment rate and the rise of part-time employment rate during recessions. These findings imply that the composition of workers significantly varies over the business cycle, and these changes in the composition of workers largely come from the workers who move between the two labor markets. The two labor markets also exhibit differential dynamics to structural shocks. Motivated by these facts, the next section introduces a model that can generate these facts and studies implications of the changes in the composition of work arrangements over the business cycle.
A New Keynesian Model with Regular and Irregular Work Arrangements

This section presents a New Keynesian model featuring two types of labor\footnote{Because the model builds on \textcite{Christiano, Trabandt, and Walentin, 2020}, notations and functional specifications closely follow theirs.} Unlike the standard New Keynesian model embedding a theory of unemployment \cite{Gali, 2011; Gali, Smets, and Wouters, 2011; Christiano, Trabandt, and Walentin, 2020}, my model features two types of labor: (i) regular workers whose productivity is high but are difficult to hire/fire and (ii) irregular workers who have low productivity but are easier to hire/fire\footnote{Difficulty of adjustment comes from a number of sources: (i) firms need to pay high physical costs to hire and fire regular workers, (ii) regular workers are more attached to firms as exogenous separation rates are lower, (iii) the fact that they have higher productivity makes firms reluctant to fire them, and (iv) it takes time for promoted irregular workers to exhibit full productivity as other regular workers.}

On the labor supply side, workers endogenously decide which type of jobs they would like to have. On the labor demand side, I introduce multiple margins of adjustment for firms to change the total amount of labor input, particularly by changing the composition of the two types: not just hiring and firing each type of workers, but transferring one type to the other via promotions and/or demotions. Because there is more than one type of labor, firms now can adjust the total amount of labor input by changing the composition of job types by creating one type of jobs but destroying the other type of jobs and/or via transferring one type to the other type. I call this “composition margin,”\footnote{It is true that the composition margin is not separable from the other two margins, extensive (the number of workers) and intensive margins (hours worked per worker). The composition margin is, rather, closely connected to the other two margins and helps to better understand the behaviors of the other two margins over the business cycle. See Appendix B.10 for the relationships between the composition margin and the other two margins.} and examine the relevance of composition margin for understanding labor market dynamics and welfare and policy implications over the business cycle.

The model has the following agents: (i) Infinitely many workers within a representative family, who decide whether to participate in the labor market or not. If participating, they decide \textit{which} labor market to enter and how much job search effort to exert; (ii) a representative family that supplies two types of labor, consumes, invests in physical capital, chooses the degree of capital utilization, leases capital services to intermediate-goods-producing firms, and saves using nominal bonds\footnote{It is easy to extend the model to incorporate wage rigidities for regular workers. Appendix B.14 presents such extensions.} (iii) intermediate-goods-producing-firms that use capital services and two types of labor to produce differentiated intermediate goods and set prices by paying price adjustment costs,
(iv) final-goods-producing-firms that bundle differentiated intermediate goods into a final good;
(v) a central bank conducting a Taylor-rule-type monetary policy and a fiscal authority, which does spending exogenously and finances it with lump-sum taxes and debt. The next subsections explain the environments of each economic agent and their optimization problems.

3.1 Workers

There are infinitely many identical families $h \in [0, 1]$ consisting of infinitely many workers. Figure 9 summarizes the environment of a worker within each family.

Figure 9: Summary of Workers’ Problems

Each worker within a family draws the level of work aversion, $\ell^{i.i.d.} \sim U[0, 1]$, at the beginning of every period. After drawing $\ell^{i.i.d.} \sim U[0, 1]$, the worker decides to either work or not. If

---

Note: This figure illustrates an individual worker’s problem within the representative family. Each worker in the family has three problems to solve. After her type $\ell$ is drawn at the beginning of the period, she decides whether to participate in the labor force, by comparing expected utility from participating in any labor market and from not participating. If she decides to enter any labor market, she needs to choose which labor market to enter, again by comparing ex-ante expected utility from entering each labor markets. Upon entering a labor market, she determines the level of effort to exert by taking into account the fact that it increases the probability to find a job at the quadratic utility costs.

---

The assumption that workers draw their type, $\ell^{i.i.d.} \sim U[0, 1]$ every period makes the model tractable. Specifically, this assumption makes each worker’s problem static and reduces the indirect utility of the representative family to the standard functional form often used in many dynamic macroeconomic models. (See Section 3.2.) This assumption is also necessary for imperfect consumption insurance provision by the representative family, so that the level of con-
she decides to participate in any labor market, she in turn determines which market to enter: (i) Regular labor market \((m = R)\) and (ii) Irregular labor market \((m = IR)\). Upon entering a labor market, she chooses the optimal level of effort to exert to find a job.

A worker who draws \(\ell\) and decides to enter the labor market, \(m = R\) or \(IR\), faces the following disutility of working:

\[
F^m + (1 + \sigma^m_L)\ell^m, \quad m = R, IR.
\]

(2)

I assume that \(F^R \leq F^IR\), so that working in the irregular labor market generates higher disutility. I also assume \(1 < \sigma^R_L \leq \sigma^IR_L\), implying that disutility from working increases at a higher rate in the regular labor market than in the irregular labor market.\(^{15}\) When workers decide whether to work or not and which labor market to enter, they know the values of these parameters.

If a worker with \(\ell\) chooses to enter a labor market \(m\), she decides how much effort, \(e^m_{\ell,t}\), to exert to find a job. The more effort she exerts, the higher the probability that she finds a job, but at some quadratic utility costs defined as \((e^m_{\ell,t})^2/2\), for \(m = R, IR\). The probability of finding a job in a labor market \(m\) when a worker exerts effort, \(e^m_{\ell,t}\), is given by the following linear function:

\[
p(e^m_{\ell,t}) = \eta^m + a^m e^m_{\ell,t}, \quad \eta^m, a^m \geq 0, \quad m = R, IR.
\]

(3)

I assume \(0 \leq \eta^R \leq \eta^IR\), and \(0 \leq a^R \leq a^IR\), to capture the fact that it is easier to find an irregular job than a regular job, given the same level of effort.\(^{16}\)

\textit{Ex-ante}, a worker with work aversion of \(\ell \in [0,1]\) considering to enter a labor market \(m\) has

\(^{15}\)The assumption of \(1 < \sigma^R_L \leq \sigma^IR_L\) reflects the fact that regular workers tend to work overtime and have much more responsibilities than irregular workers (see Mas and Pallais 2020). Therefore the higher the inherent level of private cost of working is, the higher their disutility becomes when participating in the regular labor market.

\(^{16}\)These parameter values capture the fact that firms might be more cautious to hire regular workers than irregular workers, as the former tends to be more attached to firms than the latter. Firing costs of regular workers are much higher than those of irregular workers due to this firm-specific human capital and severance payment. In fact, firms tend to have more screening processes for hiring regular workers than hiring irregular workers. (see, for example, Houseman, Kalleberg, and Erickcek 2003) It is more likely to randomly encounter vacancy postings for irregular jobs than those for regular jobs. For example, it is easy to encounter job postings for restaurant/cafe servers when we visit those restaurants/cafes, and servers are one of the popular irregular jobs. (e.g. part-timers) This parameter choice is consistent with the estimated transition probabilities in Hall and Kudlyak (2019) that the transition probability from unemployment to the short-term job is higher than the transition probability from unemployment to the longer-term job for men.
the following expected utility:

\[
p(e^m_{\ell,t}) \times \begin{cases} 
\text{utility from consumption when she finds a job} & \log(c^m_t) \\
\text{Disutility from working} & (F^m + (1 + e^m_{\ell,t}) e_{\ell,t}^2) \\
\text{utility from consumption when she cannot find a job} & \log(c^{\text{U}m}_t) \\
\text{effort cost} & \frac{1}{2} (e^m_{\ell,t})^2 
\end{cases}, \quad (4)
\]

where \(c^m_t\) is the level of consumption when this worker gets a job in a labor market \(m\), and \(c^{\text{U}m}_t\) is the level of consumption if she fails to find a job in a labor market, \(m = R, \text{IR}\). Here, I allow for the possibility of \(c^{\text{U}R}_t \geq c^{\text{U}IR}_t\) to capture the fact that unemployed workers in the irregular labor market typically do not have access to unemployment benefits. While the level of \(c^R_t, c^{\text{IR}}_t, c^{\text{U}IR}_t\) do vary over time, I assume that \(c^{\text{U}IR}_t / c^{\text{U}IR}_t\) is a fixed fraction to derive a tractable functional form for the family-level utility. A worker with inherent work aversion \(\ell\), considering to enter a labor market \(m\), then optimally chooses the effort level \(e^m_{\ell,t}\) which maximizes her ex-ante utility given by Equation (4)\(^{17}\).

Meanwhile, if a worker decides not to enter any labor market, her utility is simply given by

\[
\log(c^{\text{U}IR}_t), \quad (5)
\]

regardless of the value of \(\ell \in [0, 1]\). I assume that the level of consumption for those out of the labor force is the same as the level of consumption for the unemployed in the irregular labor market.

A worker with \(\ell\) that considers entering a labor market \(m\) compares the ex-ante expected utilities of participating in any labor market as in Equation (4) given the optimal level of effort \(e^m_{\ell,t}\), and the utility of being out of labor force given by Equation (5). Upon deciding to participate in any labor market, she compares her expected utility of participating in the regular labor market and that of participating in the irregular labor market. If the former is higher, then she enters the regular labor market, and if the latter is higher, she enters the irregular labor market.

\(^{17}\)For the functional form of the optimal level of effort, see Appendix B.1.
Remark 1. For $1 \leq \sigma^R_L \leq \sigma^IR_L$, we can define the following two thresholds, $\bar{b}_t$ and $\bar{d}_t$ with the following rankings:

$$0 < \bar{d}_t \leq \bar{b}_t < 1,$$

where $\bar{b}_t$ is the value of $\ell$ at which the ex-ante expected utility of a worker with $\ell = \bar{b}_t$ when entering the irregular labor market is the same as the utility of being out of labor force, and $\bar{d}_t$ is the value of $\ell$ at which the ex-ante expected utilities of participating in the regular and in the irregular labor market for a worker with $\ell = \bar{d}_t$ are the same.

Given the two thresholds, then a worker with $\ell$ makes the following decision regarding her labor market status:

$$\begin{align*}
\text{if } 0 \leq \ell \leq \bar{d}_t & \Rightarrow \text{Participate in the regular labor market,} \\
\text{if } \bar{d}_t < \ell \leq \bar{b}_t & \Rightarrow \text{Participate in the irregular labor market,} \\
\text{if } \bar{b}_t < \ell \leq 1 & \Rightarrow \text{Out of the labor force.}
\end{align*}$$

Therefore, workers with $\ell$ close to $\bar{b}_t$ are marginally attached to the labor market (mostly irregular labor market), in a sense that a slight decrease of $\bar{b}_t$ pushes her out of labor force. Similarly, a slight decrease in $\bar{d}_t$ makes workers with $\ell$ close to $\bar{d}_t$ move from the regular labor market to the irregular labor market. Therefore, these workers are marginally attached to the regular labor market because they move between the two labor markets, as $\bar{d}_t$ changes over time.

Because of the unit measure of workers ($\ell \sim \mathcal{U}[0,1]$) within the representative family, these two thresholds correspond to the size of each type’s labor forces: $\bar{b}_t$ corresponds to the size of the total labor force; $\bar{d}_t$ corresponds to the size of the regular labor force; $\bar{b}_t - \bar{d}_t$ is the size of the irregular labor force.

---

18This assumption makes sure to generate that those tend to participate in the irregular labor market are the ones who sometimes are out of labor force. Therefore, they are the ones who are marginally attached to the total labor market, which is consistent with one of the findings from [Hall and Kudlyak (2019)](HallKudlyak2019) that “circlings” happen between unemployment, short-term jobs, and out of labor force, considering that short-term jobs are close to irregular jobs than regular jobs.

19See Appendix B.1 for the formal equations related to these two thresholds, i.e. incentive compatibility conditions.

20For example, in response to a negative demand shock (a contractionary monetary policy shock or a negative government spending shock), labor demand decreases. This in turn decreases $\bar{b}_t$ with lower wages. See Section 5.

21This is consistent with empirical analysis using the microdata from the CPS. Flows between the part-time labor market and out-of-labor-force are cyclically more important than those between the full-time labor market and out-of-labor-force. See Appendix A.5.
The two thresholds $\bar{b}_t$ and $\bar{d}_t$ and hence each labor market’s labor forces vary as consumption premiums ($c_R^t/c_{UR}^t$, $c_I^t/c_{UIR}^t$, $c_R^t/c_I^t$) change over time.\footnote{22} Intuitively, a worker’s labor market participation decision, given $\ell$, depends on the level of consumption which differs across the labor markets and the labor market status. If the level of consumption for a regular worker is much higher than the level of consumption for an irregular worker, more workers want to enter the regular market, which would increase $\bar{d}_t$. Similarly, if the gap between the levels of consumption for any worker and for a non-worker is much higher, then more workers are willing to participate in the labor force. The next section explains how the representative family sets consumption premiums, which in turn determines the sizes of labor forces in each labor market.

### 3.2 A Representative Family’s Problem

Figure 10: Summary of a Representative Family’s Problem

Within a family, $h$, $l$

- Devise an incentive system to make sure they supply $n_R^l$ and $n_{IR}^l$ to each labor market.
- A family wisely chooses consumption premiums, $c_R^l/c_{IR}^l$, $c_I^l/c_{UIR}^l$ and $c_R^l/c_{IR}^l$ to provide incentive to workers to participate in each labor market.

---

Note: This figure illustrates the representative family’s problem. The representative family selects the level of family-wide consumption, saves, supplies regular and irregular subject to budget constraint, and capital accumulation process. For infinitely many workers within the family, she devises an incentive system by setting corresponding consumption premiums in each labor market to supply the desired amount of regular and irregular labor.

In this section, I describe a representative family’s problems. The representative family’s problem is summarized in Figure 10. This extends Section 2.3. of CTW to the case of two types of labor. I begin by explaining the incentive provision problem given asymmetric information between the

\footnote{22}Consumption premiums are derived explicitly in Appendix B.1.
workers and the family, by taking the amount of family-level consumption and the number of regular and irregular workers to supply as given. I then derive a family’s indirect utility function. Given this family-level indirect utility function, I present the problem that the representative family solves to select the level of family-wide consumption, the amount of regular and irregular workers to supply, and the amount of savings from the family-level optimization problem.

There is asymmetric information between the representative family and workers within the family. In particular, each worker’s type, \( \ell \) which determines her private cost of working, and the level of effort, \( e_{\ell,t} \) each worker exerts to find a job, are private information. Under this environment, the family cannot perfectly insure against each worker’s idiosyncratic risk about her labor market status. This is because if the family provides perfect insurance, workers will not exert any effort to find a job and enjoy the same level of consumption (\( c^R_t = c^IR_t = c^UIR_t \)). Therefore, the family needs to devise an alternative insurance arrangement for workers to provide incentives. Specifically, the family sets consumption premiums of working as a regular and as an irregular type (\( c^R_t / c^IR_t \), \( c^IR_t / c^UIR_t \), and \( c^R_t / c^IR_t \)) to supply exactly \( n^R_t \) and \( n^IR_t \) numbers of workers to each labor market, and to satisfy the feasibility condition, given the level of total consumption, \( C_t \). This level of total consumption, \( C_t \) will be endogenously determined later from the representative family’s optimization problem.

Consider first the relationship between the two thresholds, \( \bar{b}_t \) and \( \bar{d}_t \), and the number of each type of worker to supply at the family level. The number of regular workers, \( n^R_t \), and the number of irregular workers, \( n^IR_t \), are obtained by integrating the probability of finding a job given the optimal level of effort up to the thresholds determining its labor market’s sizes as follows:

\[
\begin{align*}
    n^R_t &= \int_0^{\bar{d}_t} p(e_{\ell,t}^{R,*}) \, d\ell, \\
    n^IR_t &= \int_{\bar{d}_t}^{\bar{b}_t} p(e_{\ell,t}^{IR,*}) \, d\ell.
\end{align*}
\]  

(7)

From the two thresholds and the number of workers of each type, the unemployment rates for each labor market, and the total unemployment rate can be written as follows:

\[
\begin{align*}
    u^R_t &= \frac{\bar{d}_t - n^R_t}{\bar{d}_t}, \\
    u^IR_t &= \frac{(\bar{b}_t - \bar{d}_t) - n^IR_t}{(\bar{b}_t - \bar{d}_t)}, \\
    u_t &= \frac{\bar{b}_t - n^R_t - n^IR_t}{\bar{b}_t}.
\end{align*}
\]

---

\( ^{23} \)Workers within the family draws her type \( \ell \sim U[0, 1] \) at the beginning of each period and this determines her private cost of working. While there are no search frictions in this model, unemployment is generated from pure frictions. In this regard, there is a reduced-form labor market frictions summarized by the job finding probability function: Not all the job seekers are able to find jobs, and job seekers need to exert some efforts to find jobs.
The close relationship between the number of workers and the two thresholds shows that if the representative family wants to supply \( n_t^R \) (\( n_t^{IR} \)) number of workers to the regular (irregular) labor market, the family needs to adjust the two thresholds, \( \bar{b}_t \) and \( \bar{d}_t \), accordingly. Because \( \bar{b}_t \) and \( \bar{d}_t \) change in response to the changes of consumption premiums in each labor markets (\( c_t^m / c_t^{lm} \) for \( m = R, IR \) and \( c_t^R / c_t^{IR} \)), the family needs to set consumption premiums correspondingly, to make sure \( \bar{b}_t \) amount of workers participate in the labor market, of whom \( \bar{d}_t \) amount of workers enter the regular labor market, and the rest participate in the irregular labor market.

When setting these consumption premiums, the family has to consider the following feasibility conditions as well:

\[
n_t^R c_t^R + (\bar{d}_t - n_t^R) c_t^{IR} + n_t^{IR} c_t^R + (1 - \bar{d}_t - n_t^{IR}) c_t^{UIR} = C_t, \tag{8}
\]

where \( C_t \) is total amount of family-wide consumption, which will be determined later from the family’s optimization problem. Formal derivations of insurance provision problem of the representative family can be found in Appendix B.2.

Combining all, we can derive the indirect utility function for the representative family, which extends the indirect utility function for a representative family in CTW to the case with two types of labor:

**Proposition 1.** The representative family’s indirect utility is reduced to the following simple expression:

\[
u(C_t, n_t^R, n_t^{IR}) = \log C_t - n(n_t^R, n_t^{IR}), \tag{9}\]

with \( n(n_t^R, n_t^{IR}) \) summarizing disutility generated from supplying \( n_t^R \) and \( n_t^{IR} \) to each labor market. Appendix B.3 provides the functional form of \( n(n_t^R, n_t^{IR}) \) with proofs.

Provided this indirect utility function for the representative family, we can write the representative family’s optimization problem. The representative family determines the level of family-wide consumption \( C_t \); the amount of regular-type labor \( n_t^R \) to supply; the amount of irregular-type labor \( n_t^{IR} \) to supply; capital utilization rate, \( v_t \); next period’s capital stock, \( K_{t+1} \); the amount

\footnote{Again for \( c_t^{IR} \geq c_t^{UIR} \), it needs to be that \( c_t^{IR} / c_t^{UIR} = C > 1 \), a fixed fraction which does not vary over time. \( c_t^R, c_t^{IR}, c_t^{IR} \), however, do vary over time in response to aggregate shocks.}
of investment, $I_t$; the amount of a nominal bond, $B_{t+1}$, subject to budget constraint and capital accumulation process, and the cost of capital utilization rates which accelerates the depreciation of existing capital, $\delta(v_t)$.

Formally, the family’s problem can be written as follows:

$$\max_{\{C_t, n_t^R, n_t^{IR}, B_{t+1}, v_t, K_{t+1}, I_t\}} \mathbb{E}_0^{\infty} \sum_{t=0}^{\infty} \beta^t u(C_t, n_t^R, n_t^{IR})$$

subject to

$$P_tC_t + P_I I_t + B_{t+1} \leq (1 + i_{t-1}) B_t + W_t^R n_t^R + W_t^{IR} n_t^{IR} + R_t K_t v_t + \text{Profits, Taxes, and Transfers}_t,$$

$$K_{t+1} = \left[ 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right]^2 I_t + (1 - \delta(v_t)) K_t,$$

where $P_t$ denotes the aggregate price level and $W_t^R$ ($W_t^{IR}$) is nominal wage of regular (irregular) workers, and $R_t$ is nominal rental rate of the capital services. The representative family takes these prices as given when solving the problem.

Because the indirect utility function for the representative family reduces to the standard functional form often used in dynamic macroeconomic models, it is easy to embed the rich structure of the labor supply decisions presented in Section 3.1 and 3.2 into the standard (medium-scale) New Keynesian model, yet it can still explain various labor market variables such as labor force participation rates, unemployment rates, and job search efforts in each labor market. Moreover, we can easily extend the model to incorporate more complicated structures such as habit formation, and nominal wage rigidity for a particular type of labor. Appendix B.14 presents these extensions.

3.3 Production

As is standard in New Keynesian models, there are two production sectors: intermediate goods sector and final goods sector. This section describes the firms’ decisions in each sector. Final goods firms are standard. Intermediate-goods-producing firms are non-standard, as they face multiple ways of adjusting the total amount of labor input, given the two types of labor.
3.3.1 Final Goods Production

A final good, $Y_t$, is produced using a continuum of intermediate goods using Dixit and Stiglitz (1977) aggregator as follows:

$$Y_t = \left( \int_0^1 Y_t(j) \frac{\epsilon_p - 1}{\epsilon_p} \, dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \quad \epsilon_p > 1,$$

with $\epsilon_p$ denoting the degree of substitutability between intermediate goods $j \in [0, 1]$. Final goods are produced by a competitive, representative final-goods-producing firm. Profit maximization by this firm generates the following downward-sloping demand curve for each intermediate good, $j \in [0, 1]$:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t, \quad P_t = \left( \int_0^1 P_t(j)^{1-\epsilon_p} \, dj \right)^{\frac{1}{1-\epsilon_p}},$$

where $P_t(j)$ is the price for an intermediate good $j \in [0, 1]$ and $P_t$ is the aggregate price index.

3.3.2 Intermediate Goods Production

There are infinitely many firms, $j \in [0, 1]$, in the intermediate goods sector and each firm produces a fixed variety under the monopolistic competition. Intermediate-goods-producing firms use two types of labor: (i) regular types and (ii) irregular types, and use an effective unit of capital to produce intermediate goods using the following production technology:

$$Y_t(j) = \left( A^P_t A^T_t n_t(j) \right)^{1-\alpha} \hat{K}_t(j)^{\alpha}, \quad (10)$$

where $A^P_t$ denotes economy-wide persistent productivity process, $A^T_t$ denotes transitory productivity shock, $\hat{K}_t(j)$ is effective unit of capital, and $n_t(j)$ is effective unit of labor employed by a firm $j$ which is given by:

$$n_t(j) = \left( (\eta_n)^{\frac{1}{\alpha_n}} \left( n^R_t(j) \right)^{\frac{\alpha_n - 1}{\alpha_n}} + (1 - \eta_n)^{\frac{1}{\alpha_n}} \left( n^I_t(j) \right)^{\frac{\alpha_n - 1}{\alpha_n}} \right)^{\frac{\alpha_n}{\alpha_n - 1}}, \quad \eta_n > 0.5. \quad (11)$$
This functional form reflects the fact that the two types of labor are imperfectly substitutable with the degree of substitutability represented as $\epsilon_n > 0$. I assume that regular types are more productive than irregular types, i.e. $\eta_n > 0.5$.

Because there is more than one type of labor, firms can adjust the total labor input in a number of ways. Firms can create/destroy each type of job. On top of this, firms can transfer one type of labor to the other type, by promoting some portion of irregular jobs to regular jobs or by demoting some portion of regular jobs to irregular jobs. Hence, firms can adjust the composition of job types. This “composition margin” differentiates my model from most of the other dynamic models with more than one type of labor. This transfer is the key to generate differential responses of the two labor markets documented in Section 2.

Given these multiple ways of adjusting the total amount of labor, the evolution of the stock of regular and irregular jobs for a firm $j$ can be written as follows:

$$n_t^R(j) = (\rho^R + x_t^R(j) - p_t^{R \rightarrow IR}(j))n_{t-1}^R(j) + \lambda_1 p_t^{IR \rightarrow R}(j)n_{t-1}^R(j), \quad \rho^R > 0, \lambda_1 < 1,$$

$$n_t^I(j) = (x_t^I(j) - p_t^{I \rightarrow R}(j))n_{t-1}^I(j) + \lambda_2 p_t^{R \rightarrow IR}(j)n_{t-1}^R(j), \quad \lambda_2 > 1,$$

where $x_t^R$ ($x_t^I$) is the net hiring rate of regular (irregular) workers with the net creation of regular (irregular) jobs and $p_t^{I \rightarrow R}$ ($p_t^{R \rightarrow IR}$) is the promotion rate (demotion rate), as a share of previous period’s regular (irregular) jobs. Regular jobs are destroyed at rate $1 - \rho^R$, while all the non-transferred irregular jobs are destroyed after one period. This reflects stickier nature of regular types and flexible nature of irregular types. $\lambda_1 < 1$ captures the fact that irregular types are less productive than regular types, and therefore when they are just transferred, their productivity will be discounted during the first period of the promotion. Similarly, $\lambda_2 > 1$ denotes that regular types are more productive than irregular types, so their productivity will be higher in the case of

25For the intermediate goods producing firms, they consider each type of labor in terms of “jobs,” not in terms of workers. For example, when firms keep some portion, $\rho^R$ of regular jobs after one period, firms in my model do not care if those are taken by the same workers from the previous period or not. This distinction is due to the assumption that workers in my model draw their work aversion level, $l$ every period, which enables the differences in consumption according to labor market status and makes my model tractable and solvable.

26Here are some examples of demotions: If firms furlough full-time workers and recall them but as part-timers during economic downturns, or as fixed-term contract workers, then this could be considered as demotion, in particular, if this happens within a quarter. (For instance, Fujita and Moscarini [2017] document that the average duration from the first separation to the first recall is two and a half months.) As another example, consider the case where firms suggest workers who are close to the retirement clock to retire early during economic downturns and re-hire them as temporary/fixed-term part-time workers or short-time workers.
demotion. \( \lambda_1 < 1 \) and \( \lambda_2 > 1 \) are necessary to have non-zero promotion and demotion rates in the steady-state. Results in Section 5 hold with \( \lambda_1 = 1, \lambda_2 = 1 \), and zero promotion and demotion rates in the steady state.

All the adjustment margins are subject to costs. Importantly, I assume that the stock of regular jobs is costlier to adjust than irregular jobs. Specifically, in order to hire/fire regular workers (create/destruct regular jobs), firms need to pay large adjustment costs. In contrast to this, firms can easily hire/fire irregular workers (create/destruct irregular jobs) and promote/demote each type to the other type, by paying only negligible cost. I assume the following functional forms for the adjustment costs:

\[
C(x^{R}; n^{R-1}) = \frac{1}{\kappa} \left( e^{-x^{R}} + \kappa x^{R} - 1 \right) n^{R-1} Y_t, \quad C(x^{IR}; n^{IR-1}) = \frac{1}{\gamma} \left( e^{\gamma x^{IR}} - \gamma x^{IR} - 1 \right) n^{IR-1} Y_t, \quad \kappa, \tilde{\kappa}, \gamma, \tilde{\gamma} > 0,
\]

\[
C(p^{R \to IR}; n^{R-1}) = \frac{\tilde{\theta}}{2} (p^{R \to IR})^2 n^{R-1} Y_t, \quad C(p^{IR \to R}; n^{IR-1}) = \frac{\tilde{\nu}}{2} (p^{IR \to R})^2 n^{IR-1} Y_t, \quad \text{for } p^{R \to IR}, p^{IR \to R} > 0.
\]

The adjustment cost functions for hiring/firing regular and irregular workers (creating/destructing regular and irregular jobs) are approximated to be commonly-used quadratic adjustment costs.

\[
C(x^{R}; n^{R-1}) \approx \frac{k^2}{2k} (x^{R})^2 n^{R-1} Y_t, \quad C(x^{IR}; n^{IR-1}) \approx \frac{\gamma^2}{2\gamma} (x^{IR})^2 n^{IR-1} Y_t.
\]

I assume a large enough number for \( \tilde{\gamma} \) with small values for \( \gamma \), and small enough numbers for \( \tilde{\theta} \) and \( \tilde{\nu} \) to denote that the costs of hiring/firing irregular workers, promotion, and demotion are negligible. In contrast to this, I use a small enough number for \( \tilde{\kappa} \) with large values for \( \kappa \), which reflects much higher adjustment costs for hiring/firing regular workers. Moreover, I assume different asymmetries for the costs of creating/destructing regular and irregular jobs. In particular, I assume that it is relatively costlier to destroy regular jobs than to create them. Particularly, \( \kappa \) represents the degree of this asymmetry. Meanwhile, I assume that, for irregular types, while those costs are still negligible, it is relatively more costly to create them than to destruct them. The degree of this different asymmetry will be governed by \( \gamma \). For promotion and demotion rates, I assume quadratic adjustment costs with positive domains.

On top of these labor adjustment frictions, intermediate-goods-producing firms are subject to Rotemberg (1982)-type nominal frictions. Each intermediate-goods-producing firms can freely adjust its prices every period, but instead, they need to pay quadratic price-adjustment-costs de-

---

27 As documented in Woodford (2005), because intermediate-goods-producing firms have firm-specific state variables, it would be tricky to use Calvo (1983)-type frictions.
fined as below:

\[
\frac{\phi_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t,
\]

where \(\phi_p\) governs the degree of nominal frictions. Full representation of intermediate-goods-producing firms’ problems and first order conditions are in Appendix B.4.2.

3.4 Policy, Exogenous Shocks, and Market Clearing Conditions

Monetary policy follows an inertial interest rate rule with zero inflation rate in the steady-state as follows:

\[
i_t = (1 - \rho_i)i^* + \rho_i i_{t-1} + (1 - \rho_i) (\phi_\pi \pi_t + \phi_y y_t) + \epsilon_{i,t},
\]

where \(y_t = \log Y_t - \log \bar{Y}\) is log-deviations of real output from its steady state.

The government consumes an exogenous share \(\omega^g_t\) of output,

\[
G_t = \omega^g_t Y_t, \quad \omega^g_t = (1 - \rho^g) \omega^g + \rho^g \omega^g_{t-1} + \epsilon^g_{t}.
\]

The government balances its budget each period with lump sum taxes, \(T_t = G_t\).

The exogenous process for the persistent technology shock, \(A^p_t\) is unit-root process with a zero mean in logs:

\[
\log A^p_t = \log A^p_{t-1} + \epsilon_{A^p,t}, \quad \mathbb{E}[\epsilon_{A^p,t}^2] = (\sigma^{A})^2
\]

Finally, clearing in the loan market requires \(B_{t+1} = 0\), for all periods, and clearing in the market for final goods requires:

\[
\underbrace{C_t + I_t + G_t + C(x^R_t; n^R_t) + C(x^R_{t-1}; n^R_{t-1}) + C(p^{IR}_t; n^{IR}_t) + C(p^{IR}_{t-1}; n^{IR}_{t-1}) + \frac{\phi_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t}_\text{=Sum of all the adjustment costs}
\]

4 Calibration

In this section, I describe the parameterization of the model. Table [] summarizes the choice of structural parameter values set to the standard values.
### Table 1: Parameter Values (1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.9987</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Power on capital in production function</td>
<td>0.33</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Production scale parameter</td>
<td>0.9</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Substitutabilities across the intermediate goods</td>
<td>10</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Price Stickiness (Price Adjustment Cost)</td>
<td>$(\epsilon_p-1)/0.75$ $(1-0.75\beta)$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Investment Adjustment Cost</td>
<td>5</td>
</tr>
<tr>
<td>$\delta_0, \delta_1, \delta_2$</td>
<td>Capital Depreciation Function Parameters</td>
<td>$\delta(v) = \delta_0 + \delta_1(v - 1) + \delta_2/2(v - 1)^2$</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>steady-state value of depreciation rates</td>
<td>0.025</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>steady-state value of utilization rate = 1</td>
<td>$1/\beta - (1 - \delta_0)$</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>Curvature on Capital Depreciation Function</td>
<td>0.0361</td>
</tr>
<tr>
<td>$\omega_g$</td>
<td>Steady-state government spending-GDP ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Policy weight on inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Policy weight on output gap</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Autocorrelation, government spending shock</td>
<td>0.97</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Autocorrelation, monetary policy shock</td>
<td>0.87</td>
</tr>
<tr>
<td>$\sigma_{A_t}$</td>
<td>Standard deviations, transitory technology shock</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{A_p}$</td>
<td>Standard deviations, persistent technology shock</td>
<td>0.0056</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>Standard deviations, government spending shock</td>
<td>0.0115</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Standard deviations, monetary policy shock</td>
<td>0.004</td>
</tr>
</tbody>
</table>

**Note:** This table summarizes parameter values related to non-labor market variables that are set to conventional values following the literature: $\beta$ is set so that the risk-free rate is 1%; $\alpha$ and $\nu$ are set so that the labor share becomes 60%; $\epsilon_p$ is set to be 10 following Christiano, Trabandt, and Walentin (2010); $\phi_p$ is set so that it corresponds to the Calvo parameter of 0.75 which corresponds to the expected price duration of 4 quarters; for the parameter in the adjustment cost for investment and the parameters in the capital depreciation function, I follow Sims and Wolff (2018); for the steady-state value of the government spending to GDP ratio, policy weight on inflation and output gap, I use those from Sims and Wolff (2017). The choice of parameter values related to exogenous shock processes are loosely based on those estimated in the literature (see, for example, Sims and Wolff (2017, 2018); Ireland (2001, 2004).

The remaining parameters are non-standard. On the firm side, I first set the exogenous separation rate for the regular type of jobs, $1 - \rho^R$, to 2 percent. This choice follows from the fact that firms without employment growth lose 0.891 percent of its workforce with reasons other than lay-off and discharges per month according to Job Openings and Labor Turnover Survey (see Baydur (2017)). This corresponds to 2.67 percent in a quarter. Because $1 - \rho^R$ is the exogenous separation rate for regular types who are more attached to firms than the other type, I set this value to be 2 percent which is slightly less than 2.67. The relative productivity between the regular and irregular types, $\eta''$, is set to 0.7 so that the relative marginal productivity between the two types captures the relative median earnings between the full-time and the part-time workers (of men of

---

28Results presented in the next section are robust to the value of this parameter from 1 percent to 3 percent.
age over fifteen). For the substitutability between regular and irregular workers, I set $\epsilon_n$ to be 3 and confirm if the results in Section 5 are robust to $\epsilon_n \in \{3, 5, 7\}$.

On the family side, the ratio of the consumption for the unemployed in the irregular labor market and of those in the regular labor market, $c_{UIR} / c_{UR}$ is set to 0.85, i.e. the level of consumption for the unemployed who do not have an access to unemployment benefits is about 85 percent of the level of consumption for the unemployed with unemployment insurance in the regular labor market. This number reflects the analysis from Ganong and Noel (2019) showing that household spending on non-durables after UI exhaustion drops by 15.1 percent compared to the pre-exhaustion period.

The remaining parameters are those related to disutility from working, $F_m, \sigma_m^L$, those in the probability of finding jobs, $\eta_m^R, a_m, \eta_m^I$, adjustment costs parameters, $\lambda_1$ and $\lambda_2$ in the law of motions for regular and irregular jobs. They jointly determine the relative size of labor forces in each labor market, the total labor force participation rates, the number of workers in each labor market, the unemployment rates in each labor market, and the total unemployment rate. Because I assume different asymmetries for hiring/firing regular and irregular types, they also affect some higher moments of the above variables.

I calibrate the remaining parameter values by moment matching. Table 2 reports the calibrated parameter values. Specifically, I target means and standard deviations of each labor market’s participation rates ($E[\bar{b}_t], E[\bar{d}_t], \sigma(\bar{b}_t)$, and $\sigma(\bar{d}_t)$), means and standard deviations of the total unemployment rate and of each labor market’s unemployment rates ($E[u_t], E[u_{IR}^R], E[u_{IR}^I], \sigma(u_t), \sigma(u_{IR}^R)$, and $\sigma(u_{IR}^I)$), means and standard deviations of the share of workers in each labor market ($E[n_{IR}^R], E[n_{IR}^I], \sigma(n_{IR}^R)$, and $\sigma(n_{IR}^I)$), and means of the promotion and demotion rates ($E[p_{IR}^R \rightarrow R] and E[p_{IR}^R \rightarrow IR]$).

---

29 There is no empirical estimate for the elasticity of substitution between regular and irregular workers. As a proxy, I examine the elasticity of substitution between workers with education levels of high school equivalents and workers with college equivalents. The estimates for this value in the literature range from 1/0.7 to 1/0.3 (see, for example, Card, 2009).

30 Results presented in the next section are robust to the value of this parameter from 0.75 to 1. Note however that in the model, other consumption premiums, $c_{IR}^R / c_{UIR}^R, c_{IR}^I / c_{UIR}^I$, and $c_{IR}^L / c_{IR}^R$ are time-varying.

31 Results presented in the next section are robust to the values of $\lambda_1$ and $\lambda_2$, as long as $\lambda_1 \lambda_2 > 1$, which is necessary to have non-negative promotion and demotion rates in the steady state.

32 For the details in the targeted moments and calculated moments from the baseline model, refer to Appendix E.6.

33 For the United States, there is no good-quality data for net hiring rates for regular and irregular workers. In this regard, I did not target moments of $x_{IR}^R$ and $x_{IR}^I$. Another reason is due to the assumption that I made that irregular workers are all separated after one period, which makes $x_{IR}^I$ higher in the model.
Table 2: Parameter Values related to Labor Market Variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{IR}$</td>
<td>Disutility for irregular workers</td>
<td>0.45</td>
</tr>
<tr>
<td>$\sigma_{IR}$</td>
<td>Disutility for irregular workers, curvature</td>
<td>8.69</td>
</tr>
<tr>
<td>$\eta_{IR}$</td>
<td>Probability to find a job in the irregular labor market</td>
<td>0.84</td>
</tr>
<tr>
<td>$a_{IR}$</td>
<td>Probability to find a job in the irregular labor market</td>
<td>0.53</td>
</tr>
<tr>
<td>$F_{R}$</td>
<td>Disutility for irregular workers</td>
<td>0.42</td>
</tr>
<tr>
<td>$\sigma_{R}$</td>
<td>Disutility for irregular workers, curvature</td>
<td>3.47</td>
</tr>
<tr>
<td>$\eta_{R}$</td>
<td>Probability to find a job in the regular labor market</td>
<td>0.72</td>
</tr>
<tr>
<td>$a_{R}$</td>
<td>Probability to find a job in the regular labor market</td>
<td>0.53</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Adjustment cost of hiring/firing regular workers</td>
<td>18.08</td>
</tr>
<tr>
<td>$\tilde{\kappa}$</td>
<td>Adjustment cost of hiring/firing regular workers</td>
<td>25.02</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Adjustment cost of hiring/firing irregular workers</td>
<td>5.03</td>
</tr>
<tr>
<td>$\tilde{\gamma}$</td>
<td>Adjustment cost of hiring/firing irregular workers</td>
<td>100.00</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Adjustment cost of promotion</td>
<td>1.51</td>
</tr>
<tr>
<td>$\tilde{\varphi}$</td>
<td>Adjustment cost of demotion</td>
<td>0.85</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Discount factor of the just promoted workers’ productivity</td>
<td>0.90</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Scaling factor of the just demoted workers’ productivity</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Note: This table summarizes parameter values calculated by minimizing the distance between the moments from the baseline model and the targeted data counterparts. Targeted moments are the mean of total labor force participation rates, mean of full-time labor forces, means and standard deviations of the total unemployment rates, means and standard deviations of the full-time and part-time unemployment rates, means and standard deviations of the full-time and the part-time employment rates, and the means of promotion and demotion rates.

For the data counterparts, I use full-time workers and part-time workers in the CPS as proxies for regular workers and irregular workers, respectively, as described in Section A. I use the share of full-time workers (part-time workers) out of the population of age over fifteen as a proxy for $n_t^R$ ($n_t^{IR}$). To get proxies for $u_t^R$ and $u_t^{IR}$ from the data, I use the unemployment rates of the full-time and the part-time labor markets, respectively. I then calculate $\bar{d}_t$ by dividing the sum of full-time workers and the unemployed mostly seeking full-time jobs by population of age greater than fifteen. $\bar{b}_t$ corresponds to the total labor force participation rates. Lastly, I calculate the proxy for the demotion rate, $p_t^{R\rightarrow IR}$ by dividing the number of workers who work part-time for economic reasons but usually work full-time within the same employer by the number of full-time workers in the previous period. Similarly, I calculate the proxy for the promotion rate, $p_t^{IR\rightarrow R}$ by dividing the number of workers who work full-time for economic reasons but usually work part-time within the same employers by the number of part-time workers in the previous period.
5 Labor Market Dynamics

This section studies the labor market dynamics in the baseline model with two types of labor. Specifically, I show that my model replicates the changes in the composition of labor types and differential responses of regular and irregular labor market variables to structural shocks, as documented in Section 2. I focus on monetary policy shocks and relegate impulse responses to other shocks to Appendix B.8.

Figure 11 shows that my model successfully replicates the differential responses of the two labor markets. In response to a contractionary monetary policy shock, the total number of workers and the number of regular workers decrease, while the number of irregular workers increases. At first glance, this could be puzzling because it is much easier for firms to hire/fire irregular workers than regular workers, given that the adjustment costs for hiring and firing irregular types are much smaller. As can be seen from the second panel of Figure 11, this is actually the case. The responses of net hiring rates for irregular types, \( x_{IR} \), are much larger than those for regular types, \( x_R \), and they both decrease in response to a contractionary monetary policy shock.

What derives the differential responses of the two labor markets is the ability of firms to transfer one type of labor to the other type. Firms actively exploit this margin to adjust the total amount of labor input to use for production at the time of the shock. For example, in response to a contractionary monetary policy shock, firms want to decrease the total labor input. This reaction could be achieved by firing regular workers. To do so, however, firms need to pay high firing costs. Therefore, firms instead increase demotion rates and decrease promotion rates, which increases the share of irregular types with lower productivity, and therefore decreases the total amount of labor input to use for production at relatively lower costs. Utilization of this composition margin explains the differential responses of the number of workers in each labor type.

---

34 Appendix B.9 presents the responses of non-labor market variables to structural shocks.
35 In order to generate differential responses of each labor market, either promotion or demotion is necessary. As long as firms can either promote irregular workers to be regular workers or demote regular workers to be irregular workers, the numbers of regular and irregular workers move in the opposite direction to structural shocks. See Appendix B.7.
36 Active utilization of this transfer via promotion and/or demotion is consistent with Borowczyk-Martins and Lalé (2019) that flows from full-time to part-time employment are significant within the sample employees in the United States and in the United Kingdom.
Figure 11: Responses of Labor Market Variables to a Contractionary Monetary Policy Shock

Note: This figure shows the responses of labor market related variables to a one-standard deviation of contractionary monetary policy shock from the baseline model. The first panel in the first row shows the responses of the number of workers as black solid lines, those of the number of regular workers as red dash-dot lines, and those of the number of irregular workers as blue dash lines. The second panel shows the responses of net hiring rates of regular (red dash-dot lines) and irregular workers (blue dashed lines). The third panel shows the responses of promotion and demotion rates with the blue dash line and the red dash-dot line, respectively. In the second row, the first panel shows the responses of the share of irregular workers, the second panel shows the responses of the aggregate unemployment rate (black solid line), those of the unemployment rate in the regular labor market (red dash-dot line), and those of the unemployment rate in the irregular labor market (blue dash line). The last panel shows the responses of the sizes of the labor force. The black solid line shows the responses of the total labor force, the red dash line shows the responses of the regular labor force, and the blue dash line shows the responses of the irregular labor force.
One implication from firms’ use of composition margin is that conventional aggregate labor market variables may not be enough to understand how firms adjust the total labor input in response to shocks. If firms mostly utilize the composition margin, then the total amount of labor input, \( n_t = \left( \eta^n \right)^{\frac{1}{2}} (n^R_t)^{\frac{e_n}{e_n}} + (1 - \eta^n) \left( n^I_t \right)^{\frac{e_n}{e_n}} \), changes substantially, even if there is no change in the number of workers, \( L_t \equiv n^R_t + n^I_t \). For instance, if the number of newly hired irregular workers and the number of fired regular workers are the same or if firms do not create or destruct either type of jobs, but use promotion and/or demotion to change the composition of job types, there is no change in the headcount of workers, but this stability of headcounts masks a change in the total labor input. This implies that the headcount of workers can underestimate the cyclicity of the actual amount of labor input used by firms. Table 3 illustrates this point by showing that the correlation of the headcount of workers with output is much smaller than the correlation of actual labor input with output. The changes in the composition of worker types help explain the gap between the two. This insight further suggests the importance of accounting for the composition of worker types when measuring the total factor productivity (TFP).  

Table 3: Correlation of Labor Market Variables with Output

<table>
<thead>
<tr>
<th>Correlation with</th>
<th>Headcount ( L_t )</th>
<th>Total Labor Input ( n_t )</th>
<th>Share of Irregular Types ( s^I_t )</th>
<th>Total LFP ( \bar{b}_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output (( Y_t ))</td>
<td>0.65</td>
<td>0.84</td>
<td>-0.87</td>
<td>0.48</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation with</th>
<th>Headcount Growth ( S_{L_t} )</th>
<th>Total Labor Input Growth ( S_{n_t} )</th>
<th>Share of Irregular Types Growth ( S_{s^I_t} )</th>
<th>Total LFP Growth ( S_{\bar{b}_t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Growth (( g_{Y_t} ))</td>
<td>0.58</td>
<td>0.88</td>
<td>-0.96</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Note: This table calculates the correlations of labor market variables with output. The first row calculates the correlations of HP-filtered labor market variables with the HP-filtered output (smooth parameter, \( \lambda = 1600 \)). The second row calculates the correlations of log-changes in labor market variables with log-changes of output. Labor market variables of interest are the total number of workers \( (L_t = n^R_t + n^I_t) \), the total amount of labor input to use \((n_t)\), the share of irregular workers out of the total number of workers \((s^I_t = n^I_t / (n^R_t + n^I_t))\), and the total labor force participation \((\bar{b}_t)\).

Moreover, the baseline model successfully replicates the relative rankings of the magnitudes of the changes in the unemployment rates: the responses of the unemployment rate in the regular labor market are the largest and those in the irregular labor market are the smallest, which is consistent with the empirical analysis in Section 2. This pattern is due to the responses of labor force participation.
participation rates. As can be seen from the last panel in Figure 11, the total labor force participation rate responds only modestly to a contractionary monetary policy shock. It exhibits slight procyclicality. This mild response, however, masks heterogeneous responses of the two labor market’s labor forces, which move in opposite directions, with substantially larger amplitudes. The size of the regular labor force moves in the same direction as the total labor force.

Through the lens of the model, this is because $\bar{d}_t$, the threshold determining the regular and the irregular labor markets, responds much more to aggregate shocks than $\bar{b}_t$, the threshold determining labor force participation. Because $\bar{b}_t$ moves only modestly to exogenous shocks, the total labor force responds only modestly to shocks. Meanwhile, larger movements of $\bar{d}_t$ contributes to the differential responses of the size of each type’s labor force. For instance, a contractionary monetary policy shock decreases $\bar{d}_t$, increasing the number of workers participating in the irregular labor market and decreasing the size of the regular labor force. Consider the role of the changes of $\bar{b}_t$ and the changes of $\bar{d}_t$ in the model. If only $\bar{b}_t$ changes, then both the number of irregular types and regular types move in the same direction. Meanwhile, if $\bar{d}_t$ changes with no change in $\bar{b}_t$, then $n_t^R$ and $n_t^{IR}$ move in opposite directions. See Appendix B.5 for formal comparative statics analysis. As the response of the share of irregular workers to a contractionary monetary policy shock illustrates, firms actively utilize the composition margin by changing the relative share of regular and irregular workers to adjust the total amount of labor input. These changes in the relative labor demand between the two then generate more volatile $\bar{d}_t$ over the business cycle. This result is consistent with the fact that the total labor force participation rates are modestly procyclical (see Elsby, Hobijn, and Şahin 2015) and with my analysis from the CPS that gross worker flows between the full-time labor market and the part-time labor market are much larger and more cyclical than gross flows between the part-time labor market and out of labor force (see Section 2 and Appendix A.5).

Because aggregate variables mask heterogeneous experiences of each type of labor, aggregate labor market variables may underestimate the volatility that individual workers experience within the total labor force over the business cycle. As documented in Section 2 many workers move between the two labor markets within the total labor force. This implies that a lot more workers experience changes in their labor income by moving across the two labor markets and hence by switching their job types than those who move in and out of the labor force. The variation in
aggregate labor market variables fails to capture this important source of income fluctuations generated by switching job types. In contrast, the changes in the composition of worker types can capture these risks in labor income fluctuations of workers, in particular, of those who are marginally attached to the regular labor market and hence frequently change their job types by moving across the two labor markets. These switchers manifest themselves as frequent and large changes in \( \bar{d}_t \) in my model, which leads to many workers with inherent work aversion of \( \ell \) near \( \bar{d}_t \) (marginally attached “regular workers”) frequently switch their job types. Section 6 explores the implications of this to the welfare and monetary policy.

6 Welfare and Policy Implications

This section explores welfare and policy implications by building on the insights from empirical and model analysis. I first calculate the welfare costs of economic fluctuations by individual workers and see if a certain group of workers tend to pay larger costs of economic fluctuations. It shows that switchers who frequently move between the regular and the irregular labor markets and move in and out total labor force are the most vulnerable to economic fluctuations. Based on the welfare cost calculations, I then discuss what these findings imply for optimal monetary policies, in particular, towards more inclusive monetary policy.

6.1 Who Bears the Cost of Business Cycle?

In this section, I calculate the welfare costs of the business cycle borne by each worker with \( \ell \in [0, 1] \) within the representative family. In my model, individual workers draw their work aversion level, \( \ell \) every period. This assumption is necessary to have limited consumption insurance, but \( \ell \) summarizing workers’ inherent characteristics is likely slow-moving. Given the assumption that workers draw their \( \ell \) every period from \( i.i.d. \ U[0,1] \), I calculate the welfare costs of economic fluctuations for a worker who by chance draws the same \( \ell \in [0,1] \) every period.
Following the approach in [Lucas (1987)], I calculate welfare costs of the business cycle by individual workers for each $\ell$'s, $\Lambda^{\ell}$, as follows:

$$
\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U \left( (1 + \Lambda^\ell) c_t(\ell), \ell \right) \right] = \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[ U \left( \bar{x}(\ell), \ell \right) \right],
$$

(14)

where $\bar{x}$ denotes the steady-state value of a variable, $x$. For logarithmic utility, we have

$$
\Lambda^\ell = \exp \left( - (1 - \beta) \left( \mathbb{V}^\ell - \overline{\mathbb{V}}^\ell \right) \right) - 1,
$$

(15)

with $\mathbb{V}^\ell \equiv \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U \left( c_t(\ell), \ell \right) \right]$ and $\overline{\mathbb{V}}^\ell = \sum_{t=0}^{\infty} \beta^t \mathbb{E} \left[ U \left( \bar{x}(\ell), \ell \right) \right]$. Here, $\Lambda^\ell$ represents the welfare costs of business cycle for individual workers with $\ell \in [0, 1]$ in consumption equivalents, as it denotes the value that needs to be in order to make utility in the stochastic equilibrium equal to the utility in the steady state (that is non-stochastic equilibrium). In other words, $\Lambda^\ell$ denotes how much consumption an individual worker with a work aversion $\ell$ is willing to forgo in order to avoid economic fluctuations.

Figure 12: Five Groups of Types in the Model

---

**Note:** This figure illustrates the five groups of types in the baseline model: (i) Those who always participate in the regular labor market, i.e., workers with $\ell \in [0, \bar{d} - \epsilon)$, (ii) Those who always participate in the irregular labor market, i.e., workers with $\ell \in (\bar{d} + \epsilon, \bar{b} - \epsilon)$, (iii) Those who always not in the labor force, i.e., workers with $\ell \in (\bar{b} + \epsilon, 1]$, (iv) contingent “regular workers,” i.e., workers with $\ell \in [\bar{d} - \epsilon, \bar{d} + \epsilon]$, and (v) contingent “irregular workers,” i.e., workers with $\ell \in [\bar{b} - \epsilon, \bar{b} + \epsilon]$.  

37
Types in my model can be divided into five groups for some positive number $\epsilon > 0$: (i) Those who always participate in the regular labor market, i.e. workers with $\ell \in [0, \bar{d} - \epsilon)$, (ii) Those who always participate in the irregular labor market, i.e. workers with $\ell \in (\bar{d} + \epsilon, \bar{b} - \epsilon)$, (iii) Those who are always not in the labor force, i.e. workers with $\ell \in (\bar{b} + \epsilon, 1]$, (iv) contingent “regular workers,” i.e. workers with $\ell \in [\bar{d} - \epsilon, \bar{d} + \epsilon]$, and (v) contingent “irregular workers,” i.e. workers with $\ell \in [\bar{b} - \epsilon, \bar{b} + \epsilon]$. The first three groups are “stayers,” in a sense that they always stay in one labor market or in not-in-the-labor-force (NLF). On the other hand, the last two groups are “switchers” or “contingent workers,” in the sense that their current labor types are not expected to last for a long time. For example, contingent regular workers move between the regular and the irregular labor markets over the business cycle. One can say that they are marginally attached to the regular labor market. On the other hand, contingent irregular workers move frequently in and out of the labor force, so they are marginally attached to the irregular labor market.

Figure 13 shows the welfare costs of economic fluctuations borne by each worker with $\ell \in [0, 1]$. Among the stayers, those who always participate in the regular labor market, with lower values of $\ell$, pay the smallest welfare costs of economic fluctuations; workers who are always NLF pay the largest welfare costs; workers who always participate in the irregular labor market pay the amount between the two. This ranking of the costs between the workers among the stayers is associated with the differences in the marginal utilities. Limited consumption insurance generates that the average level of consumption for those who are always in the regular labor market is the highest, that for those who are always NLF is the smallest, and that for workers who are always in the irregular labor market is in between the two. Logarithmic utility from consumption implies the reverse ranking for the marginal utilities, making consumption volatilities for those always NLF more costly.

For the simulated stream of consumption and disutility from supplying labor for a worker in each group, see Appendix B.11.
Figure 13: Welfare Costs of the Business Cycle by Workers’ Type, $\ell$, $\Lambda^\ell$ in %

Panel (a): Welfare Costs (in %)

Panel (b): Welfare Costs (in Level)

Panel (c): Welfare Costs of Contingent Regular Workers

Panel (d): Welfare Costs of Contingent Irregular Workers

Panel (e): Labor Market, Contingent Regular Workers

Panel (f): Labor Market, Contingent Irregular Workers

Note: Panel (a) shows the scatter plot of the welfare costs of business cycles by individual workers’ type, $\ell$, $\Lambda^\ell$ in the percentage of consumption equivalent when I feed in all the shocks to the economy. That is, $\Lambda^\ell$ is how much consumption in percentage an individual worker is willing to forgo in order to avoid economic fluctuations for each type $\ell$. x-axis denotes individual workers’ type, $\ell$ which determines the degree of work aversion, and therefore the type of labor market that these workers are likely to choose, and y-axis denotes the welfare costs, $\Lambda^\ell$. Panel (b) shows the scatter plot of the welfare costs in the level (in the unit of real consumption) with all the shocks in the model. To calculate this, I multiply the average level of consumption per each worker, $E_c(t(\ell))$ to $\Lambda^\ell$. The two vertical lines in the two panels correspond to the two thresholds. The red vertical line is $\bar{d}$ and the blue vertical line is $\bar{b}$. Panel (c) zooms in the welfare cost of business cycles in the percentage of consumption equivalent for contingent regular workers with $\ell \in [\bar{d} - \epsilon, \bar{d} + \epsilon]$ for some $\epsilon > 0$, and Panel (d) zooms in the welfare cost of business cycles in the percentage of consumption equivalent for contingent irregular workers with $\ell \in [\bar{b} - \epsilon, \bar{b} + \epsilon]$ for some $\epsilon > 0$. Panel (e) presents the probability of participating in the regular labor market (red solid line) and in the irregular labor market (blue dash-dot line) for workers with $\ell \in [0.5, 0.55]$, which includes contingent regular workers. The red solid vertical line denotes the threshold of $\bar{d}$ in the steady state. Panel (f) shows the probability of participating in the irregular labor market (blue dash-dot line) and NLF (black dotted line) for workers with $\ell \in [0.625, 0.675]$, which includes contingent irregular workers. The blue solid vertical line denotes the threshold of $\bar{b}$ in the steady state.

More importantly, there are spikes near $\bar{d}$ and $\bar{b}$, the steady state values of the two thresholds. This illustrates that “switchers” pay significantly higher welfare costs of business cycles than stayers. Because switchers are marginally attached to either the regular or the irregular labor market,
they frequently move between the regular labor market and the irregular labor market, or between the irregular labor market and NLF. This makes their consumption and disutility from supplying labor substantially volatile over the business cycle, making them pay the largest costs of economic fluctuations.\footnote{For the stream of consumption and disutility from supplying labor for a worker in each group, see Appendix B.11.} For instance, as \( \bar{b}_t \) decreases in response to a contractionary monetary policy shock (\( \bar{b}_t < \bar{b} \)), contingent irregular workers with \( \ell \) right below \( \bar{b} \) exit the labor force. The changes in \( \bar{b}_t \) in response to aggregate shocks make these contingent irregular workers’ labor market status fluctuates over the three states: (i) employed in the irregular labor market and consume \( c^I_{IR} \), (ii) unemployed in the irregular labor market and consume \( c^{UIR} \), and (iii) NLF and consume \( c^{UIR} \) where the levels of consumption, \( c^I_{IR} \) and \( c^{UIR} \) themselves vary over time. As they are marginally attached to the labor market, they experience larger fluctuations in their consumption with imperfect consumption insurance and disutility from working. This captures one of the findings from \textit{Hall and Kudlyak (2019)} that frequent “circling” happens between unemployment, short-term jobs, and out of labor force, considering that short-term jobs are much closer to the irregular type than the regular type in my model.

Similarly, the decreases of \( \bar{d}_t \) in response to a contractionary monetary policy shock (\( \bar{d}_t < \bar{d} \)) make workers with \( \ell \) right below \( \bar{d} \) move from the regular labor market to the irregular labor market. With the changes in \( \bar{d}_t \), contingent regular workers face the largest risks over their labor market status: (i) employed in the regular labor market and consume \( c^R \), (ii) unemployed in the regular labor market and consume \( c^{UIR} \), (iii) employed in the irregular labor market and consume \( c^I_{IR} \), and (iv) unemployed in the irregular labor market and consume \( c^{UIR} \), where the levels of consumption, \( c^R \), \( c^I_{IR} \), \( c^{UIR} \), and \( c^{UIR} \) themselves change in response to structural shocks. Because contingent regular workers’ risk regarding labor market status is the largest, they pay the largest costs of economic fluctuations over the business cycle with larger volatilities of their consumption and disutility from supplying labor.\footnote{See again Appendix B.11 for the comparison of the volatilities of consumption and disutility from supplying labor between the groups of workers.} Not only the magnitude but also the mass of contingent workers are larger near the threshold \( \bar{d}_t \) than \( \bar{b}_t \). That is, the number of contingent “regular” workers is much greater than the number of contingent “irregular” workers. This is because \( \bar{d}_t \) is about three times more volatile than \( \bar{b}_t \) in the baseline model, making those workers with \( \ell \) close to \( \bar{d} \) move between the regular and the irregular labor markets much more often than those with...
Moreover, larger changes in $\bar{d}_t$ generate many more contingent regular workers than contingent irregular workers. In the next subsection, I examine if the central bank can achieve higher overall welfare by targeting these vulnerable groups of workers (that is, contingent workers).

6.2 Towards More Inclusive Monetary Policy: An Alternative Interest Rate Rule

To study if monetary policy can be more inclusive, I examine if the monetary authority can do better by targeting a specific group of workers (switchers or contingent workers) who are more exposed to economic fluctuations. Specifically, I consider alternative specifications of the Taylor (1993) rule that considers deviations of the two thresholds, the threshold determining labor force participation ($\bar{b}_t$) and the threshold determining participation of either the regular or the irregular labor markets ($\bar{d}_t$) from their steady-state values, and hence stabilizing the composition of labor types:

$$i_t = \left(1 - \rho_i\right)i^* + \rho_i i_{t-1} + (1 - \rho_i) \left[ \phi_{\pi_t} \pi_t + \phi_b \left(\bar{b}_t - \bar{b}\right) + \phi_d \left(\bar{d}_t - \bar{d}\right) \right] + \epsilon_t. \quad (16)$$

The results from Section 6.1 show that contingent workers face larger risks regarding their labor market status as the two thresholds, $\bar{b}_t$ and $\bar{d}_t$, change over the business cycle. In this regard, if the central bank can stabilize the two thresholds, and hence the composition of labor types, then it can substantially reduce the labor market risks that these contingent workers face. For example, the stabilization of $\bar{b}_t$ can minimize the movement of contingent irregular workers who frequently enter and exit the labor force. More importantly, the stabilization of the composition of labor types, and hence the stabilization of $\bar{d}_t$, can minimize the movement of contingent regular workers across the two labor markets and can decrease the number of contingent regular workers, which is significantly larger than the number of contingent irregular workers.

Therefore, I consider the variant of Taylor (1993) rule of Equation (16). The two thresholds correspond to the size of labor forces: $\bar{b}_t$ is the size of the total labor force and $\bar{d}_t$ is the size of the regular labor force. However, the responses of the total labor force participate rate, $\bar{b}_t$, are smaller than the responses of the regular labor force, $\bar{d}_t$.

---

41 See, for example, the last panel in the second row of Figure 11. Responses of total labor force corresponds to responses of $\bar{b}_t$ and responses of regular labor force corresponds to responses of $\bar{d}_t$.

42 This is consistent with the fact that the total labor force participation rates are only modestly procyclical (see Elsby, Hobijn, and Sahin, 2015) and with the analysis from the CPS that gross flows between the full-time labor market and the part-time labor market are much higher and more cyclical than gross flows between the part-time labor market and out of labor force. Analysis from the CPS in Section A also shows that the across-market flows are strongly cyclical and of large magnitudes at the quarterly frequency. The baseline model in Section 3 generates that the responses of the total labor force participation rate, $\bar{b}_t$, are smaller than the responses of the regular labor force, $\bar{d}_t$. 

41
ular labor force, both of which are based on observables. I then compare the maximum welfare of the representative family (or equivalently, minimum welfare loss compared to the non-stochastic equilibrium) achieved from the alternative specification for the interest rate rule of Equation (16) with those achieved from the other two policy rules:

$$i_t = (1 - \rho_i)i^* + \rho_i i_{t-1} + (1 - \rho_i) \left[ \phi_i \pi_t + \phi_u (u_t - \bar{u}) \right] + \epsilon_i$$  (17)

$$i_t = (1 - \rho_i)i^* + \rho_i i_{t-1} + (1 - \rho_i) \left[ \phi_i \pi_t + \phi_u^R \left( u_t^R - \bar{u}_R \right) + \phi_u^{IR} \left( u_t^{IR} - \bar{u}_R^{IR} \right) \right] + \epsilon_i$$  (18)

The first one is the conventional Taylor (1993) rule, which targets the inflation rate and the aggregate unemployment gap, and the other one targets each labor market’s unemployment gap, separately as in Equation (18).

Table 4: Comparison of the Welfare Losses across Different Interest Rate Rules

<table>
<thead>
<tr>
<th></th>
<th>Aggregate Unemployment Rate $u$</th>
<th>Two Unemployment Rates $u^R$ and $u^{IR}$</th>
<th>Two Thresholds $\bar{b}$ and $\bar{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption Equivalent (%)</td>
<td>1.35</td>
<td>1.33</td>
<td>0.83</td>
</tr>
<tr>
<td>Relative Costs (%)</td>
<td>100</td>
<td>98.8</td>
<td>62.8</td>
</tr>
</tbody>
</table>

Note: This table shows the minimum welfare losses compared to the non-stochastic equilibrium that are achieved from different specifications for Taylor (1993) rules. The minimum welfare losses are represented in terms of consumption equivalent in percentage in the first row, in a similar fashion to the welfare cost calculations, but at the family level. Recall that the family’s welfare is defined by integrating all individual workers’ utilities. This utilitarian family puts the same weights on each workers’ welfare. The second row calculates the relative welfare losses with the other two interest rate rules compared to the minimum welfare loss achieved from the conventional Taylor rule with the aggregate unemployment gap.

Table 4 compares family-level welfare under the different monetary policy rules. It shows that the alternative interest rate rule which reacts to the deviations of the two thresholds from their steady-state values significantly reduces the overall welfare loss from aggregate fluctuations. Compared to the family-level welfare in the non-stochastic equilibrium, the minimum welfare loss achieved from the conventional interest rate rule with the aggregate unemployment gap is the largest. That is, the cost of the business cycle in terms of consumption equivalent with the

---

43Indirect utility of the representative family is derived by integrating all the individual workers’ utilities. This utilitarian family puts the same weights on each workers’ welfare.

44This holds true regardless of the nature of shocks. When the economy is shocked with only technology shocks or monetary policy shocks, or government spending shocks, the minimum welfare losses obtained from the alternative specification of the policy rule are the lowest.
optimal weight on the aggregate unemployment gap in the conventional monetary policy rule is the largest among the three specifications.\footnote{This is associated with different cyclicalities of headcount and total labor input in Section 5 While the actual output gap is associated with actual labor input used by firms, the aggregate unemployment rate is calculated from the headcount of workers, which underestimates the cyclicalities of total labor input. Appendix B.12 derives the second-order approximation of the family’s welfare in terms of the volatilities of the output gap and compares this with the formulation with the aggregate unemployment gap.} When the central bank instead stabilizes each labor market’s unemployment rate separately, the minimum welfare loss in consumption equivalent decreases, but by a modest 1.2 percent. On the other hand, the minimum welfare loss achieved from the alternative Taylor rule specification with the stabilization of the two thresholds is about 40 percent lower than that with the conventional Taylor rule. Compared to the specification of Equation (18), which considers the overall stance of each labor market, this alternative interest rate rule of Equation (16) directly targets contingent workers who are the most vulnerable to aggregate fluctuations. Because the labor market risks of those contingent workers are related to the changes in the two thresholds, the stabilization of the two thresholds can directly lower the burden of them who pay the largest costs of the business cycle. Therefore, targeting contingent workers with the stabilization of the two thresholds generates the largest welfare gains.\footnote{For the improvement of the welfare from the alternative specification of the interest rate rule with the stabilization of the two thresholds, and hence the composition of labor types by individual workers with \( \ell \in [0,1] \), see Appendix B.11.}

Figure 14 describes how the weights, \( \phi_b \) and \( \phi_d \), affect welfare (holding \( \phi_\pi = 1.5 \)). The left panel shows the level of the representative family’s welfare for each combination of \((\phi_b, \phi_d)\) from \(-1\) to 2.5, and the right panel plots the iso-welfare curve that shows the combinations of \((\phi_b, \phi_d)\) generating the same level of welfare. Maximum welfare is achieved with \( \phi_b^* = -0.85 \) and \( \phi_d^* = 2.10 \). That is, the central bank wants to put higher weight on stabilizing \( \bar{d}_t \) than stabilizing \( \bar{b}_t \). This is because contingent regular workers who are marginally attached to the regular labor market pay the highest welfare costs of economic fluctuations according to the analysis from Section 6.1. Moreover, there is a larger mass of contingent regular workers than contingent irregular workers with more volatile \( \bar{d}_t \). Therefore, the central bank wants to put higher weight on the stabilization of \( \bar{d}_t \) than the stabilization of \( \bar{b}_t \).
Figure 14: Optimal vs. Actual Monetary Policy

Note: The left panel shows the level of the representative family’s welfare per each combination of \((\phi^*_b, \phi^*_d)\) from \(-1\) to 2.5 in the policy rule of Equation (16) and the right panel plots the iso-welfare curve that shows the combinations of \((\phi^*_b, \phi^*_d)\) generating the same level of welfare. The red stars in each panel denote the optimal policy that achieves the maximum welfare of the representative family and the black circles in each panel denote the current policy estimates. (see Table 5)

This is consistent with the analytical formulation of the representative family’s life-time utility. As is standard in New Keynesian models, the representative family’s utility, in a simpler version of the model, can be approximated up to the second order as the welfare loss associated with the variance of inflation and output gap. Because the output gap is a function of the two types of labor input and the two thresholds are functions of the two types of labor, the output gap, in turn, can be represented in terms of the two thresholds as follows:

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - \overline{U}}{U \overline{C}} \right) \approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi \pi^2_t + \left( \Phi^*_b \tilde{b} + \Phi^*_d \tilde{d} \right)^2 \right\} + \text{t.i.p.} + \text{h.o.t.},
\]

where a variable with check denotes log-deviation from its value in the steady state, t.i.p. stands

\footnote{See Appendix B.12 for formal derivations.}
for terms independent from policy, and h.o.t. means higher order terms. Under the baseline calibration, $\Phi_d > 0$ and $\Phi_b > 0$. This means the volatilities of the two thresholds generate larger welfare losses. Therefore, the central bank wants to stabilize the two thresholds to achieve smaller welfare losses. Moreover, $\Phi_d$ is greater than $\Phi_b$, meaning that the volatility of $\bar{d}_t$ generates larger welfare losses than the volatility of $\bar{b}_t$. This translates into higher weights on the stabilization of $\bar{d}_t$. Meanwhile, because $\Phi_d > 0$ and $\Phi_b > 0$, the opposite direction of the changes in the two thresholds may generate smaller welfare losses. The signs of the optimal weight, $(\phi^*_b, \phi^*_d)$, reflect this point. The optimal weight suggests that the central bank can achieve higher welfare by raising interest rates when the size of the regular labor force, $\bar{d}_t$, increases and by lowering nominal interest rates when the total labor force size, $\bar{b}_t$, increases. The opposite responses to the two thresholds contributes to the stabilization of irregular labor force size, $\bar{b}_t - \bar{d}_t$.

This formulation also suggests that the stabilization of the aggregate unemployment rate is not sufficient to achieve higher overall welfare. The second-order approximation of the life-time utility of the representative family can be re-written with the aggregate unemployment gap as follows:\footnote{See Appendix B.12 for formal derivations.}

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - \bar{U}}{U_C} \approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_{\pi} \pi_t^2 + \left( \Phi_u \bar{u}_t + \Delta^{u,R} \bar{R}_t + \Delta^{u,IR} \bar{R}_t \right)^2 \right\} + t.i.p. + h.o.t., \quad (20)$$

or equivalently, in terms of $\bar{b}_t$ and $\bar{d}_t$,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - \bar{U}}{U_C} \approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_{\pi} \pi_t^2 + \left( \Phi_u \bar{u}_t + \Delta^{u,R} \bar{R}_t + \Delta^{u,IR} \bar{R}_t \right)^2 \right\} + t.i.p. + h.o.t. \quad (21)$$

The presence of the extra-terms, such as, $\Delta^{u,R} \bar{R}_t + \Delta^{u,IR} \bar{R}_t$ or $\Delta^{u,R} \bar{R}_t + \Delta^{u,IR} \bar{R}_t$, illustrates that central banks need to account for more than the stabilization of aggregate unemployment rates to achieve higher welfare. Because aggregate unemployment rates do not account for the differential dynamics of the two labor markets, which tend to be more volatile, the stabilization of the aggregate unemployment rate does not translate into the stabilization of the two labor markets.\footnote{See Section 5 for the comparison of impulse responses of aggregate labor market variables and each labor market’s variables.} Moreover because unemployment rates are calculated based on the headcount of the labor, which
tends to underestimate the cyclicality of the total labor input (see Section 5), the stabilization of aggregate unemployment rates is not enough to stabilize the economy.

How does the current policy look like compared to the optimal inclusive monetary policy specified as above? To answer this question, I estimate the interest rate rule with the two thresholds, \( \bar{b}_t \) and \( \bar{d}_t \), i.e. with the size of total labor force and the size of the full-time labor force from the CPS. Specifically, I consider the following three regression equations:

(i) \( i_t = \alpha + \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1 - \rho_1 - \rho_2) \left[ \phi_\pi \pi_t + \phi_u (u_t - u^*_t) \right] + \epsilon_i \),

(ii) \( i_t = \alpha + \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1 - \rho_1 - \rho_2) \left[ \phi_\pi \pi_t + \phi_b (\bar{b}_t - \bar{b}) + \phi_d (\bar{d}_t - \bar{d}) \right] + \epsilon_i \),

(iii) \( i_t = \alpha + \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1 - \rho_1 - \rho_2) \left[ \phi_\pi \pi_t + \phi_u (u_t - u^*_t) + \phi_b (\bar{b}_t - \bar{b}) + \phi_d (\bar{d}_t - \bar{d}) \right] + \epsilon_i \).

I estimate the above regressions by using nowcasts of GDP-deflator based inflation rates (for \( \pi_t \)), nowcasts of unemployment rates (for \( u_t \)) from the Greenbook dataset maintained by the Federal Reserve Bank of Philadelphia, and short-term natural rate of unemployment (NAIRU, for \( u^*_t \)) retrieved from the Federal Reserve Economic Data (FRED). To calculate the deviations of the two thresholds from their steady-state values, I calculate their sample means across the sample horizon from the first quarter of 1976 to the last quarter of 2007 and subtract them from the series of total labor force participation rates and the share of the full-time labor force out of the total population. As alternatives, I also consider the HP-filtered series of them with a large enough smoothing parameter of \( 10^7 \).

---

50 The Federal Reserve does not explicitly take into account the compositional changes of worker types when adjusting nominal interest rates. In this regard, this exercise backs out the weights that the Federal Reserve could have put on if it follows the alternative specification of interest rate rule of Equation 16.

51 Taylor rule estimates in Table 5 are robust to the forecasts of inflation rates in other horizons (one to three-quarters ahead forecasts of GDP-deflator based inflation rates and the four-quarter average of expected inflation rates. They are also robust to the use of the long-run natural rate of unemployment from the FRED.

52 Taylor rule estimates in Table 5 are robust to a range of smoothing parameters from \( 10^5 \) to \( 10^9 \).
Table 5: Taylor Rule Estimation from Data

<table>
<thead>
<tr>
<th>Optimal Monetary Policy</th>
<th>Data Deviation from Mean</th>
<th>HP filtered</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
</tr>
<tr>
<td>( \phi_\pi )</td>
<td>1.5</td>
<td>1.51***</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>( \phi_u )</td>
<td>-0.82***</td>
<td>-0.83***</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>( \phi_b )</td>
<td>-0.85</td>
<td>1.03***</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>( \phi_d )</td>
<td>2.10</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.19)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>( RMSE )</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>Welfare Loss (in %)</td>
<td>0.83</td>
<td>3.61</td>
</tr>
<tr>
<td></td>
<td>std. dev of ( \bar{b}_t ) (%)</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>std. dev of ( \bar{d}_t ) (%)</td>
<td>0.61</td>
</tr>
<tr>
<td>Welfare Loss (in %, ( \phi_\pi = 1.5 ))</td>
<td>0.83</td>
<td>3.69</td>
</tr>
<tr>
<td></td>
<td>std. dev of ( \bar{b}_t ) (%)</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>std. dev of ( \bar{d}_t ) (%)</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Note: This table presents the estimates of interest rate rules (i)-(iii). I use nowcasts of GDP-deflator based inflation rates and unemployment rates. I use total labor force participation rates and the share of the full-time labor force out of population from the CPS for \( \bar{b}_t \) and \( \bar{d}_t \), respectively. To calculate the deviations of the two thresholds from their steady-state values, I subtract their sample means from their time series (the fourth and the fifth columns). Alternatively, I consider the HP-filtered series of them with the smoothing parameter of 10^7 (the last two columns). The first column presents the estimates of the weight on the inflation rate and unemployment gap for the conventional Taylor rule of (ii). The second and the fourth columns show the estimates of the weights on inflation rates and the two thresholds from the interest rate rule of (ii). The third and the fifth columns show the estimates of the weights on inflation rates, unemployment rate gaps, and on the two thresholds from the interest rate rule of (iii). The sample period is from the first quarter of 1976 to the last quarter of 2007. Statistical significance at the 90/95/99% confidence level indicated with */**/***, respectively. Newey-West robust standard errors are reported in parenthesis. I also calculate welfare loss in terms of consumption equivalent in percentage with all the shocks and standard deviations of \( \bar{b}_t \) and \( \bar{d}_t \) in percentages calculated from the baseline model under different policy rules with the estimates of weights on inflation, unemployment, and the two thresholds, \( \hat{\phi}_\pi \), \( \hat{\phi}_u \), \( \hat{\phi}_b \), and \( \hat{\phi}_d \). The last three rows present the case when I fix \( \phi_\pi = 1.5 \), but use the estimates from the data for other weights, \( \hat{\phi}_u \), \( \hat{\phi}_b \), and \( \hat{\phi}_d \).

Table 5 presents the estimates of the three interest rate rules considered above. Compared to the optimal weights on each threshold calculated above, the Federal Reserve has been putting either the wrong sign on the stabilization of \( \bar{b}_t \) and \( \bar{d}_t \), and/or wrong relative weights on each threshold. This could have decreased not only the economy-wide welfare (see welfare loss in Table 5) but also the contingent workers’ welfare, by making the two thresholds more volatile.
(see standard deviations of $\tilde{b}_t$ and $\tilde{d}_t$ in Table 5). For example, welfare costs under the current policy rules are about 1.67-4.35 times (2.52-6.00 times with $\phi_\pi = 1.5$) greater than the welfare cost under the optimal inclusive monetary policy rule. Under the current policy rules, $\tilde{b}_t$ is about 1.2-1.37 times more volatile and $\tilde{d}_t$ is about 1.3-2.36 times more volatile than $\tilde{b}_t$ and $\tilde{d}_t$ under the optimal policy, respectively. In particular, the current policy could have generated a lot more contingent workers and made labor market risks of the contingent workers greater. Therefore, contingent workers who frequently move either between the regular and irregular labor markets or between the irregular labor markets and NLF would have experienced even greater volatility in their consumption and disutility from supplying labor. In sum, the current Federal Reserve’s policy may be harsher on contingent workers, making them even more vulnerable to economic fluctuations. If it instead considers the alternative specification of the policy rule, which targets these contingent workers, then it can substantially ease the burden of the vulnerable, and achieve higher overall welfare.\textsuperscript{53}

7 Concluding Remarks

Central bankers traditionally emphasize that their job is to take care of aggregate fluctuations while redistributive aspects of the business cycles should be addressed by fiscal policy instruments. However, there is a growing consensus that monetary policy can have important redistributive effects (in fact, monetary policy transmission can rely on redistribution, see, for example, \textcite{Auclert}, and in the current environment of rising inequality and polarization, central banks are under increasing pressure to explore new ways to help the “average” citizen and especially the least-protected groups in the economy.

Mainstream monetary models are poorly equipped to shed light on these new tasks because these models largely rely on representative agents or (nearly) perfect insurance and so have little (if any) heterogeneity. To make progress, I depart from this tradition and develop a tractable New Keynesian model featuring two types of labor where workers and firms make endogenous decisions over labor types. My model generates that “switchers” or “contingent workers,” who frequently move between the regular and the irregular labor market or between the irregular

\textsuperscript{53}See Appendix B.13 for the comparison of historical Federal Funds Rates with the implied interest rates from the optimal inclusive monetary policy rules with $\tilde{b}_t$ and $\tilde{d}_t$. 48
labor market and not-in-the-labor-force, incur substantially larger costs of economic fluctuations. In particular, contingent regular workers who are marginally attached to the regular labor market pay the largest cost. These workers face the largest idiosyncratic risks regarding their labor market status, which generates larger volatilities in their consumption and disutility from labor supply. This makes them more vulnerable to business cycles. Can monetary policy ease the burden of these contingent workers? I show that an alternative interest rate rule to stabilize the composition of workers, based on the size of each labor force, can achieve a significantly higher aggregate welfare.

While my model focuses on a specific dimension of heterogeneity, labor market arrangements, the lesson is broader. Welfare and policy implications from my model raise a warning flag on monetary policy frameworks that focus only on “aggregate” labor market variables. The distribution of business cycle costs is unevenly distributed in the economy. By understanding the welfare implications of this heterogeneity and fine-tuning policies to respond to this heterogeneity, monetary policy can have disproportionally large benefits for the vulnerable groups such as contingent workers and thus deliver superior welfare at the aggregate level.

Much work remains ahead to understand and develop inclusive monetary policy. To gain tractability, my model makes several simplifying assumptions and future work can introduce additional realistic features. For example, in my model, workers gather into the family and they do not have access to any credit markets other than through the arrangements with the representative family, who serves as a stand-in for all the possible arrangements that workers deal with their idiosyncratic labor market risks. While convenient, this assumption misses the heterogeneity in the ability of different workers to insure against their labor market risks. It is plausible that irregular types and those not in the labor force build up smaller amounts of wealth and their borrowing constraints may be more binding than regular workers, and hence they have less opportunity to self-insure. Accounting for this heterogeneity in asset holdings and the interaction of this heterogeneity with the different labor market risks for various worker types can generate even larger costs of economic fluctuations for a certain group. Incorporating this dimension of heterogeneity is left for future work.
References


## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>The Share of Full-time and Part-time Workers and Permanent and Temporary Workers</td>
<td>58</td>
</tr>
<tr>
<td>A2</td>
<td>The Share of Temporary Workers and Involuntary Part-Timers, Raw Series</td>
<td>59</td>
</tr>
<tr>
<td>A3</td>
<td>The Share of Temporary Workers and Involuntary Part-Timers, HP-Filtered ((smoothing parameter of 100))</td>
<td>60</td>
</tr>
<tr>
<td>A4</td>
<td>$f_{FT, PT}$ of Hourly-Paid Workers, Not-Hourly-Paid Workers, and Non-respondents</td>
<td>61</td>
</tr>
<tr>
<td>A5</td>
<td>$f_{FT, PT}$ of Workers Who Do the Same Job and Work for the Same Employer, and Those Who Either a Do Different Job or Work for a Different Employer, and Non-respondents</td>
<td>62</td>
</tr>
<tr>
<td>A6</td>
<td>Flows from Employment to Employment</td>
<td>63</td>
</tr>
<tr>
<td>A7</td>
<td>Flows from Employment to Unemployment</td>
<td>64</td>
</tr>
<tr>
<td>A8</td>
<td>Flows from Unemployment to Employment</td>
<td>65</td>
</tr>
<tr>
<td>A9</td>
<td>Flow Decomposition of the Full-Time and the Part-Time Labor Forces</td>
<td>67</td>
</tr>
<tr>
<td>A10</td>
<td>Flow Decomposition of the Total Labor Force Participation: Contributions of Churn</td>
<td>68</td>
</tr>
<tr>
<td>A12</td>
<td>Flow Decomposition of the Not-in-the-Labor Force: Contributions of Exit and Entry</td>
<td>70</td>
</tr>
<tr>
<td>A13</td>
<td>Responses of Labor Market Variables When Firms Cannot Demote</td>
<td>86</td>
</tr>
<tr>
<td>A14</td>
<td>Responses of Labor Market Variables When Firms Cannot Promote</td>
<td>87</td>
</tr>
<tr>
<td>A15</td>
<td>Responses of Labor Market Variables When Firms Cannot Promote and Demote</td>
<td>87</td>
</tr>
<tr>
<td>A16</td>
<td>Responses of Labor Market Variables to Demand Shocks I.</td>
<td>88</td>
</tr>
<tr>
<td>A17</td>
<td>Responses of Labor Market Variables to Demand Shocks II.</td>
<td>89</td>
</tr>
<tr>
<td>A18</td>
<td>Responses of Labor Market Variables to a Contractionary Monetary Policy Shock</td>
<td>89</td>
</tr>
<tr>
<td>A19</td>
<td>Responses of Labor Market Variables to Supply Shocks II.</td>
<td>90</td>
</tr>
<tr>
<td>A20</td>
<td>Responses to Persistent Technology Shocks</td>
<td>90</td>
</tr>
<tr>
<td>A21</td>
<td>Responses to Transitory Technology Shocks</td>
<td>91</td>
</tr>
<tr>
<td>A22</td>
<td>Responses to Monetary Policy Shocks</td>
<td>91</td>
</tr>
<tr>
<td>A23</td>
<td>Responses to Government Spending Shocks</td>
<td>92</td>
</tr>
<tr>
<td>A24</td>
<td>Responses of Three Margins of Labor Adjustment to Exogenous Shocks</td>
<td>93</td>
</tr>
<tr>
<td>A25</td>
<td>The Stream of Consumption for a Worker in Each Group</td>
<td>94</td>
</tr>
<tr>
<td>A26</td>
<td>The Stream of Utility from Consumption for a Worker in Each Group</td>
<td>95</td>
</tr>
<tr>
<td>A27</td>
<td>The Stream of Disutility from Supplying Labor for a Worker in Each Group</td>
<td>95</td>
</tr>
<tr>
<td>A28</td>
<td>Welfare Gains by Individual Workers</td>
<td>96</td>
</tr>
<tr>
<td>A29</td>
<td>Comparison of Historical Federal Funds Rates with the Implied Interest Rates from the Non-Intertial Specification of the Alternative Monetary Policy Rule of Equation (A64)</td>
<td>114</td>
</tr>
<tr>
<td>A30</td>
<td>Comparison of Historical Federal Funds Rates with the Implied Interest Rates from the Non-Intertial Specification of the Alternative Monetary Policy Rule of Equation (A65)</td>
<td>115</td>
</tr>
</tbody>
</table>
List of Tables

<table>
<thead>
<tr>
<th>A1</th>
<th>Average Statistics of the Labor Market Statuses in CPS</th>
<th>...</th>
<th>57</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2</td>
<td>Correlation between Real GDP and Flows between the Full-time, the Part-time Labor Force and Out of Labor Force</td>
<td>...</td>
<td>70</td>
</tr>
<tr>
<td>A3</td>
<td>Target Moments and Model Counterparts</td>
<td>...</td>
<td>85</td>
</tr>
</tbody>
</table>
Appendix

Appendix A supplements the analysis from the micro-Current Population Survey (CPS) data from January 1976 to December 2019 in Section A, and Appendix B supplements the model in Section 3.

A Analysis from the CPS data

This section supplements Section 2 which examines the basic macroeconomic facts about the full-time and part-time labor markets, and the changes in the composition of workers over the business cycle, by using the microdata from the Current Population Survey (CPS) from January 1976 to December 2019.

To that end, I first divide the labor market statuses into five groups: (i) Employed in the full-time \((e^{FT})\), (ii) Employed in the part-time \((e^{PT})\), (iii) Unemployed in the full-time \((u^{FT})\), (iv) Unemployed in the part-time \((u^{PT})\), and (v) Not in the labor force \((n)\), following the CPS distinction of the full-time and the part-time labor forces. I divide the unemployed into the unemployed full-time and the unemployed part-time from the kind of a job current job-seekers are looking for. If they are mainly looking for full-time (part-time) jobs as their “next” jobs, they are classified into the unemployed full-time, \(u^{FT}\) (the unemployed part-time, \(u^{PT}\)). I then examine the stocks of those labor market statuses and gross flows across them. I use full-time workers as a proxy for “regular-type” labor, and part-time workers as a proxy for “irregular-type” labor. Table A1 presents summary statistics of the stocks and the flows between them.

Table A1: Average Statistics of the Labor Market Statuses in CPS

<table>
<thead>
<tr>
<th></th>
<th>(e^{FT})</th>
<th>(e^{PT})</th>
<th>(u^{FT})</th>
<th>(u^{PT})</th>
<th>(n)</th>
<th>stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^{FT})</td>
<td>86.62</td>
<td>3.87</td>
<td>0.91</td>
<td>0.07</td>
<td>1.50</td>
<td>49.31</td>
</tr>
<tr>
<td>(e^{PT})</td>
<td>16.99</td>
<td>65.47</td>
<td>3.29</td>
<td>0.92</td>
<td>8.04</td>
<td>11.66</td>
</tr>
<tr>
<td>(u^{FT})</td>
<td>14.83</td>
<td>8.04</td>
<td>53.68</td>
<td>0.93</td>
<td>11.73</td>
<td>4.03</td>
</tr>
<tr>
<td>(u^{PT})</td>
<td>5.00</td>
<td>20.49</td>
<td>5.23</td>
<td>35.04</td>
<td>25.73</td>
<td>0.68</td>
</tr>
<tr>
<td>(n)</td>
<td>1.71</td>
<td>3.49</td>
<td>1.10</td>
<td>0.52</td>
<td>87.64</td>
<td>35.00</td>
</tr>
</tbody>
</table>

Source: CPS microdata from January 1976 to December 2019.

1All the results in this section come from the sample with all workers of age over fifteen. The results are almost identical if I limit the sample to all the male workers of age from 15 to 64.
2Microdata from the CPS starts from January 1976.
3The CPS distinguishes full-time workers who work over 35 hours or more per week, and part-time workers who work less than 35 hours per week.
A.1 Are Full-time and Part-time Workers Good Proxies for Regular and Irregular Workers?

Before moving on, this section evaluates if it is okay to use full-time and part-time workers as proxies for regular and irregular types in the model. To preview the results, this section concludes that it makes sense to use the full-time and part-time classifications, which are only available at the business cycle frequency from the publicly available data including the CPS.

Figure A1: The Share of Full-time and Part-time Workers and Permanent and Temporary Workers

Note: This figure shows the share of full-time and part-time workers and the share of permanent and temporary workers in four countries, the United Kingdom, Germany, Greece, and Italy, from 1983 to 2019. The shares of full-time workers are denoted as red dotted lines and the shares of part-time workers are denoted as blue dotted lines. The shares of permanent workers are denoted as the red solid lines with circles, while the shares of temporary workers are represented as the blue solid lines with circles. Sources: OECD Statistics

For the United States, I use full-time workers as proxies for regular types and part-time workers as proxies for irregular types in the model developed in Section 3. In the model, regular and irregular workers are different in two aspects. First, regular workers have higher productivity than irregular workers. Second, regular workers are more attached to firms than irregular workers. While the exogenous separation rates for regular workers are low, irregular workers are all separated after one period. Moreover, hiring/firing regular workers are much more expensive than hiring/firing irregular workers. What this implies is that it would be better if I use some other classifications of labor-types. For example, I might use permanent workers and temporary workers, instead of full-time and part-time workers. Or, I can classify all the workers who work full time and are subject to social security benefits into regular workers and all the other types into
irregular workers. All the publicly available data that are available at the business cycle frequency (at least quarterly) including the CPS, however, do not have such classifications. Therefore, I instead use full-time and part-time classifications.

To figure out if using full-time and part-time workers as proxies for regular and irregular types, I first compare the relative share of the full-time workers and the part-time workers with those of permanent and temporary workers in the countries where both series are available. Figure A1 shows the shares of workers according to each classification for the United Kingdom, Germany, Greece, and Italy. As can be seen from the figure, while the share of permanent and temporary workers fluctuate much more than the share of full-time and part-time workers, they are of similar magnitudes.

Figure A2: The Share of Temporary Workers and Involuntary Part-Timers, Raw Series

Note: This figure shows the share of temporary workers (y-axis on the left), involuntary part-timers out of labor force, and out of total employment (y-axis on the right) in four countries, the United Kingdom, Germany, Greece, and Italy, from 1983 to 2019. The shares of temporary workers are represented as the blue solid lines with circles, the shares of involuntary part-time workers out of total labor force are denoted as blue dotted lines, and the shares of involuntary part-time workers out of total employment are denoted as the blue dash lines. Sources: OECD Statistics

When I focus on involuntary part-time workers, then they are more in line with temporary workers. Figure A3 illustrates this. As can be seen from this figure, the share of involuntary part-timers out of total labor force and/or out of total employment closely tracks the changes in the share of temporary workers. Given that these involuntary part-time workers are those who tend to experience fluctuations over the business cycles, this figure and the similar magnitudes of the
shares according to the two classifications in Figure A1 at least partly confirm the quality of using full-time and part-time workers as proxies for regular and irregular workers.

Figure A3: The Share of Temporary Workers and Involuntary Part-Timers, HP-Filtered ((smoothing parameter of 100)

Note: This figure shows the HP-filtered (smoothing parameter of 100) series of the share of temporary workers (y-axis on the left), involuntary part-timers out of labor force, and out of total employment (y-axis on the right) in four countries, the United Kingdom, Germany, Greece, and Italy, from 1983 to 2019. The shares of temporary workers are represented as the blue solid lines with circles, the shares of involuntary part-time workers out of total labor force are denoted as blue dotted lines, and the shares of involuntary part-time workers out of total employment are denoted as the blue dash lines. Sources: OECD Statistics

A.2 Flows from Full-Time Employment to Part-Time Employment

This section examines if the flows from the full-time employment to part-time employment, $f_{eFT,ePT}$ are entirely driven by firms’ adjustments of intensive margins or not. To this end, I first compare the flows of $f_{eFT,ePT}$ for hourly-paid workers, and those who are not paid hourly. Second, I compare the flows of $f_{eFT,ePT}$ for workers who work for the same employer, and do the same job with those who either work for different employers or do different jobs. Figure A4 and A5 illustrate that the flows from the full-time employment to part-time employment are not entirely driven by firms’ adjustment of intensive margins, but show the compositional changes of workers over the business cycle.
Figure A4: $f_{\text{FT,PT}}$ of Hourly-Paid Workers, Not-Hourly-Paid Workers, and Non-respondents

**Note:** The first panel shows the flow from full-time employment to part-time employment for hourly-paid workers with the black solid line in %. The second panel shows the same flows for those who are not paid hourly with the blue solid line. The last panel plots the same flows for the non-respondents to this question. *Source:* CPS microdata from January 1994 to December 2019. Flows are seasonally adjusted.

After the re-design of the CPS in 1994, there is a question asking if a worker is paid hourly or not. Because firms can easily adjust only hours each worker works if they are paid hourly, if the countercyclicality of the flows from the full-time employment to the part-time employment is entirely driven by hourly-paid workers, then this does not necessarily mean the compositional changes of worker types over the business cycle. In this regard, I examine if the countercyclicality of $f_{\text{FT,PT}}$ holds for workers not paid hourly. Figure A4 shows that $f_{\text{FT,PT}}$ is countercyclical for not only workers who are paid hourly but also for those who are not paid hourly and for non-respondents.

I can also examine if the flows from full-time employment to part-time employment are driven by the same employers for the same job. If workers who moved from the full-time employment status to the part-time employment status do exactly the same job for the same employer, then it could be considered as the adjustment of intensive margins. To examine this, I compare the flows, $f_{\text{FT,PT}}$ for workers who do the same job for the same employer with those who either do a different job or work for the different employers. Figure AS shows that not only the flows of $f_{\text{FT,PT}}$ for workers who work for the same employers and do exactly the same job are small, but

---

4Because there are many missing values for the question asking if he or she is paid hourly or not, I also report the case for non-respondents.
also they do not exhibit strong countercyclicality. Most of the flows, \( f_{FT,PT} \) are from workers who either do not work for the same employer or do different jobs than before, and the flows of these workers show strong countercyclicality.

Figure A5: \( f_{FT,PT} \) of Workers Who Do the Same Job and Work for the Same Employer, and Those Who Either a Do Different Job or Work for a Different Employer, and Non-respondents

---

**Note:** The first panel shows the flows of workers who do the same job and work for the same employer with the black solid line in %. The second panel shows the same flows for those who either do a different job or work for a different employer with the blue solid line. The last panel plots the same flows for the non-respondents to this question.  
Source: CPS microdata from January 1994 to December 2019. Flows are seasonally adjusted.

### A.3 Evidence from CPS: Flows within the Labor Force

This section examines flows between more granular labor market statuses. Conventional studies have looked at labor market flows between employment and unemployment over the business cycle. This section investigates labor market flows among more granular labor market statuses. In particular, I focus on the importance of labor market flows across the two different labor markets: the full-time labor market and the part-time labor market.

All the flows calculated using the microdata from the CPS denote the gross flows from one state to the other state from the previous month to the next month. These flows are obtained by using a rotating-panel element in the CPS design that those who participated in the survey remain in the sample for four months in a row, rotate out for eight months, and then rotate back in for another four months. This feature allows some samples in a given month to be linked longitudi-
nally to those in the subsequent month. From this longitudinally linked microdata, the transition probabilities to estimate flows are then simply calculated by the fraction of those in one state in a given month who report that they are in the other state. This becomes the transition probabilities from one state to the other state. All the flows reported here are adjusted for margin errors, classification errors, temporal aggregation errors, seasonally adjusted, and converted to quarterly by averaging out for three months in a given quarter. Details about margin error adjustments, classification adjustments using “deNUNified” following Elsby, Hobijn, and Sahin (2015), and temporal aggregation bias correction following Shimer (2012) are laid out in subsequent subsections of A.6.

Figure A6: Flows from Employment to Employment

Note: The first panel shows the flow from employment to employment with the black solid line in %. The second panel shows the flow from employment to employment within the same labor market. The light blue dashed line is the flows from full-time employment to full-time employment with the y-axis on the left in % and the light red dotted line plots the flows from part-time employment to part-time employment with the y-axis on the right in %. The last panel plots flow from the employment to employment across the labor market. The blue solid line shows the flows from full-time employment to part-time employment with the y-axis on the left in % and the red dash-dot line is the flows from part-time employment to full-time employment with the y-axis on the right in %. All the flows are margin error adjusted, deNUNfied following Elsby, Hobijn, and Sahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1976 to December 2019. The black vertical line is January 1994 to denote the re-design of CPS.

Figure A6 shows the flows from the employment (E) to the employment within the same labor market and across the labor markets. As is well known, E to E transition is procyclical. This procyclicality comes from the full-time labor market, as the middle panel shows. While the cyclicality of the E to E transition within the part-time labor market (the light red dotted line) does not stand

---

out (It is almost acyclical), the $E$ to $E$ transition within the full-time labor market (the light blue dash line) is procyclical.

**Figure A7: Flows from Employment to Unemployment**

The first panel shows the flow from employment to unemployment with the black solid line in %. The second panel shows the flow from employment to unemployment within the same labor market. The light blue dash line plots the flows from part-time employment to part-time unemployment with the y-axis on the right in %. The last panel plots flow from the employment to unemployment across the labor market. The blue solid line shows the flows from part-time employment to full-time unemployment with the y-axis on the left in % and the red dash-dot line is the flows from full-time employment to part-time unemployment with the y-axis on the right in %. All the flows are margin error adjusted, deNUNfied following Elsby, Hobijn, and Sahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1976 to December 2019. The black vertical line is January 1994 to denote the re-design of CPS.

The $E$ to $E$ transitions across the two labor markets are shown in the last panel. Flows from full-time employment to part-time employment are denoted as the blue solid line and flows from part-time employment to full-time employment are denoted as the red dash-dot line. Among the two, the transition from full-time employment to part-time employment is important over the business cycle. While the transition from part-time employment to full-time employment does not exhibit any regular cyclicality, the transition from full-time employment to part-time employment shows strong countercyclicality. It sharply rises during recessions and gradually decreases during booms.

The magnitudes of these flows are also large. One important countercyclical flow over the business cycle is the transition from employment to unemployment, which is about one percent of the full-time workers. The transition from full-time employment to part-time employment is about five percent of the full-time workers, which is four times larger in terms of the magnitude,
showing the importance of this flow in terms of magnitudes.

Figure A8: Flows from Unemployment to Employment

Note: The first panel shows the flow from unemployment to employment with the black solid line in %. The second-panel shows the flow from unemployment to employment within the same labor market. The light blue dash line is the flows from full-time unemployment to full-time employment with the y-axis on the left in % and the light red dotted line plots the flows from part-time unemployment to part-time employment with the y-axis on the right in %. The last panel plots flow from the unemployment to employment across the labor market. The blue solid line shows the flows from full-time unemployment to part-time employment with the y-axis on the left in % and the red dash-dot line is the flows from part-time unemployment to full-time employment with the y-axis on the right in %. All the flows are margin error adjusted, deNUNfied following Elsby, Hobijn, and Sahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1976 to December 2019. The black vertical line is January 1994 to denote the re-design of CPS.

Figure A7 shows the flows from employment to unemployment within and across the labor markets. As is well documented in the literature, the transition from employment (E) to unemployment (U) is strongly countercyclical. It sharply rises during recessions and gradually drops during booms. Similar to E to E transitions, this cyclicity stands out only within the full-time labor market. The E to U transition within part-time labor market does not show any cyclicity. For the flows across the labor markets, only flows from part-time employment to full-time unemployment show similar countercyclicity, but not for the flows from the full-time employment to part-time unemployment.

We can observe similar patterns, but in the opposite directions for the flows from the unemployment (U) to the employment within and across the labor markets. Figure A8 shows that the U to E transition is strongly procyclical. This procyclicality comes mostly from the U to E transition within the full-time labor market, while the transition shows weak procyclicality within the part-time labor market as well. Similar patterns are observed for the flows across the two labor
To summarize findings for the flows from the CPS microdata flows across the full-time labor market and the part-time labor market are important in terms of the magnitudes and cyclical-ity. Among the flows across the labor market, the flows from full-time employment to part-time employment seem to be the most interesting and important ones. They are significantly large, in particular, they are five times larger than the flows from the employment to unemployment within the full-time labor market, and strongly countercyclical over the business cycle. The other flows across the labor market which is notable are the flows between the part-time employment and full-time unemployment. They behave similarly to $E \rightarrow U$ and $U \rightarrow E$ transitions within the full-time labor market. Flows from part-time employment to full-time unemployment are countercyclical and flows from full-time unemployment to part-time employment are procyclical.

### A.4 Evidence from CPS: Flow Decomposition within the Labor Force

Previous section and Section A.3 document the importance of the flows across the two labor markets. Among them, in particular, I show the importance of the flows from full-time employment to part-time employment. As a way of examining the importance of these across-market flows, this section investigates their contributions to explain changes in the stock of each labor market states. Following Elsby et al. (2019), this section calculates the contributions of the flows across the labor markets to explain the behaviors of full-time labor forces, part-time labor forces, and total labor force participation rates, which are not presented in Section 2. For details about the method regarding flow decompositions, see Appendix A.7.

The left panel of Figure A.9 shows the contributions of the changes in the flows within and across the labor markets to explain full-time labor force changes. The grey line with crosses shows the cumulative changes of the stock of full-time labor forces, $e_{FT} + u_{FT}$ from January 1996 to December 2019. As is expected, the $E \rightarrow U$ and $U \rightarrow E$ transitions within the full-time labor market explain the significant amount of the changes in full-time employment labor forces. We can observe this from the black solid line, which closely tracks the full-time labor force participation rates denoted as the grey line with crosses. The contributions of the flows from the full-time employment to the part-time employment denoted as the blue solid line with circles are as large as the $E \rightarrow U$ and $U \rightarrow E$ transitions within the full-time labor market, showing the importance of this across-market flows. For example, the full-time employment rate could have dropped by about two and a half percent during the Great Recession, if only the $E \rightarrow U$ and $U \rightarrow E$ transitions within the full-time labor market have changed. As the transition rates from full-time employment to part-time employment sharply rose during the Great Recession, however, the size of the full-time labor force has dropped even further. Meanwhile, the green solid line with diamonds denotes that because the transition rate from the full-time unemployment to the part-time employment is procyclical, the decreases in the full-time labor force size are partially offset by the changes in these flows. That is, had the flows between the full-time unemployment to part-time employment not dropped, the size of the full-time labor force could have dropped even further.
Note: In the left panel, the grey line with crosses plots the cumulative changes of full-time labor forces. The blue solid line with circles is the cumulative contributions of the changes of the flows from full-time employment to part-time employment to explain the full-time labor force changes. The black solid line is the cumulative contributions of the changes of the flows from the full-time employment to the part-time employment to explain the full-time labor force changes, and the green solid line with diamonds is the cumulative contributions of the changes of the flows between the part-time employment and the full-time employment rate. Lastly, the red dash line is the cumulative contribution of the changes in the flows from part-time employment to full-time employment. In the right panel, the grey line with crosses plots the cumulative changes of part-time labor forces. The blue solid line with circles is the cumulative contributions of the changes of the flows from full-time employment to part-time employment to explain the part-time labor force changes. The black solid line is the cumulative contributions of the changes of the flows from the full-time employment to the part-time employment to explain the part-time labor force changes, and the green solid line with diamonds is the cumulative contributions of the changes of the flows between the part-time employment and the full-time employment rate. Lastly, the red dash line is the cumulative contribution of the changes in the flows from part-time employment to full-time employment. Recession periods following the NBER classifications are denoted as grey shaded areas. All the flows are margin error adjusted, deNUNfied following Elsby, Hobijn, and Sahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1996 to December 2019.

The importance of the flows across the two labor markets stand out more in the flow decomposition of the part-time employment rates. In the right panel of Figure A9, the grey line with crosses show the cumulative changes of the part-time labor forces, $e^{PT} + u^{PT}$ from January 1996 to December 2019. What contributes the most to the changes in the part-time labor force is the blue solid line with circles, which shows the contributions of the flows from the full-time employment rate to the part-time employment rate. The blue solid line with circles closely tracks the changes in part-time labor forces. Another important source to explain the evolution of the part-time labor forces is the changes of flows between part-time employment and full-time unemployment, which is denoted as the green solid line with diamonds. For example, during the Great Recession, the part-time labor forces could have risen by two and a half percent, if only the transition probabil-
ities from full-time employment to part-time employment have risen. However, it has risen by one and a half percent. This is because more part-time workers exit the full-time unemployment state (they are now looking for full-time jobs), and fewer full-time unemployed workers enter the part-time employment state. Due to these flow rate changes, the increases in the part-time employment rate during the Great Recession have been partially muted a little bit. In contrast to these flows across the labor market, flows within the part-time labor market do not explain the part-time employment rates much, as is clear from the black solid line.

Figure A10: Flow Decomposition of the Total Labor Force Participation: Contributions of Churn

More broadly, Figure A10 shows that the flows across the two labor markets significantly contributes to the evolution of the total labor force participation rates. The red line with circles plots the cumulative changes of the total labor force participation rates from January 1996 to December 2019. As is documented in Elsby et al. (2019), “churn” that is, the E to U and U to E transitions contributes significantly to the behaviors of the total labor force participation rates. In particular, the green dash line shows the contribution of such flows within the full-time labor market to explain the changes in the total labor force participation rates. It closely tracks the red line with circles, showing the importance of this flow. The magnitudes of the other two lines showing the contributions of the flows across the two labor markets in explaining total labor force participation rate changes are quite significant as well. The blue solid line shows the contributions of the

---

Note: The red line with circles plots the cumulative changes of total labor force participation rates. The blue solid line is the cumulative contributions of the changes of the flows from full-time employment to part-time employment to explain total labor force participation rate changes. The green dash line is the cumulative contributions of the changes of the E to U and U to E transitions within the full-time labor market, and the green dash-dot line is the cumulative contributions of the changes of the transition probabilities between the part-time employment and the full-time unemployment rate. Recession periods following the NBER classifications are denoted as grey shaded areas. All the flows are margin error adjusted, deNUNfied following Elsby, Hobijn, and Sahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1996 to December 2019.
flows from full-time employment to part-time employment, and the black dash-dot line shows the flows between part-time employment and full-time unemployment. Even though they are not as large as the green dash line, their contributions are quite large as well. For instance, the changes in the transition probabilities from full-time employment to part-time employment additionally drop about one more percent of the total labor force participation rate during the Great Recession.


Figure A11: Flows between the Labor Force and the Not-in-the-Labor Force

---

**Note:** The upper panel shows the flow between the part-time labor force and not-in-the labor force. The black dashed line is the transition from the not-in-the labor force to the part-time labor force with the y-axis on the left in %, and the blue solid line is the transition from the part-time labor force to the not-in-the labor force with the y-axis on the right in %. The lower panel shows the flow between the full-time labor force and not-in-the labor force. The green dotted line is the flows from the full-time labor force to the not-in-the labor force with the y-axis on the left in % and the red dash line plots the flows from the not-in-the labor force to the full-time labor force with the y-axis on the right in %. All the flows are margin error adjusted, deNUNfied following Elsby, Hobijn, and Sahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1976 to December 2019. The black vertical line is January 1994 to denote the re-design of CPS.

So far, we have examined stocks and flows of the employed and the unemployed for the two labor markets. This subsection investigates flows between the labor force and not-in-the labor force. In particular, this section examines which labor market is more important to understand the behavior of the not-in-the labor force over the business cycle.
Table A2: Correlation between Real GDP and Flows between the Full-time, the Part-time Labor Force and Out of Labor Force

<table>
<thead>
<tr>
<th></th>
<th>Full-time</th>
<th>Part-time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit to NLF</td>
<td>-0.0187</td>
<td>0.5572</td>
</tr>
<tr>
<td>Entry from NLF</td>
<td>0.0803</td>
<td>0.2731</td>
</tr>
</tbody>
</table>

Note: This table calculates the correlation between real GDP and the flows in from and out to not-in-the-labor-force. Source: CPS microdata from January 1996 to denote the re-design of CPS and Real GDP from FRED. All series are quarterly and band-pass filtered.

Figure A12: Flow Decomposition of the Not-in-the-Labor Force: Contributions of Exit and Entry

Note: The two panels show the flow decomposition of total labor force participation rates. In both panels, the red line with circles shows cumulative changes in not-in-the-labor force rates. The left panel shows the contribution of the flows related to exits to the not-in-the-labor force and the right panel shows the contribution of the flows related to entry from the not-in-the-labor force to explain the evolution of total labor force participation rates. In the left panel, the black dash line shows the contributions of the total exit, i.e. those of the exits from both full-time and part-time labor markets, the blue solid line is the contributions of the flows of exit from the part-time labor force, and the green dotted line is those from the full-time labor force. In the right panel, the black dash line shows the contributions of the total entry, i.e. those of the entries from the not-in-the-labor force to both full-time and part-time labor markets, the blue solid line is the contributions of the flows of entry to the part-time labor force, and the green dotted line is those to the full-time labor force. Recession periods according to the NBER classifications are denoted as grey shaded areas. All the flows are margin error adjusted, deNUNfied following [Elsby, Hobijn, and Sahin (2015)], corrected for temporal aggregation bias following [Shimer (2012)], and seasonally-adjusted using X-13-ARIMA. Source: CPS microdata from January 1996 to denote the re-design of CPS.

Figure A11 first shows the flows between each labor market’s labor force and not-in-the-labor force. The upper panel shows those between the part-time labor force and not-in-the labor force.
with the blue solid line as the exit to the not-in-the labor force with the y-axis on the left and with
the black dash line as the entry from not-in-the labor force with the y-axis on the right. The two
lines exhibit procyclical behaviors. Both entry from and exit to the not-in-the labor force decrease
during recessions. Meanwhile, the lower panel shows those between the full-time labor force and
not-in-the labor force with the green dotted line as the exit to the not-in-the labor force with the
y-axis on the left, and with the red dash line as the entry from not-in-the labor force with y-axis
on the right. In contrast to the upper panel, the two lines in the lower panel do not exhibit strong
cyclicality. Moreover, the correlations of the real gross domestic product (GDP) with the flows
between the part-time labor force and not-in-the labor-force are higher than those with the flows
between the full-time labor force and not-in-the-labor-force (Table A2). Therefore, the part-time
labor force is cyclically more important to explain the exit to and entry from the not-in-the-labor
force.

Figure A12 further corroborates this finding that the part-time labor force seems to be more
important in explaining the behavior of not-in-the-labor force over the business cycle. In both
panels, cumulative changes of not-in-the-labor-force-state are plotted as the red line with circles.
The left panel shows the contributions of the exits from both labor markets to not-in-the-labor
force to explain total labor force participation rates, and the right panel shows the contributions
of entries to both labor markets. The black dash line in the left panel is the contribution of total
exits from both labor markets, and the black dash line in the right panel is the contribution of total
entries to both labor markets. The blue solid lines in each panel, which plot exit from and entry to
the part-time labor force, respectively, closely track the black dash lines, showing the importance
of the part-time labor force in explaining not-in-the labor force participation rates. Meanwhile,
the green dotted lines in each panel that show exit from and entry to the full-time labor force,
respectively are quite apart from the black solid lines.

To summarize the findings from the micro CPS data so far: (i) flows across the two labor mar-
kets are important in terms of the magnitudes and cyclicalities; (ii) Among them, the flows from
the full-time employment to the part-time employment are strongly countercyclical and four times
larger than the E to U transitions within the full-time labor market; (iii) The flows between the
part-time employment and the full-time unemployment are the other important flows across the
two labor markets; (iv) Part-time labor force is more important in explaining not-in-the-labor force
than full-time labor force.

A.6 Adjustments and Decompositions

This section lays out methods for margin-error adjustments, classification-error adjustments, tem-
poral aggregation bias adjustments, and flow decompositions.
A.6.1 Margin Error Adjustments

Let $s_t$ be the vector of labor market statuses:

$$s_t = [e_{FT,t}, e_{PT,t}, u_{FT,t}, u_{PT,t}]',$$

where $e_{FT,t}$ ($e_{PT,t}$) is the full-time (part-time) employment to population (of age higher than 16) ratio, $u_{FT,t}$ ($u_{PT,t}$) is the full-time (part-time) unemployment to population ratio. Then not in the labor force participation rate, $n_t$ is simply the residual, $n_t = 1 - e_{FT,t} - e_{PT,t} - u_{FT,t} - u_{PT,t}$.

Now, let $p_{i,j,t}$ be the probability of transitioning from state $i$ to state $j$. Then we can write the evolution of the labor market statuses as follows:

$$\Delta s_t = s_t - s_{t-1} = \hat{P}_t s_{t-1} = X_{t-1} p_t,$$

where $\hat{P}_t$ equals to

$$\hat{P}_t[1, 1: 5] = \begin{bmatrix} -p_{FT,FT,t} & -p_{FT,PT,t} & -p_{FT,0,t} & p_{PT,FT,t} & p_{PT,PT,t} & p_{PT,0,t} \\ -p_{PT,FT,t} & -p_{PT,PT,t} & -p_{PT,0,t} & p_{FT,FT,t} & p_{FT,PT,t} & p_{FT,0,t} \\ -p_{0,FT,t} & -p_{0,PT,t} & -p_{0,0,t} & p_{FT,0,t} & p_{FT,0,t} & p_{FT,0,t} \end{bmatrix},$$

$$\hat{P}_t[2, 1: 5] = \begin{bmatrix} p_{FT,FT,t} & p_{FT,PT,t} & p_{FT,0,t} & p_{PT,FT,t} & p_{PT,PT,t} & p_{PT,0,t} \\ p_{PT,FT,t} & p_{PT,PT,t} & p_{PT,0,t} & p_{FT,FT,t} & p_{FT,PT,t} & p_{FT,0,t} \\ p_{0,FT,t} & p_{0,PT,t} & p_{0,0,t} & p_{FT,0,t} & p_{FT,0,t} & p_{FT,0,t} \end{bmatrix},$$

$$\hat{P}_t[3, 1: 5] = \begin{bmatrix} p_{FT,FT,t} & p_{FT,PT,t} & p_{FT,0,t} & -p_{PT,FT,t} & -p_{PT,PT,t} & -p_{PT,0,t} \\ p_{PT,FT,t} & p_{PT,PT,t} & p_{PT,0,t} & -p_{FT,FT,t} & -p_{FT,PT,t} & -p_{FT,0,t} \\ p_{0,FT,t} & p_{0,PT,t} & p_{0,0,t} & -p_{FT,0,t} & -p_{FT,0,t} & -p_{FT,0,t} \end{bmatrix},$$

and $X_{t-1}$ equals to

$$X_{t-1}[3, 1: 20] = \begin{bmatrix} -e_{FT,t-1} & -e_{PT,t-1} & -e_{0,t-1} & e_{FT,t-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$X_{t-1}[2, 1: 20] = \begin{bmatrix} e_{FT,t-1} & 0 & 0 & 0 & -e_{PT,t-1} & -e_{0,t-1} & e_{FT,t-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$X_{t-1}[1, 1: 20] = \begin{bmatrix} 0 & e_{FT,t-1} & 0 & 0 & 0 & -e_{PT,t-1} & -e_{0,t-1} & e_{FT,t-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$X_{t-1}[4, 1: 20] = \begin{bmatrix} 0 & 0 & e_{FT,t-1} & 0 & 0 & 0 & -e_{PT,t-1} & -e_{0,t-1} & e_{FT,t-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and $p_t$ is

$$p_t[1: 10, 1] = \begin{bmatrix} p_{FT,FT,t} & p_{FT,PT,t} & p_{FT,0,t} & p_{PT,FT,t} & p_{PT,PT,t} & p_{PT,0,t} & p_{0,FT,t} & p_{0,PT,t} & p_{0,0,t} \end{bmatrix},$$

$$p_t[11: 20, 1] = \begin{bmatrix} p_{FT,FT,t} & p_{FT,PT,t} & p_{FT,0,t} & p_{PT,FT,t} & p_{PT,PT,t} & p_{PT,0,t} & p_{0,FT,t} & p_{0,PT,t} & p_{0,0,t} \end{bmatrix}.$$

Then following Elsby, Hobijn, and Şahin (2015), I use a weighted-restricted-least-squares method to choose the vector of transition probabilities subject to the labor market statuses vector. In specific, I choose $p_t$ which solves

$$\min_{p_t} (p_t - \hat{p}_t)' W_t (p_t - \hat{p}_t), \text{ subject to } \Delta s_t = X_{t-1} p_t.$$
The solution of the above could be calculated by solving

\[
\begin{bmatrix}
\mathbf{p}_t \\
\mu_t
\end{bmatrix} = \begin{bmatrix}
\mathbf{W}_t & \mathbf{X}_{t-1}' \\
\mathbf{X}_{t-1} & 0
\end{bmatrix} \begin{bmatrix}
\Delta \mathbf{s}_t
\end{bmatrix},
\]

with \( \mu_t \) as the associated lagrangian multiplier on the constraint. For the weighting matrix, I use the following where non-zero entries are:

\[
\mathbf{W}_t[1 : 4, 1 : 4] = \begin{bmatrix}
\beta_{FT,FT}(1 - \beta_{FT,FF}) & \beta_{FT,PT} & \beta_{PT,FF} & \beta_{PT,FT} \\
\beta_{FT,FT} & \beta_{FT,FF} & \beta_{PT,FF} & \beta_{PT,FT} \\
\beta_{FT,FT} & \beta_{FT,FF} & \beta_{PT,FF} & \beta_{PT,FT} \\
\beta_{FT,FT} & \beta_{FT,FF} & \beta_{PT,FF} & \beta_{PT,FT}
\end{bmatrix},
\]

\[
\mathbf{W}_t[5 : 8, 5 : 8] = \begin{bmatrix}
\beta_{PT,FT}(1 - \beta_{PT,FF}) & \beta_{PT,FF} & \beta_{PT,FF} & \beta_{PT,FF} \\
\beta_{PT,FT} & \beta_{PT,FF} & \beta_{PT,FF} & \beta_{PT,FF} \\
\beta_{PT,FT} & \beta_{PT,FF} & \beta_{PT,FF} & \beta_{PT,FF} \\
\beta_{PT,FT} & \beta_{PT,FF} & \beta_{PT,FF} & \beta_{PT,FF}
\end{bmatrix},
\]

\[
\mathbf{W}_t[9 : 12, 9 : 12] = \begin{bmatrix}
\beta_{FT,FT}(1 - \beta_{FT,FF}) & \beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} \\
\beta_{FT,FT} & \beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} \\
\beta_{FT,FT} & \beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} \\
\beta_{FT,FT} & \beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF}
\end{bmatrix},
\]

\[
\mathbf{W}_t[13 : 16, 13 : 16] = \begin{bmatrix}
\beta_{FT,FF}(1 - \beta_{FT,FF}) & \beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} \\
\beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} \\
\beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} \\
\beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF} & \beta_{FT,FF}
\end{bmatrix},
\]

\[
\mathbf{W}_t[17 : 20, 17 : 20] = \begin{bmatrix}
\beta_{FF,FT}(1 - \beta_{FF,FF}) & \beta_{FF,FF} & \beta_{FF,FF} & \beta_{FF,FF} \\
\beta_{FF,FT} & \beta_{FF,FF} & \beta_{FF,FF} & \beta_{FF,FF} \\
\beta_{FF,FT} & \beta_{FF,FF} & \beta_{FF,FF} & \beta_{FF,FF} \\
\beta_{FF,FT} & \beta_{FF,FF} & \beta_{FF,FF} & \beta_{FF,FF}
\end{bmatrix}.
\]
A.6.2 Classification Error Adjustments

Following Elsby, Hobijn, and Şahin (2015), I recode unemployment-nonparticipation cyclers, i.e. I “deNUNified” flows. To that end, I first identify the transitions that involve the reversal of a transition from nonparticipation to unemployment and denote them as “NUN”s, and identify the reversal of a transition from unemployment to nonparticipation and denote them as “UNU”s. Then I recode “NUN”s as “NNN”s and “UNU”s as “UUU”s. For the details, please see Table 2 of Elsby, Hobijn, and Şahin (2015).

A.6.3 Temporal Aggregation Bias Adjustments

The evolution of the labor market states, \(s_t\) can be written using the discrete-time transition probabilities as follows:

\[ s_t = \hat{P}_t s_{t-1} + d_t, \]

where \(\hat{P}_t\) equals to

\[ \hat{P}_t[1, 1 : 4] = \begin{bmatrix} 1 - p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & 0 \\ p_{EST,EST} & 1 - p_{EST,EST} & p_{EST,EST} & 0 \\ p_{EST,EST} & p_{EST,EST} & 1 - p_{EST,EST} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]

\[ \hat{P}_t[2, 1 : 4] = \begin{bmatrix} 1 - p_{EST,EST} & 0 & 0 & 0 \\ p_{EST,EST} & 1 - p_{EST,EST} & p_{EST,EST} & 0 \\ p_{EST,EST} & p_{EST,EST} & 1 - p_{EST,EST} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]

\[ \hat{P}_t[3, 1 : 4] = \begin{bmatrix} 1 - p_{EST,EST} & 0 & 0 & 0 \\ p_{EST,EST} & 1 - p_{EST,EST} & p_{EST,EST} & 0 \\ p_{EST,EST} & p_{EST,EST} & 1 - p_{EST,EST} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \]

\[ \hat{P}_t[4, 1 : 4] = \begin{bmatrix} 1 - p_{EST,EST} & 0 & 0 & 0 \\ p_{EST,EST} & 1 - p_{EST,EST} & p_{EST,EST} & 0 \\ p_{EST,EST} & p_{EST,EST} & 1 - p_{EST,EST} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \]

Similarly, we can write the analogous continuous-time Markov chain can be written as:

\[ s_t = \tilde{P}_t s_{t-1} + q_t. \]

Shimer (2012) shows that the eigenvectors of \(\hat{P}_t\) are the same as those of \(\tilde{P}_t\), and that the eigenvalues of \(\tilde{P}_t\) are equal to the exponentiated eigenvalues of \(\tilde{P}_t\). Therefore, we can infer the matrix of flow hazard rates \(\tilde{P}_t\) from the eigendecomposition.

A.7 Flow Decompositions

This section summarizes the flow decomposition of labor market states in Elsby et al. (2019). The evolution of changes of labor market states can be written as follows:

\[ \Delta s_t = P_t s_{t-1} + d_t, \quad (A2) \]

where

\[ P_t[1, 1 : 4] = \begin{bmatrix} p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (A3) \]

\[ P_t[2, 1 : 4] = \begin{bmatrix} p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (A4) \]

\[ P_t[3, 1 : 4] = \begin{bmatrix} p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (A5) \]

\[ P_t[4, 1 : 4] = \begin{bmatrix} p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ p_{EST,EST} & p_{EST,EST} & p_{EST,EST} & p_{EST,EST} \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (A6) \]
Elsby et al. (2019) shows that the evolution of the labor market states \( s_t \) can be decomposed using the following equation:

\[
\Delta s_t = P_t(I + P_{t-1})^{-1}s_{t-1} + P_t(P_t + P_{t-1})^{-1} \times [2\Delta d_t + \Delta P_t(s_t + s_{t-1})],
\]

with \( \bar{s}_t = -P_t^{-1}d_t \) calculated from Equation (A2). From this, we can calculate the contributions of each entries in \( \Delta P_t \) and \( \Delta d_t \) to \( \Delta s_t \).

B Model Supplements

B.1 Solving for Consumption Premiums

First, the optimal level of effort derived by maximizing ex-ante utility of participating in each labor market, Equation (4) is given by

\[
e^{m,*}_{\ell,t} = \max \left\{ a^m \left( \log \frac{c^m}{c^{LM}} - F^m - (1 + \sigma^m_L)\ell^m \right), 0 \right\}, \tag{A7}
\]

so that a worker with higher \( \ell \) exerts less effort, while she exerts more efforts with higher incentive, \( c^m / c^{LM} \), for \( m = R, IR \).

Given the optimal effort level in Equation (A7), the ex-ante utility that a worker with a type \( l \) considering to enter a labor market given by Equation (4) can be written as follows:

\[
\left[ \eta^m + (a^m)^2 \max \left\{ \left( \log \frac{c^m}{c^{LM}} - F^m - (1 + \sigma^m_L)\ell^m \right), 0 \right\} \right] + \log c^{LM} \\
\times \left[ \log \left( \frac{c^m}{c^{LM}} - F^m - (1 + \sigma^m_L)\ell^m \right) \right] \\
- \frac{1}{2} \left[ \max \left\{ a^m \left( \log \frac{c^m}{c^{LM}} - F^m - (1 + \sigma^m_L)\ell^m \right), 0 \right\} \right]^2, \quad m = R, IR. \tag{A8}
\]

Let \( \bar{b}_t \) be the threshold at which a worker with a type \( l = \bar{b}_t \) is indifferent between participating in the irregular labor market and being out of labor force, and \( \bar{d}_t \) be the threshold at which a worker with a type \( l = \bar{d}_t \) is indifferent between entering to the regular and irregular labor markets. That is, the ex-ante expected utility of entering the irregular labor market is the same as the utility of being in the not-in-the-labor force for a worker with \( l = \bar{b}_t \), and the ex-ante expected utilities of participating in the regular and irregular labor market are the same for a worker who draws \( l = \bar{d}_t \).

We can write the incentive compatibility conditions related to the two thresholds in equations as follows:

\[
\log \left( \frac{c^{IR}}{c^{LM}} \right) - F^{IR} = (1 + \sigma^IR_L)p^{IR}_t \ell^{IR}. \tag{A9}
\]
The second incentive compatibility condition equating the ex-ante expected utilities of entering to regular and irregular labor markets can be written as follows:

\[
\begin{align*}
\left[ \eta^R + (a^R)^2 \max \left\{ \left( \log \left( \frac{c^R_{\ell}}{c^R_{IR}} \right) - F^R - (1 + \sigma^R_{L})\ell^R \right), 0 \right\} \right]
\times \left\{ \log(c^R_{\ell}) - F^R - (1 + \sigma^R_{L})\ell^R - \frac{1}{2} \left( \max \left\{ a^R \left( \log \left( \frac{c^R_{\ell}}{c^R_{IR}} \right) - F^R - (1 + \sigma^R_{L})\ell^R \right), 0 \right\} \right)^2 \right\}
\right.
\end{align*}
\]

\[
\begin{align*}
&+ \left[ 1 - \eta^R - (a^R)^2 \max \left\{ \log \left( \frac{c^R_{\ell}}{c^R_{IR}} \right) - F^R - (1 + \sigma^R_{L})\ell^R, 0 \right\} \right]
\times \left\{ \log(c^R_{\ell}) - F^R - (1 + \sigma^R_{L})\ell^R - \frac{1}{2} \left( \max \left\{ a^R \left( \log \left( \frac{c^R_{\ell}}{c^R_{IR}} \right) - F^R - (1 + \sigma^R_{L})\ell^R \right), 0 \right\} \right)^2 \right\}
\end{align*}
\]

Equation \((A9)\) can be obtained by equating the ex-ante expected utility of participating in the irregular labor market and the utility from being out of labor force and equation \((A10)\) can be obtained by equating the ex-ante expected utility of participating in the regular labor market and that of participating in the irregular labor market.

The incentive compatibility conditions above show that because the optimal level of effort is proportional to logarithmic consumption premiums in each labor market \((c^m_i / c^{IR}_i, m = R, IR)\) per each type of workers, \(\ell\), the two thresholds \(\bar{b}_i\) and \(\bar{d}_i\) are the functions of consumption premiums given the parameter values of \(F^m, \sigma^m, \sigma^R, \eta^m, \eta^R, m = R, IR\). Next subsections show this relationships under some cases.

**B1-1. Special Case**

Under the special case of \(a^R = a^{IR} = a, \eta^R = \eta^{IR} = \eta, \) and \(c^{IR}_i = c^{IR}_i \equiv c^L_i\) so that the probability of finding a job in each regular market is the same with the same level of effort, we can derive a simple relationship between the consumption premium of working as a regular-type, \(c^R_i / c^{IR}_i\) and the two thresholds, \(\bar{b}_i\) and \(\bar{d}_i\).
To see this, consider Equation (A10). The left-hand-side of Equation (A10) is
\[
\eta \left( \log \left( \frac{c_i^R}{c_i^U} \right) - F^R - 1 + \sigma^R_L \bar{d}_t^{ER} \right) + \frac{a^2}{2} \left( \log \left( \frac{c_i^R}{c_i^U} \right) - F^R \right)^2
\]
\[-a^2 \left( \log \left( \frac{c_i^R}{c_i^U} \right) - F^R \right) (1 + \sigma^R_L \bar{d}_t^{ER} + \frac{a^2}{2} (1 + \sigma^R_L)^2 \bar{d}_t^{ER}),
\]
and the right-hand-side of Equation (A10) is
\[
\eta (1 + \sigma^R_L ) (\bar{b}_t^{ER} - \bar{d}_t^{ER}) + \frac{a^2}{2} (1 + \sigma^R_L)^2 (\bar{b}_t^{ER} - \bar{d}_t^{ER})^2.
\]
Equating the above two equations and defining \( x \equiv \log \left( \frac{c_i^R}{c_i^U} \right) - F^R \) then generates the following equation:
\[
\frac{a^2}{2} x^2 - (a^2 (1 + \sigma^R_L ) \bar{d}_t^{ER} - \eta ) x + \frac{a^2}{2} (1 + \sigma^R_L )^2 \bar{d}_t^{ER}
\]- \eta (1 + \sigma^R_L ) \bar{d}_t^{ER} - \eta (1 + \sigma^R_L ) (\bar{b}_t^{ER} - \bar{d}_t^{ER}) - \frac{a^2}{2} (1 + \sigma^R_L )^2 (\bar{b}_t^{ER} - \bar{d}_t^{ER}) = 0.
\]
Solving for \( x \) generates
\[
x^* = (1 + \sigma^R_L ) \bar{d}_t^{ER} - \frac{\eta}{a^2} \pm \sqrt{\left( 1 + \sigma^R_L (\bar{b}_t^{ER} - \bar{d}_t^{ER}) + \frac{\eta}{a^2} \right)^2}.
\]
Among the two roots, the reasonable solution for \( x \) (because consumption for workers should be higher than consumption for the unemployed) is
\[
x \equiv \log \left( \frac{c_i^R}{c_i^U} \right) - F^R = (1 + \sigma^R_L ) \bar{d}_t^{ER} + (1 + \sigma^R_L ) (\bar{b}_t^{ER} - \bar{d}_t^{ER}).
\]
Combining the above expression with the incentive compatibility condition for \( b_t \) gives
\[
\log \left( \frac{c_i^R}{c_i^U} \right) - F^R = (1 + \sigma^R_L ) (\bar{b}_t^{ER} - \bar{d}_t^{ER}) + (1 + \sigma^R_L ) \bar{d}_t^{ER}.
\]
\[\text{A11}\]

B1-2. General Case

Under more general case where \( a^R \leq a^I^R, \eta^R \leq \eta^I^R, c_i^UR \neq c_i^UIR, \), and \( C \equiv c_i^{UR}/c_i^{UIR} \), the left-hand-side of Equation (A10) is
\[
\eta^R \left( \log \left( \frac{c_i^R}{c_i^{UIR}} \right) - F^R - (1 + \sigma^R_L ) \bar{d}_t^{ER} \right) + \frac{(a^R)^2}{2} \left( \log \left( \frac{c_i^R}{c_i^{UIR}} \right) - F^R \right)^2
\]\[-(a^R)^2 \left( \log \left( \frac{c_i^R}{c_i^{UIR}} \right) - F^R \right) (1 + \sigma^R_L ) \bar{d}_t^{ER} + \frac{(a^R)^2}{2} (1 + \sigma^R_L )^2 \bar{d}_t^{ER} + \log c_i^{UR},
\]
\[77\]
and the right-hand-side of Equation (A10) is

\[ \eta^{IR}(1 + \sigma^{IR}_L)\left(\bar{b}_t^{IR} - \bar{d}_t^{IR}\right) + \frac{(a^{IR})^2}{2} \left(1 + \sigma^{IR}_L\right)^2 \left(\bar{b}_t^{IR} - \bar{d}_t^{IR}\right)^2 + \log c_t^{UIR}. \]

Equating the above two equations and defining \( x \equiv \log \left(\frac{c_t^R}{c_t^{UIR}}\right) - F^R \) then generates the following equation:

\[ \frac{(a^R)^2}{2} x^2 - \left((a^R)^2(1 + \sigma^R_L)\bar{d}_t^R - \eta^R\right)x + \frac{(a^R)^2}{2} \left(1 + \sigma^R_L\right)^2 \bar{d}_t^R + \log \frac{c_t^{UIR}}{c_t^R} \]

\[ - \eta^R(1 + \sigma^R_L)\bar{d}_t^R - \eta^{IR}(1 + \sigma^{IR}_L)\left(\bar{b}_t^{IR} - \bar{d}_t^{IR}\right) - \frac{(a^{IR})^2}{2} \left(1 + \sigma^{IR}_L\right)^2 \left(\bar{b}_t^{IR} - \bar{d}_t^{IR}\right) = 0. \]

Solving for \( x \) generates

\[ x^* = \left(1 + \sigma^R_L\right)\bar{d}_t^R - \frac{\eta^R}{(a^R)^2} \pm \frac{1}{a^R} \cdot \sqrt{\left(a^{IR}(1 + \sigma^R_L)\bar{b}_t^{IR} - \bar{d}_t^{IR}\right)^2 + \left(\frac{\eta^{IR}}{a^R}\right)^2 + \left(\frac{\eta^R}{a^R}\right)^2} - 2 \log C. \]

Among the two roots, the reasonable value (again because the consumption of workers needs to be higher than the level of consumption for the unemployed) is

\[ x \equiv \log \left(\frac{c_t^R}{c_t^{UIR}}\right) - F^R \]

\[ = \left(1 + \sigma^R_L\right)\bar{d}_t^R - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R} \cdot \sqrt{\left(a^{IR}(1 + \sigma^R_L)\bar{b}_t^{IR} - \bar{d}_t^{IR}\right)^2 + \left(\frac{\eta^{IR}}{a^R}\right)^2 + \left(\frac{\eta^R}{a^R}\right)^2} - 2 \log C. \]

\[ (A12) \]

**B.2 The Representative Family’s Incentive Provision Problem**

The number of regular workers and irregular workers are obtained by integrating out the probability of finding a job in each labor market given the optimal level of effort as follows:

\[ n_t^R = \int_0^{d_t} \left[ \eta^R + (a^R)^2 \max \left\{ \left(\log \left(\frac{c_t^R}{c_t^{UIR}}\right) - F^R - (1 + \sigma^R_L)d_t^R\right), 0 \right\} \right] dl, \]

\[ n_t^{IR} = \int_{d_t}^{b_t} \left[ \eta^{IR} + (a^{IR})^2 \max \left\{ \left(\log \left(\frac{c_t^{IR}}{c_t^{UIR}}\right) - F^{IR} - (1 + \sigma^{IR}_L)d_t^{IR}\right), 0 \right\} \right] dl. \]

So the number of regular workers, \( n_t^R \) can be represented as follows:

\[ n_t^R = \eta^R \bar{d}_t + (a^R)^2 \left(\log \left(\frac{c_t^R}{c_t^{UIR}}\right) - F^R\right) \bar{d}_t - (a^R)^2 \bar{d}_t^{R+1}. \]

\[ (A13) \]

Under the special case of \( a^R = a^{IR} = \eta^R = \eta^{IR} = \eta \), and \( c_t^{IR} = c_t^{UIR} \equiv c_t^U \), plugging in
Equation (A11) generates
\[ n_t^R = \eta \tilde{d}_t + a^2 (1 + \sigma^L_t \bar{d}_t (\tilde{b}^R_t - \tilde{d}^R_t)) + a^2 \sigma^R_t \tilde{d}_t^{R+1}, \] (A14)
and under more general case, the number of regular workers becomes
\[ n_t^R = (a^R)^2 \sigma^R_t \tilde{d}_t^{R+1} + a^R \tilde{d}_t \left( (a^R)^2 (1 + \sigma^L_t \bar{d}_t (\tilde{b}^R_t - \tilde{d}^R_t)) + \left( \frac{\eta^R_t}{a^R_t} \right)^2 \right) - 2 \log C. \] (A15)

Similarly, we can write the number of irregular workers, \( n_t^IR \) by combining the above with the incentive compatibility condition in Equation (A9) as:
\[ n_t^IR = \eta^IR_t (\tilde{b}_t - \tilde{d}_t) + (a^IR_t)^2 \sigma^L_t \bar{d}_t^{IR+1} - (a^IR_t)^2 (1 + \sigma^L_t \bar{d}_t) \tilde{d}_t + (a^IR_t)^2 \tilde{d}_t^{IR+1}. \] (A16)

The expressions for the number of regular and irregular workers in Equations (A15) and (A16) show that the two thresholds are the functions of the two number of workers, \( n_t^R \) and \( n_t^IR \):
\[ \bar{b}_t = f(n_t^R, n_t^IR), \quad \bar{d}_t = g(n_t^R, n_t^IR). \]

Combining all, if the family wants to supply \( n_t^R \) number of workers to regular labor market and \( n_t^IR \) number of workers to irregular labor market, then they need to set \( \bar{b}_t \) and \( \bar{d}_t \) accordingly as in Equation (A15) (or under the simple case, Equation (A14)) and in Equation (A16). To ensure that the family lets \( \bar{d}_t \) number of workers participate in regular labor market and \( \bar{b}_t - \bar{d}_t \) number of workers participate in irregular labor market, the family then needs to set consumption premiums correspondingly as in Equation (A9) and Equation (A12) (under the simple case, Equation (A11)). At the same time, this adjustment of consumption premiums needs to be feasible. Hence, it needs to satisfy the following feasibility condition:
\[ n_t^R c_t^R + (g(n_t^R, n_t^IR) - n_t^R) c_t^{IR} + n_t^IR c_t^{IR} + (1 - g(n_t^R, n_t^IR) - n_t^IR) c_t^{UIR} = C_t. \] (A17)

### B.3 Indirect Utility Function for a Representative Family

This section derives the indirect utility function for a representative family. This follows from integrating the utility of individual workers within the representative family. That is, the summation of the integration of the regular labor forces’ expected utilities from zero to \( \bar{d}_t \), the threshold determining the size of the regular labor force, the integration of irregular workers’ expected utilities from \( \bar{d}_t \) to \( \bar{b}_t \), the size of the irregular labor force, and the integration of the utilities for those out of labor force generates the indirect utility function for the representative family. Formally, this can
be written as follows:

\[ u(C_t, n_t^R, n_t^{IR}) = \]

\[
\int_0^{\delta_t} \left\{ p(e_{l,t}^{R,\ast}) \left( \log c_t^R - F^R - \left( 1 + \sigma_t^R \right) e_{l,t}^{R,\ast} - \frac{1}{2} \left( e_{l,t}^{R,\ast} \right)^2 \right) + \left( 1 - p(e_{l,t}^{R,\ast}) \right) \left( \log c_t^{IR} - \frac{1}{2} \left( e_{l,t}^{IR,\ast} \right)^2 \right) \right\} \, dl \\
+ \int_0^{\delta_t} \left\{ p(e_{l,t}^{IR,\ast}) \left( \log c_t^{IR} - F^{IR} - \left( 1 + \sigma_t^{IR} \right) e_{l,t}^{IR,\ast} - \frac{1}{2} \left( e_{l,t}^{IR,\ast} \right)^2 \right) + \left( 1 - p(e_{l,t}^{IR,\ast}) \right) \left( \log c_t^{IR} - \frac{1}{2} \left( e_{l,t}^{IR,\ast} \right)^2 \right) \right\} \, dl \\
+ \int_{H_t}^1 \log c_t^{IR} \, dl.
\]  

(A18)

If we combine the above with the expression for the optimal efforts in each labor market in Equation (A17), incentive compatibility conditions in Equation (A9) and (A11), and the two thresholds as functions of the number of workers, \( n_t^R \) and \( n_t^{IR} \) in Equation (A15) and (A16), and the resource constraint of Equation (A17), then we have the following simplified expression for the above equation for indirect utility (under the special case):

\[ u(C_t, n_t^R, n_t^{IR}) = \log C_t - n(n_t^R, n_t^{IR}), \]  

(A19)

where

\[ n(n_t^R, n_t^{IR}) = \log N_t - H_t, \]  

(A20)

with

\[ N_t = n_t^R \cdot \left( e^{F^R + (1 + \sigma_t^R) \left( f(n_t^R, n_t^{IR}) e_t^{IR,\ast} - g(n_t^R, n_t^{IR}) e_t^{IR,\ast} \right) + (1 + \sigma_t^R) g(n_t^R, n_t^{IR}) e_t^{IR,\ast} - 1} \right) \]

\[ + n_t^{IR} \left( e^{F^{IR} + (1 + \sigma_t^{IR}) f(n_t^R, n_t^{IR}) e_t^{IR,\ast} - 1} \right) + 1, \]  

(A21)

and

\[ H_t = \eta \sigma_t^{IR} \left( f(n_t^R, n_t^{IR}) e_t^{IR,\ast} - 1 \right) - \eta \sigma_t^{IR} g(n_t^R, n_t^{IR}) e_t^{IR,\ast} + a^2 \left( 1 + \sigma_t^{IR} \right) \left( \sigma_t^{IR} \right)^2 \left( f(n_t^R, n_t^{IR}) e_t^{IR,\ast} + 1 \right) \]

\[ + \frac{a^2 (1 + \sigma_t^{IR} \left( \sigma_t^{IR} \right)^2)}{2 \sigma_t^{IR} + 1} \left( f(n_t^R, n_t^{IR}) e_t^{IR,\ast} + 1 \right) \]

\[ + a^2 \left( 1 + \sigma_t^{IR} \left( \sigma_t^{IR} \right)^2 \right) g(n_t^R, n_t^{IR}) e_t^{IR,\ast} + a^2 (1 + \sigma_t^{IR} \left( \sigma_t^{IR} \right)^2) \left( f(n_t^R, n_t^{IR}) e_t^{IR,\ast} + 1 \right) \]

\[ \times \left( \sigma_t^{IR} g(n_t^R, n_t^{IR}) e_t^{IR,\ast} + 1 \right) - \sigma_t^{IR} g(n_t^R, n_t^{IR}) e_t^{IR,\ast} + 1 \). \]  

(A22)
Under more general case, we can simplify the indirect utility of a family as follows:

\[ u(C_t, n^R_t, n^{IR}_t) = \log C_t - n^{IR}_t - n^R_t, \]  
(A23)

where

\[ n^{IR}_t = n^R_t - (\log C_t) \cdot g(n^R_t, n^{IR}_t) - H_t, \]  
(A24)

with

\[ C = \frac{s^{IR}}{u^{IR}}, \]

\[
N_t = n^R_t \cdot C \cdot \left( e^{F^R + (1+\sigma^R)g(n^R_t, n^{IR}_t)\eta^R} - \frac{C}{\sigma^R} \cdot \frac{1}{\sqrt{2\pi}} \right) - 1
+ n^{IR}_t \left( e^{F^{IR} + (1+\sigma^{IR})f(n^R_t, n^{IR}_t)\eta^{IR}} - 1 \right) + (C - 1) \cdot g(n^R_t, n^{IR}_t) + 1,
\]
(A25)

\[ H_t = \eta^{IR} (1 + \sigma_L^{IR}) f(n^R_t, n^{IR}_t) \eta^{IR} (f(n^R_t, n^{IR}_t) - g(n^R_t, n^{IR}_t)) \]
\[ - \frac{\eta^{IR}}{2} \left( f(n^R_t, n^{IR}_t) \eta^{IR} \right)^2 - g(n^R_t, n^{IR}_t) \]
\[ + \frac{(a^{IR})^2 (1 + \sigma_L^{IR})^2 (\sigma_L^{IR})^2 f(n^R_t, n^{IR}_t) \eta^{IR} \eta^{IR} \eta^{IR}}{2 \sigma_L^{IR} + 1} - \frac{(a^{IR})^2 (1 + \sigma_L^{IR})^2 (\sigma_L^{IR})^2 g(n^R_t, n^{IR}_t) \eta^{IR}}{2 \sigma_L^{IR} + 1}
\]
\[ - \frac{(a^{IR})^2 (1 + \sigma_L^{IR})^2 \eta^{IR}}{2 \sigma_L^{IR} + 1} \]
\[ + a^{R} \sqrt{C} \left( n^R_t, n^{IR}_t \right) \eta^{IR} \eta^{IR} \eta^{IR} \eta^{IR} \]
\[ + d^R \sigma_L^{IR} \left( n^R_t, n^{IR}_t \right) \eta^{IR} \eta^{IR} \eta^{IR} \eta^{IR} \eta^{IR} + \frac{\eta^{IR}}{2} \eta^{IR} \eta^{IR} \eta^{IR} \eta^{IR} \eta^{IR}, \]
(A26)

where \( \eta_t \) is defined as below:

\[ \eta_t = \left( a^{IR} (1 + \sigma_L^{IR}) (b_L^{IR} - d_L^{IR}) + \frac{\eta^{IR}}{a^{IR}} \right)^2 + \left( \frac{\eta^{IR}}{a^{IR}} - \frac{\eta^{IR}}{a^{IR}} \right)^2 - 2 \log C. \]  
(A27)

B.4 Optimizing Conditions for Families and Firms

B.4.1 Family’s Problem

Recall that the family’s problem is given by:

\[
\max_{\{C_t, n^R_t, n^{IR}_t, R_t, \ldots, K_t \}} \sum_{t=0}^{\infty} \beta^t \frac{u(C_t, n^R_t, n^{IR}_t)}{\log C_t - n^R_t - n^{IR}_t} \]  
(A28)
subject to

\[ P_i C_t + P_i I_t + B_{t+1} \leq (1 + i_{t-1}) B_t + W^R_t n^R_t + W^{IR}_t n^{IR}_t + R_t K_t v_t + \text{Profits, Taxes, and Transfers}, \]

\[ K_{t+1} = \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] I_t + (1 - \delta(v_t)) K_t, \]

\[ \delta(v_t) = \delta_0 + \delta_1 (v_t - 1) + \frac{\delta_2}{2} (v_t - 1)^2. \]

The necessary conditions for the above optimization problem are

\[ \lambda_t \equiv P_t \Lambda_t = \frac{1}{C_t}, \]

with \( \Lambda_t \) as a lagrangian multiplier on a budget constraint,

\[ n_{n^R}(n^R_t, n^{IR}_t) = \Lambda_t W^R_t, \]

\[ n_{n^{IR}}(n^R_t, n^{IR}_t) = \Lambda_t W^{IR}_t, \]

where \( n_{n^R} \equiv \partial n(n^R_t, n^{IR}_t) / \partial n^R_t \) and \( n_{n^{IR}} \equiv \partial n(n^R_t, n^{IR}_t) / \partial n^{IR}_t \).

\[ \Lambda_t R_t K_t = \mu_t \delta'(v_t) K_t, \]

with \( \mu_t \) as a lagrangian multiplier on a capital accumulation process,

\[ \Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} (1 + i_t), \]

\[ \lambda_t = \mu_t \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right) - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_t}{I_{t-1}} - 1 \right) + \beta \mathbb{E}_t \left[ \mu_{t+1} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right) \right], \]

\[ \mu_t = \beta \mathbb{E}_t \left[ \Lambda_{t+1} R_{t+1} v_{t+1} + \mu_{t+1} (1 - \delta(v_{t+1})) \right]. \]

### B.4.2 Intermediate Firms’ Problem

Each intermediate firm chooses \{n^{IR}_{t}(j), n^R_{t}(j), x^R_{t}(j), x^{IR}_{t}(j), P_t(j), \hat{K}_t(j), p^{R\rightarrow IR}_{t}(j), p^{IR\rightarrow R}_{t}(j)\} by solving the following profit maximization problem:

\[
\max_{E_t} \sum_{k=0}^{\infty} Q_{t+k} \left[ \frac{P_{t+k}(j)}{P+k} Y_{t+k}(j) - \frac{W^R_{t+k} n^R_{t+k}(j)}{P_{t+k}} - \frac{W^{IR}_{t+k} n^{IR}_{t+k}(j)}{P_{t+k}} - \frac{R_{t+k} \hat{K}_{t+k}(j)}{P_{t+k}} - \frac{\phi}{2} \left( \frac{P_{t+k}(j)}{P_{t+k-1}(j)} - 1 \right) \right]
\]

\[
= \max_{E_t} \sum_{k=0}^{\infty} Q_{t+k} \left[ \frac{P_{t+k}(j)}{P+k} Y_{t+k}(j) - C(x^R_{t+k}(j); n^R_{t+k-1}(j)) - C(x^{IR}_{t+k}(j); n^{IR}_{t+k-1}(j)) - C(p^{R\rightarrow IR}_{t+k}(j); n^{IR}_{t+k-1}(j)) - C(p^{IR\rightarrow R}_{t+k}(j); n^{IR}_{t+k-1}(j)) \right]
\]

82
subject to

\[ Y_{t+k}(j) = \left( (A_{t+k}e_{t+k}h_{t+k}(j))^{1-a} \left( \tilde{K}_{t+k}(j) \right)^a \right)^p, \quad \text{with } \tilde{K}_{t+k} = v_{t+k}K_{t+k}, \]

\[ n_{t+k}(j) = \left( \left( \frac{\eta}{1} \right) e_{t+k}(j) \right)^{\frac{1}{1-a}} + \left( 1 - \frac{\eta}{1} \right) e_{t+k}(j) \]

\[ n^{R}_{t+k}(j) = \left( \rho + \lambda_1x^{R}_{t+k}(j) - p^{R \rightarrow IR}_{t+k}(j) \right) n^{R}_{t+k-1}(j) + \lambda_1p^{R \rightarrow IR}_{t+k}(j)n^{IR}_{t+k-1}(j), \quad \lambda_1 < 1, \]

\[ n^{IR}_{t+k}(j) = \left( x^{IR}_{t+k}(j) - p^{IR \rightarrow R}_{t+k}(j) \right) n^{IR}_{t+k-1}(j) + \lambda_2p^{IR \rightarrow R}_{t+k}(j)n^{R}_{t+k-1}(j), \quad \lambda_2 > 1. \]

\[ Y_{t+k}(j) = \left( \frac{P_{t+k}(j)}{P_{t+k}} \right)^{-\epsilon_p} Y_{t+k}, \quad \forall k. \]

The first order necessary conditions are

\[ \frac{W^{IR}_{t+k}}{P_{t+k}} + \lambda^{IR}_{t+k} = \phi_{t+k}(j)MPN^{IR}_{t+k}(j) \]

\[ - \mathbb{E}_{t+k} \left[ Q_{t+k,t+k+1} \cdot \left\{ \frac{1}{\gamma} \left( e^{\gamma x^{IR}_{t+k+1}(j)} - \gamma x^{IR}_{t+k+1}(j) - 1 \right) + \frac{\tilde{\nu}}{2} \left( p^{IR \rightarrow R}_{t+k+1}(j) \right)^2 \right\} Y_{t+k+1} \right] + \mathbb{E}_{t+k} \left[ Q_{t+k,t+k+1} \left\{ x^{IR}_{t+k+1}(j) \lambda^{IR}_{t+k+1}(j) + \left( \lambda_1 \lambda^{R}_{t+k+1}(j) - \lambda^{IR}_{t+k+1}(j) \right) p^{IR \rightarrow R}_{t+k+1}(j) \right\} \right], \]

\[ \frac{W^{R}_{t+k}}{P_{t+k}} + \lambda^{R}_{t+k} = \phi_{t+k}(j)MPN^{R}_{t+k}(j) \]

\[ - \mathbb{E}_{t+k} \left[ Q_{t+k,t+k+1} \cdot \left\{ \frac{1}{\gamma} \left( e^{-\kappa x^{R}_{t+k+1}(j)} + \kappa x^{R}_{t+k+1}(j) - 1 \right) + \frac{\tilde{\theta}}{2} \left( p^{R \rightarrow IR}_{t+k+1}(j) \right)^2 \right\} Y_{t+k+1} \right] + \mathbb{E}_{t+k} \left[ Q_{t+k,t+k+1} \left\{ \rho + x^{R}_{t+k+1}(j) \lambda^{R}_{t+k+1}(j) + \left( \lambda_2 \lambda^{IR}_{t+k+1}(j) - \lambda^{R}_{t+k+1}(j) \right) p^{R \rightarrow IR}_{t+k+1}(j) \right\} \right], \]

where \( MPN^R \) and \( MPN^{IR} \) denote marginal productivity of regular and irregular types, respectively. That is, \( MPN^R = \partial Y_t(j)/\partial n^R_t(j) \) and \( MPN^{IR} = \partial Y_t(j)/\partial n^{IR}_t(j) \).

\[ \lambda^{IR}_{t+k}(j) = \frac{\gamma}{\gamma} \left( e^{\gamma x^{IR}_{t+k}(j)} - 1 \right) Y_{t+k}, \]

\[ \lambda^{R}_{t+k}(j) = \frac{\kappa}{\kappa} \left( 1 - e^{-\kappa x^{R}_{t+k}(j)} \right) Y_{t+k}, \]

\[ \lambda_1 \lambda^{R}_{t+k}(j) - \lambda^{IR}_{t+k}(j) = \tilde{\nu} p^{IR \rightarrow R}_{t+k}(j) Y_{t+k}, \]

\[ \lambda_2 \lambda^{IR}_{t+k}(j) - \lambda^{R}_{t+k}(j) = \tilde{\theta} p^{R \rightarrow IR}_{t+k}(j) Y_{t+k}, \]

\[ 0 = (1 - \epsilon_p) Y_{t+k}(j) - \phi \left( \frac{P_{t+k}(j)}{P_{t+k-1}(j)} - 1 \right) Y_{t+k} + \epsilon_p \phi Y_{t+k}(j) \]

\[ + \mathbb{E}_{t+k} \left[ Q_{t+k,t+k+1} \phi \left( \frac{P_{t+k+1}(j)}{P_{t+k}(j)} - 1 \right) \frac{P_{t+k+1}(j)}{P_{t+k}(j)} \right]. \]
B.5 Some Comparative Statics

This section examines the role of the two thresholds in terms of changing the number of regular and irregular workers. Under the parameterizations of this paper, when $\bar{b}_t$ increases, both the number of regular and irregular workers. In contrast to this, when $d_t$ increases, while the number of regular workers increases, the number of irregular workers decreases. Because firms adjust the total amount of labor by changing the composition of workers, that is, by increasing one type of labor and decreasing the other type of labor, it is $\bar{d}_t$ that frequently moves over the business cycle, not $\bar{b}_t$.

Under the special case of $a^R = a^{IR} = a$, $\eta^R = \eta^{IR} = \eta$, and $c^U_{IR} = c^U_{IR}$,

$$\frac{\partial n^R_t}{\partial b_t} = a^2 \sigma^R_L (1 + \sigma^R_L) b^R_t \bar{d}_t > 0,$$

$$\frac{\partial n^R_t}{\partial b_t} = \eta + a^2 \sigma^R_L (1 + \sigma^R_L) b^R_t \bar{d}_t - (\bar{b}_t - \bar{d}_t) > 0,$$

$$\frac{\partial n^R_t}{\partial d_t} = \eta + a^2 (1 + \sigma^R_L) (\bar{b}^R_t - \bar{d}_t) + a^2 \sigma^R_L (1 + \sigma^R_L) d^R_t - a^2 \sigma^R_L (1 + \sigma^R_L) d^R_t > 0$$

$$> 0, \text{ with } c^R > c^R_{IR}$$

$$\frac{\partial n^R_t}{\partial d_t} = -\eta - a^2 (1 + \sigma^R_L) (\bar{b}^R_t - \bar{d}_t) < 0.$$

Under more general case of $a^R \neq a^{IR}$, $\eta^R \neq \eta^{IR}$, and $c^U_{IR} \neq c^U_{IR}$,

$$\frac{\partial n^R_t}{\partial b_t} = a^R \sigma^R_L (1 + \sigma^R_L) b^R_t \bar{d}_t \left( \frac{a^R \bar{d}_t}{\sqrt{X_t}} \left( a^R (1 + \sigma^R_L) (\bar{b}^R_t - \bar{d}^R_t) \right) \right) > 0,$$

$$\frac{\partial n^R_t}{\partial b_t} = \eta^R + \left( a^R \right)^2 \sigma^R_L (1 + \sigma^R_L) b^R_t \bar{d}_t \bar{d}_t - (\bar{b}_t - \bar{d}_t) > 0,$$

$$\frac{\partial n^R_t}{\partial d_t} = a^R \sqrt{X_t} + \left( a^R \right)^2 \sigma^R_L (1 + \sigma^R_L) d^R_t \left( a^R (1 + \sigma^R_L) (\bar{b}^R_t - \bar{d}_t) \right) \left( a^R (1 + \sigma^R_L) \left( \frac{\bar{d}_t}{\sqrt{X_t}} + \frac{\eta^R}{\sigma^R_L} \right) \right)$$

$$> 0, \text{ with } c^R < c^R_{IR}$$

$$\frac{\partial n^R_t}{\partial d_t} = -\eta^R - \left( a^R \right)^2 (1 + \sigma^R_L) (\bar{b}^R_t - \bar{d}^R_t) < 0,$$
where
\[
X_t \equiv \left( a^{IR}(1 + \sigma_L^{IR}) (b_t^{IR} - d_t^{IR}) + \frac{\eta_R^{IR}}{a^{IR}} \right)^2 + \left( \frac{\eta_R}{a^{IR}} \right)^2 - \left( \frac{\eta_R^{IR}}{a^{IR}} \right)^2 - 2 \log C
\]

\[
\geq 0, \quad \text{with } b_t > d_t
\]

\[
\geq 0, \quad \text{with parameterizations of this paper}
\]

under the parameterizations of this paper.

### B.6 Supplements to Calibration

**Table A3: Target Moments and Model Counterparts**

<table>
<thead>
<tr>
<th></th>
<th>(E[\bar{b}_t])</th>
<th>(E_t[\bar{d}_t])</th>
<th>(E[n_t^R])</th>
<th>(E[n_t^{IR}])</th>
<th>(\sigma(n_t^R))</th>
<th>(\sigma(n_t^{IR}))</th>
<th>(E[p_t^{IR-R}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target</td>
<td>64.71%</td>
<td>52.39%</td>
<td>0.4910</td>
<td>0.1174</td>
<td>0.0207</td>
<td>0.0041</td>
<td>1.08%</td>
</tr>
<tr>
<td>Model</td>
<td>64.71%</td>
<td>52.39%</td>
<td>0.4910</td>
<td>0.1174</td>
<td>0.0208</td>
<td>0.0040</td>
<td>1.08%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(E[u_t^R])</th>
<th>(E[u_t^{IR}])</th>
<th>(E[u_t])</th>
<th>(\sigma(u_t^R))</th>
<th>(\sigma(u_t^{IR}))</th>
<th>(\sigma(u_t))</th>
<th>(E[p_t^{R-IR}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target</td>
<td>6.30%</td>
<td>4.71%</td>
<td>6.01%</td>
<td>2.16%</td>
<td>0.54%</td>
<td>1.82%</td>
<td>2.00%</td>
</tr>
<tr>
<td>Model</td>
<td>6.30%</td>
<td>4.71%</td>
<td>6.01%</td>
<td>2.16%</td>
<td>0.54%</td>
<td>1.82%</td>
<td>2.00%</td>
</tr>
</tbody>
</table>

*Notes:* This table shows the target moments and their model counterparts used to calibrate the following parameters: parameters related to workers’ disutility, \(F_m\), \(\sigma_m^L\) for \(m = R, IR\), parameters in the probability of finding jobs, \(\eta_m\), \(a_m\), \(\eta_m\) for \(m = R, IR\), the degree of substitutability between regular and irregular workers, \(\sigma_n\), the parameter denoting the relative productivity between the regular and irregular types, \(\eta_n\), and the parameters related to the adjustment costs, \(\kappa, \beta\), and \(\bar{v}\). The target moments are calculated from the Current Population Survey from the first quarter of 2001 to the fourth quarter of 2019. I use full-time jobs as proxies for regular type and part-time jobs as proxies for irregular jobs.

### B.7 The Role of Promotion and Demotion

This section examines the responses of labor market related variables to structural shocks (persistent and transitory technology shocks, monetary policy shock, government spending shock) when (i) firms cannot demote (Figure A13), (ii) firms cannot promote (Figure A14), and (iii) firms cannot transfer one type to the other, i.e. promotion and demotion (Figure A15). By comparing the impulse responses from the three models, this section illustrates that the ability for firms to transfer one type to the other type via *either* promotion or demotion is the key to generate differential responses of the two labor markets.
Given the two types of labor, not only firms can newly create and/or destruct each type of job, firms can transfer one type of labor to the other type at negligible costs. Hence, firms actively exploit this margin to adjust the total amount of labor input at the time of the shock. For example, if a firm wants to increase the number of regular workers by one unit, then this firm has two options: i) hire one regular worker ii) promote one irregular worker to be the regular worker. The first option incurs direct hiring costs and it also increases the future stream of regular types’ adjustment costs (hiring/firing costs and demotion costs) but no change in the future stream of irregular types’ adjustment costs. In contrast, direct promotion cost is a lot cheaper. Moreover, it changes the future stream of regular types’ adjustment costs and that of irregular types’ adjustment costs in the opposite direction. It might increase the future stream of regular types’ adjustment costs, but it decreases the future stream of irregular types’ adjustment costs. This makes the responses of transfer larger than the net hiring of each type, and these immediate changes in the transfer generate differential responses of the number of regular and irregular workers.

Figure A13: Responses of Labor Market Variables When Firms Cannot Demote

Note: This figure plots the responses of labor market variables (the total number of workers, the numbers of regular and irregular workers, total labor force participation rates, each labor market’s labor force, net hiring rates for each type, the total unemployment rate, the unemployment rate in the regular labor market, and the unemployment rate in the irregular labor market) in response to one standard deviation of structural shocks (persistent and technology shocks, monetary policy shock, and government spending shocks) when firms cannot demote (but can promote).
Figure A14: Responses of Labor Market Variables When Firms Cannot Promote

Note: This figure plots the responses of labor market variables (the total number of workers, the numbers of regular and irregular workers, total labor force participation rates, each labor market’s labor force, net hiring rates for each type, the total unemployment rate, the unemployment rate in the regular labor market, and the unemployment rate in the irregular labor market) in response to one standard deviation of structural shocks (persistent and technology shocks, monetary policy shock, and government spending shocks) when firms cannot promote (but can demote).

Figure A15: Responses of Labor Market Variables When Firms Cannot Promote and Demote

Note: This figure plots the responses of labor market variables (the total number of workers, the numbers of regular and irregular workers, total labor force participation rates, each labor market’s labor force, net hiring rates for each type, the total unemployment rate, the unemployment rate in the regular labor market, and the unemployment rate in the irregular labor market) in response to one standard deviation of structural shocks (persistent and technology shocks, monetary policy shock, and government spending shocks) when firms cannot transfer one type to the other. That is, firms cannot promote and demote.
B.8 Responses of Labor Market Variables to Other Shocks

This section shows the responses of labor market variables (total number of workers, the number of regular and irregular workers, net hiring rates for regular and irregular types, promotion and demotion rates, the share of irregular workers, the total unemployment rate, the unemployment rates in regular and irregular labor markets, the total labor force participation rate, and the sizes of regular and irregular labor forces) to a monetary policy shock, a government spending shock (Figure A16 and A17) and to persistent and technology shocks. (Figure A18 and A19)

Figure A16: Responses of Labor Market Variables to Demand Shocks I.

Note: This figure shows the responses of labor market related variables to demand shocks from the baseline model. Each column corresponds to labor market variables, while each row corresponds to exogenous shocks. The first row shows the responses to monetary policy shocks, $\epsilon^i$, and the second row shows those to government spending shocks, $\epsilon^g$. 

88
Figure A17: Responses of Labor Market Variables to Demand Shocks II.

Note: This figure shows the responses of labor market related variables to exogenous demand shocks from the baseline model. Each column corresponds to labor market variables, while each row corresponds to exogenous shocks. The first row shows the responses to monetary policy shocks, $\epsilon^i$, and the second row shows those to government spending shocks, $\epsilon^g$.

Figure A18: Responses of Labor Market Variables to a Contractionary Monetary Policy Shock

Note: This figure shows the responses of labor market related variables to persistent and transitory technology shocks from the baseline model. Each column corresponds to labor market variables, while each row corresponds to exogenous shocks. The first row shows the responses to persistent technology shock, $\epsilon^{A_P}$ and the second row shows those to transitory technology shock, $\epsilon^{A_T}$.
Figure A19: Responses of Labor Market Variables to Supply Shocks II.

Note: This figure shows the responses of labor market related variables to exogenous supply shocks from the baseline model. Each column corresponds to labor market variables, while each row corresponds to exogenous shocks. The first row shows the responses to persistent technology shock, $\epsilon_A^p$, and the second row shows those to transitory technology shock, $\epsilon_A^t$.

B.9 Responses of Non-Labor Market Variables

Figure A20: Responses to Persistent Technology Shocks

Note: This figure plots the responses of the variables in the baseline model to a one standard deviation of persistent technology shock, $\epsilon_A^p$, from the baseline model.
Figure A21: Responses to Transitory Technology Shocks

Note: This figure plots the responses of the variables in the baseline model to a one standard deviation of transitory technology shock, $A^T$ from the baseline model.

Figure A22: Responses to Monetary Policy Shocks

Note: This figure plots the responses of the variables in the baseline model to a one standard deviation of monetary policy shock, $\epsilon^i$ from the baseline model.

91
B.10 In Relation to Extensive and Intensive Margins

This section examines the relationship between the third margin and the other two margins, extensive and intensive margins that the literature has focused on. Analysis from the CPS and the baseline model has shown the importance of the composition margins over the business cycle. When there is more than one type of labor, firms adjust the total amount of labor input to use for production, by changing the composition of workers, and I call this "the composition margin."

It is, however, true that the composition margin is not separable from the other two margins, extensive (the number of workers) and intensive margins (hours worked per worker). The third margin is, rather, closely connected to the other two margins. In fact, the third margin helps to better understand the behaviors of the other two margins over the business cycles. For example, consider the case where a firm decides to fire two full-time workers and hire two part-timers. As the total number of workers is fixed, there is no change in the extensive margin. The intensive margin, however, is likely to decrease. As another example, if a firm decides to replace two full-time workers with four part-time workers, then there is no change in total labor hours, but the number of workers increases, and the hours per worker decreases. In these cases, it is helpful to examine the composition margin to better understand the behaviors of extensive and intensive margins. Moreover, there are some cases where there is no change in both extensive and intensive margins, but the composition margins. For example, if a firm decides to replace two permanent full-time workers with two full-time interns, it is likely that there is no change in the number of workers, total hours of work, and hours per worker. But this would decrease the total amount of
labor input for firms, as interns have lower productivity than regular workers. This could have important welfare implications for workers. Section 6.1 examines this implication for welfare.

To illustrate this point, Figure A24 plots the responses of extensive, intensive, the third margins, and the total amount of labor input to each exogenous shocks. In fact, the model assumes “indivisible labor.” Workers can either work or not in each labor market. Therefore, there is no intensive margin. However, if we assume that regular workers work full time and irregular workers work part-time, then we can roughly examine the responses of “hours per worker,” and how is this related to the third margin.

Figure A24: Responses of Three Margins of Labor Adjustment to Exogenous Shocks

Note: This figure shows the responses of the three margins: (i) extensive margins (the number of workers), (ii) intensive margins (hours worked per workers), and (iii) the third margin (the composition margin, the share of regular workers) to each exogenous shocks in the baseline model. Each panel shows the responses to each exogenous shocks: persistent technology shock, $\epsilon^A_p$, transitory technology shock, $\epsilon^A_T$, monetary policy shocks, $\epsilon^i$, government spending shocks, $\epsilon^g$, and marginal efficiency to investment shocks, $\epsilon^z$. The responses of the total amount of labor, $n$ are represented as black solid lines, those of extensive margins are denoted as blue dash-dot lines, those of intensive margins are shown as red dash lines, and those of the third margins are denoted as green dotted lines.

From the figure above, first it is noticeable that most variations in the total amount of labor input reflect the variations in extensive margins, but not completely. This is consistent with previous studies on indivisible labor that variations in the total hours worked are mostly explained by the variations in extensive margins. (see, for example, Hansen [1985] Rogerson [1988] Krusell et al [2008]) Extensive margins, however, do not completely explain the changes in the total amount of labor input. The gaps between the two are explained by the changes of the third margin: when the share of regular workers rises, the total amount of labor input increases much more than the total number of workers, and vice versa. Another interesting observation is the close relationship
between the third margin and intensive margins. The responses of intensive margins are largely explained by the changes in the share of regular workers, the third margin. This is consistent with the finding from the recent study of Borowczyk-Martins and Lalé (2019) that fluctuations in part-time employment play a major role in variations of intensive margins over the business cycles in the United States and the United Kingdom. The baseline model successfully replicates this empirical observation.

B.11 Welfare Costs of the Business Cycle per Worker

Section 6.1 shows that contingent workers pay substantially larger costs of economic fluctuations than stayers and among contingent workers, contingent “regular workers” pay substantially larger costs of economic fluctuations. This is because contingent workers face larger risks regarding their labor market status over the business cycle. This section illustrates this by showing the stream (time-series of a random 2000 consecutive periods) of consumption and disutility from working for a worker in each group from the simulation.

Figure A25: The Stream of Consumption for a Worker in Each Group

Note: This figure illustrates streams of consumption for a worker in each five groups: (i) Workers who are always in the regular labor market, (ii) Contingent regular workers who frequently move between the regular and irregular labor market, (iii) Workers who are always in the irregular labor market, (iv) Contingent irregular workers who frequently in and out of labor force, and (v) Workers who are always out of labor force. This figure illustrates that contingent workers experience larger volatilities of consumption than stayers.

Figure A25 - A28 illustrate the streams (time-series of a random 2000 consecutive periods) of consumption, utility from consumption, and disutility from supplying labor for a worker in each of five groups for 2000 periods: (i) Workers who are always in the regular labor market, (ii) Contingent regular workers who frequently move between the regular and irregular labor market, (iii) Workers who are always in the irregular labor market, (iv) Contingent irregular workers who frequently in and out of labor force, and (v) Workers who are always out of labor force. All panels in Figures A25 and A28 share the same y-axes. These figures illustrate that contingent regular and
irregular workers experience substantially larger volatilities of consumption and disutilities from supplying labor than stayers. Among the two contingent workers, contingent regular workers experience the largest volatilities in consumption and labor supply disutility.

Figure A26: The Stream of Utility from Consumption for a Worker in Each Group

Note: This figure illustrates streams of consumption for a worker in each five groups: (i) Workers who are always in the regular labor market, (ii) Contingent regular workers who frequently move between the regular and irregular labor market, (iii) Workers who are always in the irregular labor market, (iv) Contingent irregular workers who frequently in and out of labor force, and (v) Workers who are always out of labor force. This figure illustrates that contingent workers experience larger volatilities of consumption than stayers.

Figure A27: The Stream of Disutility from Supplying Labor for a Worker in Each Group

Note: This figure illustrates streams of disutilities from supplying labor (the sum of disutility from working and costs of exerting efforts to find a job) for a worker in each five groups: (i) Workers who are always in the regular labor market, (ii) Contingent regular workers who frequently move between the regular and irregular labor market, (iii) Workers who are always in the irregular labor market, (iv) Contingent irregular workers who frequently in and out of labor force, and (v) Workers who are always out of labor force. This figure illustrates that contingent workers experience larger volatilities of disutilities from supplying labor than stayers.
Lastly, Figure A28 illustrates welfare gains from the alternative specification of interest rate rule where the monetary authority stabilizes the thresholds, $\bar{b}_t$ and $\bar{d}_t$, compared to the welfare from the conventional monetary policy rule with the stabilization of the aggregate unemployment gap, by individual workers with $\ell \in [0, 1]$.

Figure A28: Welfare Gains by Individual Workers

Note: This figure illustrates welfare gains from the alternative specification of monetary policy rule which stabilizes the two thresholds. In particular, this plots the ratio of $\Lambda^t$'s under the alternative specification of monetary policy rule to $\Lambda^t$'s under the conventional monetary policy rule by $\ell \in [0, 1]$.

B.12 Second-order Approximation of Welfare Function

This section presents the second-order approximation of the welfare function which is the lifetime expected utility of the representative family’s utility function. To that end, consider a simpler version of the model in the main text where there is no capital, no promotion and demotion, no adjustment costs for regular and irregular workers, and constant returns to scale of production with only productivity shocks as an exogenous force. Then the economy reduces to the following system of optimality conditions, law of motions, and market clearing conditions:

\[
C_t z_{n_t^R} = w_t^R, \quad (A29)
\]

\[
C_t z_{n_t^I} = w_I^R, \quad (A30)
\]

\[
1 = \beta E_t \left[ \frac{C_t}{C_{t+1}} \frac{1 + i_t}{1 + \pi_{t+1}} \right], \quad (A31)
\]

\[
w_t^R = \varphi_t A_t \eta^{\frac{1}{2}} \left( \frac{n_t^R}{n_t} \right)^{-\frac{1}{2}}, \quad (A32)
\]
\[ w^R_t = \varphi_t A_t (1 - \eta)^{\frac{1}{2}} \left( \frac{n_t^R}{n_t} \right)^{-\frac{1}{2}}, \]  
(A33)

\[ n_t = \left( \eta^{\frac{1}{2}} \left( \frac{n_t^R}{n_t} \right)^{-\frac{1}{2}} + (1 - \eta)^{\frac{1}{2}} \left( \frac{n_t^{IR}}{n_t} \right)^{-\frac{1}{2}} \right)^{\frac{1}{\epsilon}}, \]  
(A34)

\[ (1 - \epsilon_p) Y_t + \epsilon_p \varphi_t Y_t = \phi_p (1 + \pi_t) \pi_t - \beta \phi_p \mathbb{E}_t \left[ \frac{C_t}{C_{t+1}} \pi_{t+1} (1 + \pi_{t+1}) \right], \]  
(A35)

\[ Y_t = A_t n_t, \]  
(A36)

\[ Y_t = C_t \phi_p^2 \pi_t^2, \]  
(A37)

\[ i_t = \phi_t \pi_t + \phi_y (\log Y_t - \log Y_{t-1}), \]  
(A38)

\[ \log A_t = \rho A \log A_{t-1} + \epsilon_t^A. \]  
(A39)

Households' welfare is defined as follows:

\[ \mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - Z(n_t^R, n_t^{IR}) \right). \]  
(A40)

First, the second-order approximation of the period-by-period utility, \( \mathcal{U}_t \), can be written as follows:

\[ \frac{\mathcal{U}_t - \mathcal{U}}{\mathcal{U}_C C} \approx \tilde{\mathcal{C}}_t - \sum_{n_R^{IR} n_R^R} \left( \tilde{n}_t^R + \frac{1 + \omega_{n^R}}{2} \left( \tilde{n}_t^R \right)^2 \right) - \sum_{n_{IR} n_{IR}^R} \left( \tilde{n}_t^{IR} + \frac{1 + \omega_{n^{IR}}}{2} \left( \tilde{n}_t^{IR} \right)^2 \right) - \sum_{n_{IR} n_{IR}^R \tilde{n}_t^R \tilde{n}_t^{IR}} \tilde{n}_t^R \tilde{n}_t^{IR}, \]

where \( \tilde{x}_t \) denotes the percentage deviations from its steady-state value of \( x, \tilde{x} \) is the steady state value of \( x \), and \( \omega_{n^R} = \frac{\tilde{n}_t^R}{Z_{n^R}} \), \( \omega_{n^{IR}} = \frac{\tilde{n}_t^{IR}}{Z_{n^{IR}}} \).

For the disutility from each-type of labor, first, we need to combine each type of labor demand and supply. The log-linearized versions of the two can be then written as

\[ \tilde{\mathcal{C}}_t + \omega_{n^R} \tilde{n}_t^R = \tilde{\varphi}_t A_t - \frac{1}{\epsilon} \left( \tilde{n}_t^R - \tilde{n}_t \right), \]  
(A41)

\[ \tilde{\mathcal{C}}_t + \omega_{n^{IR}} \tilde{n}_t^{IR} = \tilde{\varphi}_t A_t - \frac{1}{\epsilon} \left( \tilde{n}_t^{IR} - \tilde{n}_t \right), \]  
(A42)

Combining the two then generates the relationship between \( \tilde{n}_t^R \) and \( \tilde{n}_t^{IR} \):

\[ \tilde{n}_t^R = \frac{1 + \epsilon \omega_{n^R}}{1 + \epsilon \omega_{n^{IR}}} \tilde{n}_t^R, \]  
(A43)

Then from Equation (A34), \( C_Z n_R n_R^R = \frac{\epsilon_p - 1}{\epsilon_p} \eta^{\frac{1}{2}} \left( \frac{n_t^R}{n_t} \right)^{\frac{1}{2}} Y, \) and \( C_Z n_{IR} n_{IR}^R = \frac{\epsilon_p - 1}{\epsilon_p} (1 - \eta)^{\frac{1}{2}} \left( \frac{n_t^{IR}}{n_t} \right)^{\frac{1}{2}} Y, \) we have

\[ \tilde{n}_t^R = \frac{\epsilon_p - 1}{\epsilon_p} \left( \frac{1 + \epsilon \omega_{n^{IR}}}{(1 + \epsilon \omega_{n^{IR})Z_{n^{IR}} n_{IR}^R} + (1 + \epsilon \omega_{n^R})Z_{n^R} n_{IR}^R) \right) \tilde{n}_t. \]  
(A44)
\[ \hat{n}_t^{IR} = \frac{\epsilon_p - 1}{\epsilon_p} \left( 1 + \epsilon \omega_{nR} \right) \left( \frac{1 + \epsilon \omega_{nR}}{\bar{Z}_{nR} n + (1 + \epsilon \omega_{nR}) \bar{Z}_{nR} n^{IR}} \right) \hat{n}_t. \]  

Moreover, if we combine (A41) after multiplying both sides by \( \frac{\epsilon_p - 1}{\epsilon_p} \) \( Y \) and (A42) after multiplying both sides by \( \frac{\epsilon_p - 1}{\epsilon_p} \) \( (\frac{\bar{Z}_{nR} n^{IR}}{\bar{Z}_{nR} n^{IR}}) \) \( Y = \bar{C}_{nR} n^{IR} \), then we have

\[ \hat{C}_t + \frac{\epsilon_p}{\epsilon_p - 1} \left( \frac{Z_{nR} n^{IR} \omega_{nR} \hat{n}_t^n + Z_{nR} n^{IR} \omega_{nR} \hat{n}_t^{IR}}{\bar{Z}_{nR} n^{IR} + (1 + \epsilon \omega_{nR}) \bar{Z}_{nR} n^{IR}} \right) = \hat{n}_t. \]  

Then from Equations (A44) and (A45), we have

\[ \frac{Z_{nR} n^{IR} \omega_{nR} \hat{n}_t^n + Z_{nR} n^{IR} \omega_{nR} \hat{n}_t^{IR}}{\bar{Z}_{nR} n^{IR} + (1 + \epsilon \omega_{nR}) \bar{Z}_{nR} n^{IR}} = \hat{n}_t. \]  

Now, combining this with the first-order approximation of the market clearing condition (Equation (A37): \( \hat{C}_t + \hat{\pi} \hat{Y}_t = \hat{Y}_t \)) and the first-order approximation of the production function (Equation (A36): \( \hat{n}_t = \hat{Y}_t - \hat{A}_t \)) generates

\[ \hat{\phi}_t = \hat{C}_t - \hat{A}_t + C_{\omega} (\hat{Y}_t - \hat{A}_t), \]

\[ = (1 + C_{\omega}) (\hat{Y}_t - \hat{A}_t). \]  

(A47)

If we combine Equation (A41) and Equation (A42) with the first-order approximation of the market clearing condition (Equation (A37): \( \hat{C}_t = \hat{Y}_t \)) and the first-order approximation of the mproduction function (Equation (A36): \( \hat{n}_t = \hat{Y}_t - \hat{A}_t \)), then we have

\[ \hat{n}_t^n = \frac{\epsilon}{1 + \epsilon \omega_{nR}} \left( \hat{\phi}_t - \hat{C}_t + \hat{A}_t + \frac{1}{\epsilon} (\hat{Y}_t - \hat{A}_t) \right) \]

\[ = \frac{1 + \epsilon C_{\omega}}{1 + \epsilon \omega_{nR}} (\hat{Y}_t - \hat{A}_t), \]  

(A48)

\[ \hat{n}_t^{IR} = \frac{\epsilon}{1 + \epsilon \omega_{nR}} \left( \hat{\phi}_t - \hat{C}_t + \hat{A}_t + \frac{1}{\epsilon} (\hat{Y}_t - \hat{A}_t) \right), \]

\[ = \frac{1 + \epsilon C_{\omega}}{1 + \epsilon \omega_{nR}} (\hat{Y}_t - \hat{A}_t). \]  

(A49)

Consider now the first-order approximation and the second-order approximation of the price-setting equation, Equation (A35). The log-linearized equation can be written as

\[ \pi_t + \beta E_t [\pi_{t+1}] = \frac{(\epsilon_p - 1) \bar{Y}}{\phi_p} \hat{\phi}_t + O_2. \]  

(A50)
Following Damjanovic and Nolan (2011), if we re-arrange this New Keynesian Phillips Curve (NKPC) with $T_t = \varphi_t Y_t$, we have

$$\phi_p \pi_t - \beta \phi_p E_t[\pi_{t+1}] = (1 - \epsilon_p) Y_t (\hat{T}_t - \tilde{T}_t) + O_2. \quad \text{(A51)}$$

Meanwhile, the second-order approximation of the NKPC can be written as

$$(\hat{Y}_t - \tilde{T}_t) + \frac{\phi_p}{(\epsilon_p - 1) Y} (\pi_t - \beta E_t[\pi_{t+1}]) + \frac{1}{2} \frac{\phi_p}{(\epsilon_p - 1) Y} (\hat{Y}_t^2 - \tilde{T}_t^2)$$

$$+ \frac{3\phi_p}{2(\epsilon_p - 1) Y} (\pi_t^2 - \beta E_t[\pi_{t+1}]) - \beta \frac{\phi_p}{(\epsilon_p - 1) Y} E_t[(\tilde{C}_t - \tilde{C}_{t+1}) \pi_{t+1}] = O_3. \quad \text{(A52)}$$

Solving forward the above equation then generates

$$\mathbb{E}_0 \sum_{t=0}^\infty \beta^t (\hat{Y}_t - \tilde{T}_t) + \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t (\hat{Y}_t^2 - \tilde{T}_t^2) = \beta \frac{\phi_p}{(\epsilon_p - 1) Y} E_t[(\tilde{C}_t - \tilde{C}_{t+1}) \pi_{t+1}] + t.i.p. + O_3, \quad \text{(A53)}$$

where t.i.p. stands for terms independent from policy. Then if we combine the second-order approximation of the market clearing condition and $\phi_t = C_\omega (\hat{Y}_t - \tilde{A}_t) + \tilde{C}_t - \tilde{A}_t$, we have

$$\tilde{T}_t = (C_\omega + 1) (\hat{Y}_t - \tilde{A}_t) + \tilde{C}_t$$

$$= (C_\omega + 1) (\hat{Y}_t - \tilde{A}_t) - \frac{\phi_p}{2Y} \pi_t^2 + \hat{Y}_t.$$ 

Therefore,

$$\tilde{T}_t - \hat{Y}_t = (C_\omega + 1) (\hat{Y}_t - \tilde{A}_t) - \frac{\phi_p}{2Y} \pi_t^2.$$ 

Moreover, from the above expression for $\tilde{T}_t$,

$$\hat{Y}_t^2 - \tilde{T}_t^2 = - ((C_\omega + 1) (\hat{Y}_t - \tilde{A}_t))^2 - 2 ((C_\omega + 1) (\hat{Y}_t - \tilde{A}_t)) \hat{Y}_t + t.i.p. + O_3.$$ 

So we have

$$\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t (\hat{Y}_t^2 - \tilde{T}_t^2) - \beta \frac{\phi_p}{(\epsilon_p - 1) Y} E_t[(\tilde{C}_t - \tilde{C}_{t+1}) \pi_{t+1}] = \mathbb{E}_0 \sum_{t=0}^\infty \beta^t (\tilde{T}_t - \hat{Y}_t) + t.i.p. + O_3$$

$$- \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t ((C_\omega + 1) (\hat{Y}_t - \tilde{A}_t))^2 - \mathbb{E}_0 \sum_{t=0}^\infty \beta^t ((C_\omega + 1) (\hat{Y}_t - \tilde{A}_t)) \hat{Y}_t$$

$$- \beta \frac{\phi_p}{(\epsilon_p - 1) Y} E_t[(\tilde{C}_t - \tilde{C}_{t+1}) \pi_{t+1}] + \frac{\phi_p}{2Y} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \pi_t^2$$

$$= \mathbb{E}_0 \sum_{t=0}^\infty \beta^t (1 + C_\omega) (\hat{Y}_t - \tilde{A}_t) + t.i.p. + O_3.$$ 

99
Now if we multiply \( \tilde{C}_t \) to the NKPC, Equation (A51), solve forward, and re-arrange then we have

\[
-\beta \frac{\phi_p}{(\epsilon_p - 1)Y} E_0 \sum_{t=0}^{\infty} \left[ (\tilde{C}_t - \tilde{C}_{t+1}) \pi_{t+1} \right] = E_0 \sum_{t=0}^{\infty} \beta^t \left( (C_\omega + 1) (\tilde{Y}_t - \tilde{A}_t) \right) \tilde{Y}_t + t.i.p. + O_3,
\]

Therefore, the above becomes

\[
E_0 \sum_{t=0}^{\infty} \beta^t (\tilde{Y}_t - \tilde{A}_t) + t.i.p. + O_3 = \frac{\phi_p}{2Y (1 + C_\omega)} E_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2 - \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (1 + C_\omega) (\tilde{Y}_t - \tilde{A}_t)^2.
\]

Combining all, the second-order approximation of the disutility from supplying labor can be written as

\[
\frac{Z_n \pi^R}{n^R} \left( \hat{n}_t^R + \frac{1 + \omega_{nR}}{2} \left( \hat{n}_t^R \right)^2 \right) + \frac{Z_n \pi^{IR}}{n^{IR}} \left( \hat{n}_t^{IR} + \frac{1 + \omega_{nIR}}{2} \left( \hat{n}_t^{IR} \right)^2 \right) + \frac{Z_n^\delta \pi^R \pi^{IR}}{n^R n^{IR}} \hat{R}_t^R \hat{R}_t^{IR} = (1 + \epsilon C_\omega) \left( \frac{Z_n \pi^R}{n^R} \left( \hat{n}_t^R + \frac{1 + \omega_{nR}}{2} \left( \hat{n}_t^R \right)^2 \right) + \frac{Z_n \pi^{IR}}{n^{IR}} \left( \hat{n}_t^{IR} + \frac{1 + \omega_{nIR}}{2} \left( \hat{n}_t^{IR} \right)^2 \right) + \frac{Z_n^\delta \pi^R \pi^{IR}}{n^R n^{IR}} \hat{R}_t^R \hat{R}_t^{IR} \right) (\tilde{Y}_t - \tilde{A}_t) + \frac{1}{2} \left( \frac{Z_n \pi^R}{(1 + \epsilon \omega_{nR})^2} + \frac{Z_n \pi^{IR}}{(1 + \epsilon \omega_{nIR})^2} \right) (1 + \epsilon C_\omega)^2 (\tilde{Y}_t - \tilde{A}_t)^2 + \frac{Z_n^\delta \pi^R \pi^{IR}}{(1 + \epsilon \omega_{nR})(1 + \epsilon \omega_{nIR})} (\tilde{Y}_t - \tilde{A}_t)^2.
\]

Therefore, the second-order approximation of the disutility from supplying labor can be written as follows:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{Z_n \pi^R}{n^R} \left( \hat{n}_t^R + \frac{1 + \omega_{nR}}{2} \left( \hat{n}_t^R \right)^2 \right) + \frac{Z_n \pi^{IR}}{n^{IR}} \left( \hat{n}_t^{IR} + \frac{1 + \omega_{nIR}}{2} \left( \hat{n}_t^{IR} \right)^2 \right) + \frac{Z_n^\delta \pi^R \pi^{IR}}{n^R n^{IR}} \hat{R}_t^R \hat{R}_t^{IR} \right\} = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{Z_n \pi^R}{n^R} + \frac{Z_n \pi^{IR}}{n^{IR}} \right) (\tilde{Y}_t - \tilde{A}_t) + \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{Z_n \pi^R}{(1 + \epsilon \omega_{nR})^2} + \frac{Z_n \pi^{IR}}{(1 + \epsilon \omega_{nIR})^2} \right\} (1 + \epsilon C_\omega)^2 + \frac{Z_n^\delta \pi^R \pi^{IR}}{(1 + \epsilon \omega_{nR})(1 + \epsilon \omega_{nIR})} (\tilde{Y}_t - \tilde{A}_t)^2
\]

\[
= \frac{\phi_p}{2Y (1 + C_\omega)} E_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2,
\]
Finally, because

\[ \tilde{\mathcal{C}}_t = \tilde{Y}_t - \frac{\phi_p}{2Y} \pi_t^2 + \mathcal{O}_3 \]

we have

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U C} - \frac{\phi_p}{2Y} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2 - \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 + C_\omega) (\tilde{Y}_t - \tilde{\lambda}_t)^2
\]

\[
- \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \frac{Z_{nR} n_R^R (1 + \omega_{nR})^2 + Z_{nIR} n_{IR}^R (1 + \omega_{nIR})^2}{(1 + \epsilon \omega_{nR})^2} \right) (1 + \epsilon C_\omega) - (1 + C_\omega) (Z_{nR} n_R^R + Z_{nIR} n_{IR}^R) \right. \\
\left. + \frac{2Z_{nR} n_R^R n_{IR}^R (1 + \epsilon C_\omega)^2}{(1 + \epsilon \omega_{nIR})^2} \right) \times (\tilde{Y}_t - \tilde{\lambda}_t)^2
\]

\[
- \phi_p \left( \frac{Z_{nR} n_R^R + Z_{nIR} n_{IR}^R}{2C (1 + C_\omega)} \right) \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2
\]

\[
= - \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_\pi \pi_t^2 + \Phi_y (\tilde{Y}_t - \tilde{\lambda}_t)^2 \right\} + t.i.p. + \mathcal{O}_3, \quad (A56)
\]

where

\[
\Phi_\pi = \frac{\phi_p}{Y (1 + C_\omega)} \bigg( C_\omega + \frac{\epsilon_p - 1}{\epsilon_p} \bigg), \quad (A57)
\]

because \( Z_{nR} n_R^R + Z_{nIR} n_{IR}^R = \frac{\epsilon_p - 1}{\epsilon_p} \).

\[
\Phi_y = \left( \frac{Z_{nR} n_R^R (1 + \omega_{nR})^2 + Z_{nIR} n_{IR}^R (1 + \omega_{nIR})^2}{(1 + \epsilon \omega_{nR})^2} \right) (1 + \epsilon C_\omega)^2 + \frac{1}{\epsilon_p (1 + C_\omega)} + \frac{2Z_{nR} n_R^R n_{IR}^R (1 + \epsilon C_\omega)^2}{(1 + \epsilon \omega_{nIR})^2},
\]

\[
\omega_{nR} = \frac{Z_{nR} n_R^R}{Z_{nIR}^R}, \quad \omega_{nIR} = \frac{Z_{nIR}^R}{Z_{nIR}^R}, \quad \text{and}
\]

\[
C_\omega = \left( \frac{Z_{nR} n_R^R (1 + \epsilon \omega_{nIR}) Z_{nIR} n_{IR}^R (1 + \epsilon \omega_{nIR})}{(1 + \epsilon \omega_{nIR}) Z_{nR} n_R^R + (1 + \epsilon \omega_{nIR}) Z_{nIR} n_{IR}^R} \right).
\]

Observe that

\[
\tilde{n}_t = (\eta^u)^{-\frac{1}{2}} \left( \frac{n_R}{n} \right)^{\frac{\epsilon_p - 1}{\epsilon_p}} \tilde{n}_t^R + (1 - \eta^u)^{\frac{1}{2}} \left( \frac{n_{IR}}{n} \right)^{\frac{\epsilon_p - 1}{\epsilon_p}} \tilde{n}_{IR}^R, \quad (A59)
\]
and
\[ \bar{n}^R \bar{\eta}_t^R = \frac{\partial \bar{n}^R}{\partial b} \bar{b}_t + \frac{\partial \bar{n}^R}{\partial d} \bar{d}_t, \quad \bar{n}^{IR} \bar{\eta}_t^{IR} = \frac{\partial \bar{n}^{IR}}{\partial b} \bar{b}_t + \frac{\partial \bar{n}^{IR}}{\partial d} \bar{d}_t. \]

Therefore, we have
\[ \bar{Y}_t - \bar{A}_t = \left\{ (\eta^n)^{\frac{1}{2}} \left( \bar{n}^R / \bar{n} \right) \frac{\partial \bar{n}^R / \bar{n}}{\partial b / b} + (1 - \eta^n)^{\frac{1}{2}} \left( \bar{n}^{IR} / \bar{n} \right) \frac{\partial \bar{n}^{IR} / \bar{n}}{\partial b / b} \right\} \bar{b}_t \]
\[ + \left\{ (\eta^n)^{\frac{1}{2}} \left( \bar{n}^R / \bar{n} \right) \frac{\partial \bar{n}^R / \bar{n}}{\partial d / d} + (1 - \eta^n)^{\frac{1}{2}} \left( \bar{n}^{IR} / \bar{n} \right) \frac{\partial \bar{n}^{IR} / \bar{n}}{\partial d / d} \right\} \bar{d}_t. \]

This implies that
\[ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi \bar{n}_t^2 + (\Phi_b \bar{b}_t + \Phi_d \bar{d}_t)^2 \right\} + t.i.p. + \mathcal{O}_3, \]

where
\[ \Phi_b \equiv \sqrt{\Phi_y} \times \left\{ (\eta^n)^{\frac{1}{2}} \left( \bar{n}^R / \bar{n} \right) \frac{\partial \bar{n}^R / \bar{n}}{\partial b / b} + (1 - \eta^n)^{\frac{1}{2}} \left( \bar{n}^{IR} / \bar{n} \right) \frac{\partial \bar{n}^{IR} / \bar{n}}{\partial b / b} \right\}, \]
\[ \Phi_d \equiv \sqrt{\Phi_y} \times \left\{ (\eta^n)^{\frac{1}{2}} \left( \bar{n}^R / \bar{n} \right) \frac{\partial \bar{n}^R / \bar{n}}{\partial d / d} + (1 - \eta^n)^{\frac{1}{2}} \left( \bar{n}^{IR} / \bar{n} \right) \frac{\partial \bar{n}^{IR} / \bar{n}}{\partial d / d} \right\}. \]

Meanwhile, the log-linearized total unemployment rate can be written as follows. Because the two thresholds, \( \bar{b}_t \) and \( \bar{d}_t \), are functions of \( \bar{n}^R_t \) and \( \bar{n}^{IR}_t \), we have
\[ \bar{u}_t = \frac{\bar{n}^R}{\bar{b}} \left( \varepsilon_{b,\bar{n}^R} - 1 \right) \bar{n}_t^R + \frac{\bar{n}^{IR}}{\bar{b}} \left( \varepsilon_{b,\bar{n}^{IR}} - 1 \right) \bar{n}_t^{IR}, \quad \text{(A60)} \]
or equivalently,
\[ \bar{u}_t = \left\{ \frac{\bar{n}^R}{\bar{b}} (1 - \varepsilon_{\bar{n}^R, b}) + \frac{\bar{n}^{IR}}{\bar{b}} (1 - \varepsilon_{\bar{n}^{IR}, b}) \right\} \bar{b}_t - \left\{ \frac{\bar{n}^R}{\bar{b}} \varepsilon_{\bar{n}^R, d} + \frac{\bar{n}^{IR}}{\bar{b}} \varepsilon_{\bar{n}^{IR}, d} \right\} \bar{d}_t, \quad \text{(A61)} \]

where
\[ \varepsilon_{b,\bar{n}^R} \equiv \frac{\partial \bar{n}^R / \bar{n}}{\partial b / b}, \quad \varepsilon_{d,\bar{n}^R} \equiv \frac{\partial \bar{n}^R / \bar{n}}{\partial d / d}, \quad \text{for } m = R, IR, \]
\[ \varepsilon_{\bar{n}^R, b} \equiv \frac{\partial \bar{n}^R / \bar{n}}{\partial b / b}, \quad \varepsilon_{\bar{n}^R, d} \equiv \frac{\partial \bar{n}^R / \bar{n}}{\partial d / d}, \quad \text{for } m = R, IR. \]

This implies that output gap can be written in terms of \( \bar{u}_t \) and either with \( \bar{\eta}_t^R \) and \( \bar{\eta}_t^{IR} \) or with
\( \bar{b}_t \) and \( \bar{d}_t \):

\[
\bar{Y}_t - \bar{A}_t = \bar{u}_t + \Delta^{u,n^R} \bar{n}^R_t + \Delta^{u,n^{IR}} \bar{n}^{IR}_t, \tag{A62}
\]

or equivalently,

\[
\bar{Y}_t - \bar{A}_t = \bar{u}_t + \Delta^{u,b} \bar{b}_t + \Delta^{u,d} \bar{d}_t, \tag{A63}
\]

where

\[
\Delta^{u,n^R} = (\eta^n)^\frac{1}{2} \left( \frac{n^R}{\bar{n}} \right)^{\frac{\epsilon - 1}{\epsilon}} - \frac{n^R}{\bar{b}} \left( \mathcal{E}_{\bar{n}} \eta^n - 1 \right),
\]

\[
\Delta^{u,n^{IR}} = (1 - \eta^n)^\frac{1}{2} \left( \frac{n^{IR}}{\bar{n}} \right)^{\frac{\epsilon - 1}{\epsilon}} - \frac{n^{IR}}{\bar{b}} \left( \mathcal{E}_{\bar{n}} \eta^n - 1 \right),
\]

\[
\Delta^{u,b} = \mathcal{E}_{\bar{n},b} \left( \left( \frac{n^R}{\bar{n}} \right)^{\frac{\epsilon - 1}{\epsilon}} + \frac{n^R}{\bar{b}} \right) + \mathcal{E}_{\bar{n},d} \left( 1 - \eta^n \right)^\frac{1}{2} \left( \frac{n^{IR}}{\bar{n}} \right)^{\frac{\epsilon - 1}{\epsilon}} + \frac{n^{IR}}{\bar{b}} - \frac{n^R + n^{IR}}{\bar{b}},
\]

\[
\Delta^{u,d} = \mathcal{E}_{\bar{n},d} \left( \left( \frac{n^R}{\bar{n}} \right)^{\frac{\epsilon - 1}{\epsilon}} + \frac{n^R}{\bar{d}} \right) + \mathcal{E}_{\bar{n},d} \left( 1 - \eta^n \right)^\frac{1}{2} \left( \frac{n^{IR}}{\bar{n}} \right)^{\frac{\epsilon - 1}{\epsilon}} + \frac{n^{IR}}{\bar{d}}.
\]

This implies that

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - \bar{U}}{U_C} \approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_t \bar{n}^2_t + \left( \Phi_t \bar{u}_t + \Delta^{u,n^R} \bar{n}^R_t + \Delta^{u,n^{IR}} \bar{n}^{IR}_t \right)^2 \right\} + t.i.p. + O_3,
\]

\[
\approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_t \bar{n}^2_t + \left( \Phi_t \bar{u}_t + \Delta^{u,b} \bar{b}_t + \Delta^{u,d} \bar{d}_t \right)^2 \right\} + t.i.p. + O_3.
\]

This shows that the stabilization of the aggregate unemployment rate is not sufficient to minimize welfare losses from aggregate fluctuations, with extra other terms, such as, \( \Delta^{u,n^R} \bar{n}^R_t + \Delta^{u,n^{IR}} \bar{n}^{IR}_t \) or \( \Delta^{u,b} \bar{b}_t + \Delta^{u,d} \bar{d}_t \).

**B.12.1 Calculating Disutility from Supplying Two Types of Labor**

Note that disutility from supplying two types of labor (in the case of simple one) is

\[
z(n^R_t, n^{IR}_t) = \log K(n^R_t, n^{IR}_t) - H_t(n^R_t, n^{IR}_t),
\]

103
where

\[
K(n^R_t, n^{IR}_t) = n^R_t \left( e^{\sigma^R_t + (1 + \sigma^R_t) b_t^{IR} - d_t^{IR}} - 1 \right) + n^{IR}_t \left( e^{\sigma^{IR}_t + (1 + \sigma^{IR}_t) b_t^{IR} - 1} - 1 \right) + 1,
\]

\[
H_t(n^R_t, n^{IR}_t) = \eta \sigma^R_t b_t^{IR} (\bar{b}_t^{IR} - d_t^{IR} + 1) + \eta \sigma^{IR}_t \bar{d}_t^{IR} + \frac{a^2 (1 + \sigma^R_t) \left( \sigma^R_t + 1 \right)}{2} \left( b_t^{IR} - \bar{b}_t^{IR} \right) \left( d_t^{IR} - \bar{d}_t^{IR} \right),
\]

with \( \bar{b}_t = f(n^R_t, n^{IR}_t) \), \( \bar{d}_t = g(n^R_t, n^{IR}_t) \).

To calculate this, we first need

\[
\frac{\partial n^R}{\partial b_t} = a^2 \sigma^R_t (1 + \sigma^R_t) \bar{b}_t^{IR}^{-1} \bar{a}_t,
\]

\[
\frac{\partial n^R}{\partial d_t} = \eta + a^2 (1 + \sigma^R_t) (\bar{b}_t^{IR} - d_t^{IR}) + a^2 \sigma^R_t (1 + \sigma^R_t) \bar{d}_t^{IR} - a^2 \sigma^R_t (1 + \sigma^R_t) \bar{d}_t^{IR},
\]

\[
\frac{\partial n^{IR}}{\partial b_t} = \eta + a^2 \sigma^{IR}_t (1 + \sigma^{IR}_t) \bar{b}_t^{IR}^{-1} (\bar{b}_t - \bar{d}_t),
\]

\[
\frac{\partial n^{IR}}{\partial d_t} = \eta + a^2 (1 + \sigma^{IR}_t) \left( \bar{b}_t^{IR} - \bar{d}_t^{IR} \right).
\]

This gives that

\[
\frac{\partial b_t}{\partial n^R} = \frac{\partial n^R}{\partial b_t}, \quad \frac{\partial b_t}{\partial n^{IR}} = -\frac{\partial n^{IR}}{\partial b_t},
\]

\[
\frac{\partial \bar{d}_t}{\partial n^R} = -\frac{\partial n^R}{\partial \bar{d}_t}, \quad \frac{\partial \bar{d}_t}{\partial n^{IR}} = -\frac{\partial n^{IR}}{\partial \bar{d}_t}.
\]
\[
\begin{align*}
\frac{\partial^2 b_t}{\partial (n_t^R)^2} &= - \left( \left( \frac{\partial n_t^R}{\partial b_t} \right)^2 H + \left( \frac{\partial n_t^R}{\partial t} \right)^2 G \right), \\
\frac{\partial^2 b_t}{\partial (n_t^R)^2} &= - \left( \left( \frac{\partial n_t^R}{\partial b_t} \right)^2 H + \left( \frac{\partial n_t^R}{\partial t} \right)^2 G \right), \\
\frac{\partial^2 d_t}{\partial (n_t^R)^2} &= - \left( \left( \frac{\partial n_t^R}{\partial d_t} \right)^2 I + \left( \frac{\partial n_t^R}{\partial t} \right)^2 I \right), \\
\frac{\partial^2 d_t}{\partial (n_t^R)^2} &= - \left( \left( \frac{\partial n_t^R}{\partial d_t} \right)^2 I + \left( \frac{\partial n_t^R}{\partial t} \right)^2 I \right), \\
\frac{\partial^2 b_t}{\partial n_t^R \partial n_t^R} &= - \frac{\partial^2 b_t}{\partial n_t^R \partial n_t^R} \frac{\partial n_t^R}{\partial b_t} \frac{\partial n_t^R}{\partial d_t} + \frac{\partial^2 b_t}{\partial n_t^R \partial n_t^R} \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t}, \\
\frac{\partial^2 d_t}{\partial n_t^R \partial n_t^R} &= - \frac{\partial^2 d_t}{\partial n_t^R \partial n_t^R} \frac{\partial n_t^R}{\partial b_t} \frac{\partial n_t^R}{\partial d_t} + \frac{\partial^2 d_t}{\partial n_t^R \partial n_t^R} \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t},
\end{align*}
\]

where

\[
\begin{align*}
H &\equiv \left( \frac{\partial^2 n_t^R}{\partial d_t^2} - \frac{\partial^2 n_t^R}{\partial d_t^2} \frac{\partial^2 n_t^R}{\partial d_t^2} \right) \left( \frac{\partial n_t^R}{\partial b_t} \frac{\partial n_t^R}{\partial d_t} + \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t} \right) - 2 \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t} \left( \frac{\partial^2 n_t^R}{\partial d_t^2} \frac{\partial^2 n_t^R}{\partial d_t^2} \right), \\
G &\equiv \left( \frac{\partial^2 n_t^R}{\partial d_t^2} - \frac{\partial^2 n_t^R}{\partial d_t^2} \frac{\partial^2 n_t^R}{\partial d_t^2} \right) \left( \frac{\partial n_t^R}{\partial b_t} \frac{\partial n_t^R}{\partial d_t} + \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t} \right) - 2 \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t} \left( \frac{\partial^2 n_t^R}{\partial d_t^2} \frac{\partial^2 n_t^R}{\partial d_t^2} \right), \\
I &\equiv \left( \frac{\partial^2 n_t^R}{\partial d_t^2} - \frac{\partial^2 n_t^R}{\partial d_t^2} \frac{\partial^2 n_t^R}{\partial d_t^2} \right) \left( \frac{\partial n_t^R}{\partial b_t} \frac{\partial n_t^R}{\partial d_t} + \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t} \right) - 2 \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t} \left( \frac{\partial^2 n_t^R}{\partial d_t^2} \frac{\partial^2 n_t^R}{\partial d_t^2} \right), \\
J &\equiv \left( \frac{\partial^2 n_t^R}{\partial d_t^2} - \frac{\partial^2 n_t^R}{\partial d_t^2} \frac{\partial^2 n_t^R}{\partial d_t^2} \right) \left( \frac{\partial n_t^R}{\partial b_t} \frac{\partial n_t^R}{\partial d_t} + \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t} \right) - 2 \frac{\partial n_t^R}{\partial d_t} \frac{\partial n_t^R}{\partial d_t} \left( \frac{\partial^2 n_t^R}{\partial d_t^2} \frac{\partial^2 n_t^R}{\partial d_t^2} \right).
\end{align*}
\]

105
Now to calculate the second derivatives, we first need

\[
\frac{\partial^2 H}{\partial t_i^2} = \eta \left( \left( 1 + c_i^R \right) b_i^{IR} - d_i^{IR} \right)^2 + a^2 \left( \left( 1 + c_i^R \right) d_i^{IR} - d_i^{IR} \right) \left( c_i^R \left( 1 + c_i^R \right) d_i^{IR} - d_i^{IR} \right) \left( 1 + c_i^R \right) d_i^{IR} - d_i^{IR} \right) \left( 1 + c_i^R \right) d_i^{IR} - d_i^{IR} \right) \left( 1 + c_i^R \right) d_i^{IR} - d_i^{IR} \right),
\]

\[
\frac{\partial^2 H}{\partial b_i \partial d_i} = a^2 \left( \left( 1 + c_i^R \right) b_i^{IR} - d_i^{IR} \right) \left( 1 + c_i^R \right) d_i^{IR} - d_i^{IR} \right),
\]

Then,

\[
\frac{\partial z(n_i^R, n_i^{IR})}{\partial n_i^R} = \frac{1}{K(n_i^R, n_i^{IR})} \frac{\partial K(n_i^R, n_i^{IR})}{\partial n_i^R} - \frac{\partial H(n_i^R, n_i^{IR})}{\partial n_i^R},
\]

\[
\frac{\partial z(n_i^R, n_i^{IR})}{\partial n_i^{IR}} = \frac{1}{K(n_i^R, n_i^{IR})} \frac{\partial K(n_i^R, n_i^{IR})}{\partial n_i^{IR}} - \frac{\partial H(n_i^R, n_i^{IR})}{\partial n_i^{IR}}.
\]
Moreover,

\[
\frac{\partial^2 K(n_R, n_R^g)}{\partial (n_R^g)^2} = 2e^{IR} + (1 + e^{IR})b_{IR}e^{IR}\left(\sigma_{IR}^L(1 + e^{IR})b_{IR}^L - \frac{\partial b_{IR}}{\partial n_R^g}\right) + \left(\sigma_{IR}^L(1 + e^{IR})d_{IR}^L - \sigma_{IR}^L(1 + e^{IR})d_{IR}^L\right) \frac{\partial d_{IR}}{\partial n_R^g}
\]

\[
+ n_R^g e^{IR} + (1 + e^{IR})b_{IR}e^{IR}\left(\sigma_{IR}^L(1 + e^{IR})b_{IR}^L - \frac{\partial b_{IR}}{\partial n_R^g}\right) + \left(\sigma_{IR}^L(1 + e^{IR})d_{IR}^L - \sigma_{IR}^L(1 + e^{IR})d_{IR}^L\right) \frac{\partial d_{IR}}{\partial n_R^g}
\]

\[
+ n_R^g e^{IR} + (1 + e^{IR})b_{IR}e^{IR}\left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)b_{IR}^L - 2 \frac{\partial b_{IR}}{\partial n_R^g}\right) + \left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L - 2 \frac{\partial d_{IR}}{\partial n_R^g}\right)
\]

\[
+ \left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L - \sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L\right) \frac{\partial^2 d_{IR}}{\partial (n_R^g)^2}
\]

\[
+ n_R^g e^{IR} + (1 + e^{IR})b_{IR}e^{IR}\left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)b_{IR}^L - 2 \frac{\partial b_{IR}}{\partial n_R^g}\right) + \left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L - 2 \frac{\partial d_{IR}}{\partial n_R^g}\right)
\]

\[
+ \left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L - \sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L\right) \frac{\partial^2 d_{IR}}{\partial (n_R^g)^2}
\]

\[
+ 2e^{IR} + (1 + e^{IR})b_{IR}e^{IR}\left(\sigma_{IR}^L(1 + e^{IR})b_{IR}^L - \frac{\partial b_{IR}}{\partial n_R^g}\right) + \left(\sigma_{IR}^L(1 + e^{IR})d_{IR}^L - \sigma_{IR}^L(1 + e^{IR})d_{IR}^L\right) \frac{\partial d_{IR}}{\partial n_R^g}
\]

\[
+ n_R^g e^{IR} + (1 + e^{IR})b_{IR}e^{IR}\left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)b_{IR}^L - 2 \frac{\partial b_{IR}}{\partial n_R^g}\right) + \left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L - 2 \frac{\partial d_{IR}}{\partial n_R^g}\right)
\]

\[
+ \left(\sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L - \sigma_{IR}^L(1 + e^{IR})(\sigma_{IR}^L - 1)d_{IR}^L\right) \frac{\partial^2 d_{IR}}{\partial (n_R^g)^2}
\]
\[ \frac{\partial^2 K(n^R_t, n^L_t)}{\partial n^R_t \partial n^L_t} = e^{R_1 + c(R)n^{IR}_t} e^{R_2 + c(L)n^{IR}_t} e^{R_0 + c(R)n^{IR}_t} \left( c^{R_1}(1 + c^{R_2}) c^{IR}_t \right) \frac{\partial^2 b_t}{\partial n^R_t \partial n^L_t} + \left( c^{R_1}(1 + c^{R_2}) c^{IR}_t \right) \frac{\partial^2 b_t}{\partial n^L_t \partial n^R_t} + \left( c^{R_1}(1 + c^{R_2}) e^{R_0} \right) \frac{\partial^2 b_t}{\partial n^R_t \partial n^L_t} + \left( c^{R_1}(1 + c^{R_2}) e^{R_0} \right) \frac{\partial^2 b_t}{\partial n^L_t \partial n^R_t} \]

Then the second-order derivatives of the disutility from supplying labor is

\[ \frac{\partial^2 z(n^R_t, n^L_t)}{\partial (n^R_t)^2} = \frac{\partial^2 K(n^R_t, n^L_t)}{\partial (n^R_t)^2} K(n^R_t, n^L_t)^2 - \left( \frac{\partial K(n^R_t, n^L_t)}{\partial n^R_t} \right)^2 - \frac{\partial^2 z(n^R_t, n^L_t)}{\partial (n^L_t)^2} = \frac{\partial^2 K(n^R_t, n^L_t)}{\partial (n^L_t)^2} K(n^R_t, n^L_t)^2 - \left( \frac{\partial K(n^R_t, n^L_t)}{\partial n^L_t} \right)^2 - \frac{\partial^2 z(n^R_t, n^L_t)}{\partial n^R_t \partial n^L_t} \]

108
Note that disutility from supplying two types of labor under more general case can be written as

\[ z(n_t^R, n_t^{IR}) = \log K(n_t^R, n_t^{IR}) - \log C - H(n_t^R, n_t^{IR}), \]

with \( C = \frac{\eta^{IR}}{\eta^{IR}} \), and

\[
K(n_t^R, n_t^{IR}) = n_t^R \left( e^{F_R + (1 + \sigma^L_{IR})d_t^R - \frac{\sigma^R}{\sigma^L} + \frac{1}{\sigma^R} \sqrt{X_t} - 1} \right) + n_t^{IR} \left( e^{F^{IR} + (1 + \sigma^L_{IR})\bar{b}_t^{IR} - 1} \right) + (C - 1) \bar{d}_t + 1,
\]

\[
H(n_t^R, n_t^{IR}) = \eta^{IR} \left( 1 + \sigma^L_{IR} \right) \bar{b}_t^{IR} (\bar{b}_t - \bar{d}_t) - \eta^{IR} \left( \bar{b}_t^{IR} - \bar{d}_t^{IR} \right) - \frac{(\eta^{IR})^2}{2 (a^{IR})^2 \bar{d}_t}
\]

\[
+ \frac{(a^{IR})^2 (1 + \sigma^L_{IR})^2 (\bar{b}_t^{IR} - \bar{d}_t^{IR})}{2 \sigma^L_{IR} + 1} + \frac{(a^{IR})^2 (1 + \sigma^L_{IR})^2 (\bar{b}_t^{IR} - \bar{d}_t^{IR})}{2 \sigma^L_{IR} + 1} \bar{d}_t^{IR} + \frac{(a^{IR})^2 (1 + \sigma^L_{IR})^2 (\bar{b}_t^{IR} - \bar{d}_t^{IR})}{2 \sigma^L_{IR} + 1} \bar{d}_t^{IR} + \frac{\bar{X}_t}{2} \sigma^L_{IR}.
\]

where

\[
\bar{X}_t = \left( a^{IR} (1 + \sigma^L_{IR} \left( \bar{b}_t^{IR} - \bar{d}_t^{IR} \right) + \frac{\eta^{IR}}{a^{IR}} \right)^2 + \left( \frac{\eta^{IR}}{a^{IR}} \right)^2 - 2 \log C.
\]

In this case,

\[
n_t^R = \left( a^{IR} \right)^2 \sigma^R_d \bar{d}_t^{IR} + a^R \bar{d}_t \sqrt{X_t},
\]

\[
n_t^{IR} = \eta^{IR} (\bar{b}_t - \bar{d}_t) + \left( a^{IR} \right)^2 \sigma^L_{IR} \bar{b}_t^{IR} + \left( a^{IR} \right)^2 \sigma^L_{IR} \bar{b}_t^{IR} - \left( a^{IR} \right)^2 (1 + \sigma^L_{IR}) \bar{d}_t \bar{b}_t^{IR} + \left( a^{IR} \right)^2 \bar{d}_t^{IR} + \frac{\bar{X}_t}{2} \sigma^L_{IR}.
\]

so we have

\[
\frac{\partial n_t^R}{\partial b_t} = \frac{a^R}{2 \sqrt{X_t}} \bar{d}_t \frac{\partial \bar{X}_t}{\partial b_t},
\]

\[
\frac{\partial n_t^R}{\partial d_t} = \left( a^{IR} \right)^2 \sigma^R_d \bar{d}_t^{IR} + a^R \bar{d}_t \sqrt{X_t} + \frac{a^R}{2 \sqrt{X_t}} \bar{d}_t \frac{\partial \bar{X}_t}{\partial d_t},
\]

\[
\frac{\partial n_t^{IR}}{\partial b_t} = \eta^{IR} + \left( a^{IR} \right)^2 \sigma^L_{IR} (1 + \sigma^L_{IR}) \bar{b}_t^{IR} + \left( a^{IR} \right)^2 (1 + \sigma^L_{IR}) \bar{d}_t \bar{b}_t^{IR},
\]

\[
\frac{\partial n_t^{IR}}{\partial d_t} = \eta^{IR} - \left( a^{IR} \right)^2 (1 + \sigma^L_{IR}) \bar{b}_t^{IR} + \left( a^{IR} \right)^2 (1 + \sigma^L_{IR}) \bar{d}_t \bar{b}_t^{IR}.
\]
where

\[
\frac{\partial X_t}{\partial b_t} = 2 \left( a^{IR} (1 + \sigma^{IR}_t) (b^{rIR}_t - \bar{d}^{rIR}_t) + \eta^{IR}_t \right) \left( a^{IR} \sigma^{IR}_t (1 + \sigma^{IR}_t) b^{rIR}_t \right),
\]

\[
\frac{\partial X_t}{\partial d_t} = 2 \left( a^{IR} (1 + \sigma^{IR}_t) (b^{rIR}_t - \bar{d}^{rIR}_t) + \eta^{IR}_t \right) \left( -a^{IR} \sigma^{IR}_t (1 + \sigma^{IR}_t) \bar{d}^{rIR}_t \right),
\]

\[
\frac{\partial H(n^{R}_t, n^{IR}_t)}{\partial b_t} = \eta^{IR} \sigma^{IR}_t (1 + \sigma^{IR}_t) b^{rIR}_t - (a^{IR})^2 (1 + \sigma^{IR}_t) \left( \sigma^{IR}_t \right) 
\]

\[
= \frac{2 a^{IR}_t}{2 \sqrt{X_t}} \sigma^{R}_t \bar{d}^{rIR}_t + \frac{1}{2} \frac{\partial \bar{d}^{rIR}_t}{\partial d_t},
\]

\[
\frac{\partial H(n^{R}_t, n^{IR}_t)}{\partial d_t} = -\eta^{IR} \sigma^{IR}_t (1 + \sigma^{IR}_t) (b^{rIR}_t - \bar{d}^{rIR}_t) - \frac{(\eta^{IR}_t)^2}{2 (a^{IR}_t)^2} + \left( a^{IR}_t \right) \frac{2 a^{IR}_t}{(1 + \sigma^{IR}_t)^2} \sigma^{rIR}_t \n
\]

\[
+ \frac{2 a^{IR}_t}{2 \sqrt{X_t}} \sigma^{R}_t \bar{d}^{rIR}_t + \frac{1}{2} \frac{\partial \bar{d}^{rIR}_t}{\partial d_t},
\]

\[
\frac{\partial K(n^{R}_t, n^{IR}_t)}{\partial n^{R}_t} = C \left( e^{F^{IR}_{R} + (1 + \sigma^{IR}_t) \bar{d}^{rIR}_t} - \eta^{IR}_t \frac{\sqrt{X_t}}{(\sigma^{IR}_t)^2} \right) + \left( e^{F^{IR}_{R} + (1 + \sigma^{IR}_t) \bar{d}^{rIR}_t} - \eta^{IR}_t \frac{\sqrt{X_t}}{(\sigma^{IR}_t)^2} \right) \frac{1}{2 a^{R}_t \sqrt{X_t}} \frac{\partial \bar{d}^{rIR}_t}{\partial n^{R}_t} + \left( \frac{1}{2 a^{R}_t \sqrt{X_t}} \frac{\partial \bar{d}^{rIR}_t}{\partial n^{R}_t} \right)
\]

\[
+ \left( e^{F^{IR}_{R} + (1 + \sigma^{IR}_t) \bar{d}^{rIR}_t} - \eta^{IR}_t \frac{\sqrt{X_t}}{(\sigma^{IR}_t)^2} \right) \frac{1}{2 a^{R}_t \sqrt{X_t}} \frac{\partial \bar{d}^{rIR}_t}{\partial n^{R}_t} + \left( \frac{1}{2 a^{R}_t \sqrt{X_t}} \frac{\partial \bar{d}^{rIR}_t}{\partial n^{R}_t} \right)
\]

Then,

\[
\frac{\partial z(n^{R}_t, n^{IR}_t)}{\partial n^{R}_t} = 1 - \frac{K(n^{R}_t, n^{IR}_t)}{\partial n^{R}_t} - \log C \frac{\partial \bar{d}^{rIR}_t}{\partial n^{R}_t} - \frac{\partial H(n^{R}_t, n^{IR}_t)}{\partial n^{R}_t},
\]

where

\[
\frac{\partial z(n^{R}_t, n^{IR}_t)}{\partial n^{IR}_t} = 1 - \frac{K(n^{R}_t, n^{IR}_t)}{\partial n^{IR}_t} - \log C \frac{\partial \bar{d}^{rIR}_t}{\partial n^{IR}_t} - \frac{\partial H(n^{R}_t, n^{IR}_t)}{\partial n^{IR}_t},
\]

110
Now to calculate second derivatives under the general case, we first need to calculate

\[
\frac{\partial^2 n^R}{\partial b_i \partial t} = -\frac{a^R}{4} \bar{X}_t^{-\frac{3}{2}} d_t \left( \frac{\partial \bar{X}_t}{\partial b_i} \right)^2 + \frac{a^R}{2 \sqrt{ \bar{X}_t } d_t} \frac{\partial^2 \bar{X}_t}{\partial b_i^2},
\]

\[
\frac{\partial^2 n^R}{\partial t^2} = (a^R)^2 \left( \sigma^R_L \right)^2 (1 + \sigma^R_L) b_i^{\text{IR}-1} + \frac{a^R}{2 \sqrt{ \bar{X}_t } d_t} \frac{\partial^2 \bar{X}_t}{\partial d_t} + \frac{a^R}{4} \bar{X}_t^{-\frac{3}{2}} d_t \left( \frac{\partial \bar{X}_t}{\partial d_t} \right)^2 + \frac{a^R}{2 \sqrt{ \bar{X}_t } d_t} \frac{\partial^2 \bar{X}_t}{\partial d_t^2},
\]

\[
\frac{\partial^2 n^R}{\partial b_i \partial d_t} = -\frac{a^R}{4} \bar{X}_t^{-\frac{3}{2}} d_t \frac{\partial \bar{X}_t}{\partial b_i} \frac{\partial \bar{X}_t}{\partial d_t} + \frac{a^R}{2 \sqrt{ \bar{X}_t } d_t} \frac{\partial^2 \bar{X}_t}{\partial b_i \partial d_t},
\]

\[
\frac{\partial^2 n^R}{\partial d_t^2} = (a^R)^2 \left( \sigma^R_L \right)^2 (1 + \sigma^R_L) b_i^{\text{IR}-1} - (a^R)^2 \sigma^R_L (1 + \sigma^R_L) (\sigma^R_L - 1) b_i^{\text{IR}-2} d_t,
\]

\[
\frac{\partial^2 n^R}{\partial b_i \partial d_t^2} = -(a^R)^2 (1 + \sigma^R_L) \sigma^R_L b_i^{\text{IR}-1}.
\]

\[
\frac{\partial^2 X_t}{\partial \sigma^R_L \partial b_i} = 2 \frac{a^R c_{\text{IR}} (1 + c_{\text{IR}}) b_i^{\text{IR}-1}}{\sigma^R_L} + 2 \frac{\partial^2 X_t}{\partial \sigma^R_L \partial d_t} + \frac{\partial^2 X_t}{\partial \sigma^R_L \partial d_t^2} - (a^R)^2 \sigma^R_L b_i^{\text{IR}-1}.
\]

\[
\frac{\partial^2 X_t}{\partial \sigma^R_L \partial d_t} = -(a^R)^2 (1 + \sigma^R_L) \sigma^R_L b_i^{\text{IR}-1}.
\]

With those, we can calculate

\[
\frac{\partial^2 \mathcal{K}(n^R, n^R)}{\partial (n^R)^2} = 2 c_{\text{IR}} c_{\text{IR}} (1 + c_{\text{IR}}) b_i^{\text{IR}-1} - \frac{a^R}{\sigma^R_L} + \frac{\partial^2 X_t}{\partial \sigma^R_L \partial b_i} + \frac{\partial^2 X_t}{\partial d_t} + \frac{\partial^2 X_t}{\partial d_t^2}.
\]

111
\[
\frac{\partial^2 K(n_1^R, n_t^R)}{\partial (n_t^R)^2} = n_t^R Ce^{\frac{R + (1 + c_l^R) \rho_n^t}{\sigma^l} + \frac{I_R}{\sqrt{X_t}}} \left( \frac{1}{2a^R \sqrt{X_t}} \frac{\partial ^2 X_t}{\partial \bar{d}_t \partial \bar{b}_t} + \left( c_l^R (1 + c_l^R) \rho_n^t - 1 + \frac{1}{2a^R \sqrt{X_t}} \frac{\partial ^2 X_t}{\partial \bar{b}_t \partial \bar{d}_t} \right)^2 \right)
\]

\[
+ n_t^R Ce^{\frac{R + (1 + c_l^R) \rho_n^t}{\sigma^l} + \frac{I_R}{\sqrt{X_t}}} \times \left[ - \frac{1}{4 a^R X_t} \frac{\partial ^2 X_t}{\partial \bar{d}_t \partial \bar{b}_t} + \frac{1}{2a^R \sqrt{X_t}} \frac{\partial ^2 X_t}{\partial \bar{d}_t \partial \bar{d}_t} \right] \frac{\partial \bar{d}_t}{\partial \bar{d}_t^R} + \frac{1}{2a^R \sqrt{X_t}} \frac{\partial ^2 X_t}{\partial \bar{d}_t \partial \bar{d}_t} \frac{\partial \bar{b}_t}{\partial \bar{d}_t^R}
\]

\[
+ \left( c_l^R (1 + c_l^R) (2 \frac{\partial ^2 X_t}{\partial \bar{d}_t \partial \bar{d}_t} - 1) \frac{\partial ^2 X_t}{\partial \bar{d}_t \partial \bar{d}_t} \right) \frac{\partial ^2 \bar{d}_t}{\partial \bar{d}_t \partial \bar{d}_t^R} + (C - 1) \frac{\partial ^2 \bar{d}_t}{\partial \bar{d}_t \partial \bar{d}_t^R}
\]
\[
\frac{\partial^2 H(n_R^R, n_t^{IR})}{\partial d_t^2} = n_R^R \frac{\partial^2}{\partial d_t^2} \left(1 + c_R^R \right) d_t^{IR-1} + 2 \left( a^R \right)^2 \left(1 + c_R^R \right) \left( c_R^R \right) d_t^{IR-1} \\
+ \left( a^R \right)^2 c_R^R (1 + c_R^R)^2 \tilde{b}_t^R d_t^{IR-1} - \left( a^R \right)^2 c_R^R (1 + c_R^R)^2 \tilde{b}_t^R d_t^{IR-1} \\
- \frac{\partial^2}{\partial d_t^2} \left( \frac{\partial X_t}{\partial d_t} \right)^2 + \frac{\partial^2 X_t}{\partial d_t^2} + \frac{\partial^2 X_t}{\partial d_t^2} (1 + c_R^R) \frac{\partial X_t}{\partial d_t} + \frac{\partial}{\partial d_t} (c_R^R)^2 (1 + c_R^R) d_t^{IR-1} \sqrt{X_t} + \frac{\partial X_t}{\partial d_t} + \frac{1}{2} \frac{\partial X_t}{\partial d_t},
\]

\[
\frac{\partial^2 H(n_R^R, n_t^{IR})}{\partial \sigma_d \partial d_t} = -n_R^R \frac{\partial^2}{\partial \sigma_d \partial d_t} (1 + c_R^R) d_t^{IR-1} - \left( a^R \right)^2 c_R^R (1 + c_R^R)^2 \tilde{b}_t^R d_t^{IR-1} + \left( a^R \right)^2 c_R^R (1 + c_R^R)^2 \tilde{b}_t^R d_t^{IR-1} \\
- \frac{\partial^2}{\partial \sigma_d \partial d_t} \left( \frac{\partial X_t}{\partial \sigma_d} \right)^2 + \frac{\partial^2 X_t}{\partial \sigma_d^2} + \frac{\partial^2 X_t}{\partial \sigma_d^2} (1 + c_R^R) \frac{\partial X_t}{\partial \sigma_d} + \frac{\partial}{\partial \sigma_d} (c_R^R)^2 (1 + c_R^R) d_t^{IR-1} \sqrt{X_t} + \frac{\partial X_t}{\partial \sigma_d} + \frac{1}{2} \frac{\partial X_t}{\partial \sigma_d}.
\]

Then,

\[
\frac{\partial^2 H(n_R^R, n_t^{IR})}{\partial (n_t^{IR})^2} = \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial b_t}{\partial n_t^{IR}} \right)^2 + 2 \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial b_t}{\partial n_t^{IR}} \right) \frac{\partial^2}{\partial (n_t^{IR})^2} + \frac{\partial^2}{\partial (n_t^{IR})^2} \frac{\partial^2}{\partial (n_t^{IR})^2} \\
+ \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial b_t}{\partial n_t^{IR}} \right)^2 + 2 \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial b_t}{\partial n_t^{IR}} \right) \frac{\partial^2}{\partial (n_t^{IR})^2} + \frac{\partial^2}{\partial (n_t^{IR})^2} \frac{\partial^2}{\partial (n_t^{IR})^2} \\
+ \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial b_t}{\partial n_t^{IR}} \right)^2 + 2 \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial b_t}{\partial n_t^{IR}} \right) \frac{\partial^2}{\partial (n_t^{IR})^2} + \frac{\partial^2}{\partial (n_t^{IR})^2} \frac{\partial^2}{\partial (n_t^{IR})^2} \\
+ \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial b_t}{\partial n_t^{IR}} \right)^2 + 2 \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial b_t}{\partial n_t^{IR}} \right) \frac{\partial^2}{\partial (n_t^{IR})^2} + \frac{\partial^2}{\partial (n_t^{IR})^2} \frac{\partial^2}{\partial (n_t^{IR})^2}.
\]

Finally, the second-order derivatives of the disutility from supplying labor under more general case is

\[
\frac{\partial^2 z(n_R^R, n_t^{IR})}{\partial (n_t^{IR})^2} = \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial K(n_t^{IR})}{\partial n_t^{IR}} \right)^2 - \log C \frac{\partial^2}{\partial (n_t^{IR})^2} - \frac{\partial^2}{\partial (n_t^{IR})^2} - \frac{\partial^2}{\partial (n_t^{IR})^2} \\
\frac{\partial^2 z(n_t^{IR})}{\partial (n_t^{IR})^2} = \frac{\partial^2}{\partial (n_t^{IR})^2} \left( \frac{\partial K(n_t^{IR})}{\partial n_t^{IR}} \right)^2 - \log C \frac{\partial^2}{\partial (n_t^{IR})^2} - \frac{\partial^2}{\partial (n_t^{IR})^2} - \frac{\partial^2}{\partial (n_t^{IR})^2} \\
\frac{\partial^2 z(n_t^{IR})}{\partial n_t^{IR} \partial n_t^{IR}} = \frac{\partial^2}{\partial n_t^{IR} \partial n_t^{IR}} \left( \frac{\partial K(n_t^{IR})}{\partial n_t^{IR}} \right)^2 - \log C \frac{\partial^2}{\partial n_t^{IR} \partial n_t^{IR}} - \frac{\partial^2}{\partial n_t^{IR} \partial n_t^{IR}} - \frac{\partial^2}{\partial n_t^{IR} \partial n_t^{IR}}.
B.13 Comparison of Federal Funds Rates with the Implied Interest Rates from the Optimal Inclusive Monetary Policy

This section compares historical Federal Funds Rates with the interest rates implied from the alternative specification of interest rate rule with the stabilization of the composition of workers, with \( \bar{b}_t \) and \( \bar{d}_t \).

I consider the two specifications of monetary policy rules with \( b_t \) and \( d_t \): one is the non-inertial rule, and the other is the intertial rule:

\[
i_t = i^* + \phi_\pi \pi_t + \phi_b (\bar{b}_t - \bar{b}) + \phi_d (\bar{d}_t - \bar{d}) + \epsilon^t, \tag{A64}
\]
\[
i_t = (1 - \rho^t) i^{*} + \rho^t i_{t-1} + (1 - \rho^t) [\phi_\pi \pi_t + \phi_b (\bar{b}_t - \bar{b}) + \phi_d (\bar{d}_t - \bar{d})] + \epsilon^t. \tag{A65}
\]

Figure A29: Comparison of Historical Federal Funds Rates with the Implied Interest Rates from the Non-Intertial Specification of the Alternative Monetary Policy Rule of Equation (A64)

Note: This figure compares historical Federal Funds Rates with the interest rates implied from the optimal inclusive monetary policy where the central bank stabilizes the two thresholds, \( \bar{b}_t \) and \( \bar{d}_t \), with the non-inertial interest rate rule of Equation (??) with the optimal weights on the two thresholds, \((\phi^*_b, \phi^*_d)\), and with \( \phi_\pi = 1.5 \). Across all panels, the black solid lines plot the mean deviation of the historical Federal Funds Rates, the red dash-dot lines show the mean deviation of the implied interest rates when \((\bar{b}_t - \bar{b})\) and \((\bar{d}_t - \bar{d})\) are calculated from the mean deviations, and the blue dash lines show the mean deviation of the implied interest rates when \((\bar{b}_t - \bar{b})\) and \((\bar{d}_t - \bar{d})\) are HP-filtered with the smoothing parameter of 10^7. Panel (a) is for the all sample periods of 1976Q1:2007Q4, Panel (b) is from Volcker to Greenspan period, 1979Q3:2005Q4, and Panel (c) is from Greenspan to Bernanke period, 1987Q3-2007Q4.
Figure A30: Comparison of Historical Federal Funds Rates with the Implied Interest Rates from the Non-Intertial Specification of the Alternative Monetary Policy Rule of Equation (A65)

Panel (a): All Sample Periods (1976Q1-2007Q4)

Panel (b): Volcker-Greenspan (1979Q3-2005Q4)

Panel (c): Greenspan-Bernanke (1987Q3-2007Q4)

Note: This figure compares historical Federal Funds Rates with the interest rates implied from the optimal inclusive monetary policy where the central bank stabilizes the two thresholds, $\bar{b}_t$ and $\bar{d}_t$, with the inertial interest rate rule of Equation (??) with the optimal weights on the two thresholds, $(\phi^*_b, \phi^*_d)$, and with $\phi^*_\pi = 1.5$. Across all panels, the black solid lines plot the mean deviation of the historical Federal Funds Rates, the red dash-dot lines show the mean deviation of the implied interest rates when $(\bar{b}_t - \bar{b})$ and $(\bar{d}_t - \bar{d})$ are calculated from the mean deviations, and the blue dash lines show the mean deviation of the implied interest rates when $(\bar{b}_t - \bar{b})$ and $(\bar{d}_t - \bar{d})$ are HP-filtered with the smoothing parameter of $10^7$. Panel (a) is for the all sample periods of 1976Q1:2007Q4, Panel (b) is from Volcker to Greenspan period, 1979Q3:2005Q4, and Panel (c) is from Greenspan to Bernanke period, 1987:Q3-2007Q4.

B.14 Model Extension

Because the indirect utility function of the representative family reduces to the standard functional form often used in the workhorse DSGE models, it is easy to extend the baseline model by embedding various other important frictions. First, it is easy to incorporate regular workers’ wage rigidities into the model. Because regular workers tend to maintain longer-term relationships with firms, it is plausible that regular workers’ wages exhibit more rigidities than those of irregular workers. This makes regular workers much stickier than irregular workers in terms of adjustments. Second, the framework in the baseline model is flexible enough to incorporate habit formation in consumption. Lastly, it is easy to augment the baseline model to include a Bernanke, Gertler, and Gilchrist (1999) financial acceleration mechanism, which enables us to examine the interaction between financial frictions and labor adjustments.

Kudlyak (2010) shows the evidence that while wages of new hires exhibit high cyclicalities, those of existing workers do not.
B.14.1 Regular Workers’ Wage Rigidities

In order to incorporate regular workers’ nominal wage rigidities, I need to assume that a continuum of families, indexed by \( h \in [0, 1] \) supply homogeneous irregular labor input to intermediate-goods-producing firms directly, but supply differentiated regular labor input to a “labor packer.” This labor packing firm then bundles the differentiated regular labor into a homogeneous regular labor input to intermediate-goods-producing firms for production.

The technology that this labor packer uses to bundle differentiated regular labor from families is:

\[
  n_t^R = \left( \int_0^1 n_t^R(h) \frac{\epsilon_w - 1}{\epsilon_w} \, dh \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad \epsilon_w > 1, \tag{A66}
\]

where \( \epsilon_w \) denotes the degree of substitutability between differentiated regular labor. Then the labor packer maximizes its profit by solving the following problem:

\[
  \max_{\{n_t^R(h)\}_h} W_t^R \left( \int_0^1 n_t^R(h) \frac{\epsilon_w - 1}{\epsilon_w} \, dh \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_t^R(h) n_t^R(h) \, dh,
\]

where \( W_t^R \) is the aggregate nominal wage for regular workers, and \( W_t^R(h) \) is the nominal wage for regular labor supplied by a family, \( h \).

The first order necessary condition of the above problem generates the following downward-sloping demand for each variety of regular-type labor:

\[
  n_t^R(h) = \left( \frac{W_t^R(h)}{W_t^R} \right)^{-\epsilon_w} n_t^R, \quad \forall \ h \in [0, 1]. \tag{A67}
\]

where \( W_t^R \) is the aggregate wage index derived in a similar way to the aggregate price index as follows:

\[
  W_t^R = \left( \int_0^1 W_t^R(h) \frac{1}{1 - \epsilon_w} \, dh \right)^{\frac{1}{1 + \epsilon_w}}.
\]

This downward-sloping demand curve for each variety of regular-type labor due to imperfect substitutabilities across varieties gives the family, \( h \) some wage-setting power. This modifies the family \( h \)’s optimization problem slightly as below.

In addition to \( \{C_t, n_t^R(h), n_t^{IR}, v_t, K_t, I_t, B_{t+1}\} \), a family \( h \) sets the wage for regular-type labor, \( W_t^R(h) \), subject to regular workers’ wage adjustment cost \( C(W_t^R(h); W_{t-1}^R(h)) \) and downward-sloping demand curve for regular-type labor for a family, \( h \). Therefore, a family \( h \)’s problem can now be written as follows:

\[
  \max_{\{C_t, n_t^R(h), W_t^R(h), n_t^{IR}, v_t, K_t, I_t, B_{t+1}, h\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( u(C_t, n_t^R(h), n_t^{IR}) \right) \tag{A68}
\]

\[
  \log C_t - n_t^R(h), n_t^{IR})
\]

116
subject to
\[
P_t C_t + P_t I_t + B_{t+1} \leq (1 + i_{t-1}) B_t + W_t^R(h) n_t^R(h) + W_t^{IR} n_t^{IR} + R_t K_t v_t
\]
\[- C(W_t^R(h), W_{t-1}^R(h)) + \text{Profits, Taxes, and Transfers}_t,
\]
\[
K_{t+1} = Z_t \left[ 1 - \frac{\tau}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right]^2 I_t + (1 - \delta(v_t)) K_t,
\]
\[
n_t^R(h) = \left( \frac{W_t^R(h)}{W_t^R} \right)^{-\epsilon_w} n_t^{R,d}, \forall \ h \in [0,1].
\]

where the Rotemberg (1982)-type adjustment cost\(^7\) for regular-type labor’s nominal wage is defined as:
\[
C(W_t^R(h), W_{t-1}^R(h)) = \frac{\phi_w}{2} \left( \frac{1}{\Pi_t W_t^R} W_t^R(h) - 1 \right)^2 P_t Y_t. \tag{A69}
\]

Because the family \( h \)'s indirect utility function features separability between consumption and leisure, families will be identical along the all margins except for the supply for regular-type of labor and its wages. Therefore, I suppress the dependence on \( h \) except for those two margins (see Erceg, Henderson, and Levin 2000).

The necessary conditions for the above optimization problem are
\[
\lambda_t \equiv P_t \Lambda_t = \frac{1}{C_t},
\]
with \( \Lambda_t \) as a lagrangian multiplier on a budget constraint,
\[
\epsilon_w n_{n^R} n_t^R(h) + \Lambda_t \left\{ (1 - \epsilon_w) n_t^R(h) - \phi_w \left( \frac{1}{\Pi_t} W_t^R(h) - 1 \right) \frac{P_t Y_t}{\Pi_t W_t^R(h)} \right\}
\]
\[
+ \beta \varepsilon_t \left[ \lambda_{t+1} \phi_w \left( \frac{1}{\Pi_{t+1}} W_{t+1}^R(h) - 1 \right) \frac{W_{t+1}^R(h) P_{t+1} Y_{t+1}}{W_t^R(h)^2} \right] = 0,
\]
where \( n_{n^R} \) denotes \( \partial n(n_t^R, n_t^{IR})/\partial n_t^R \). With the symmetry of the equilibrium, hence suppressing the dependence on \( h \), and if we define \( \Pi_t^{R,w} \equiv W_t^R / W_{t-1}^R \), the above condition becomes
\[
\epsilon_w n_{n^R} n_t^R + P_t \Lambda_t \left\{ (1 - \epsilon_w) n_t^R W_t^R(P_t) - \phi_w \left( \frac{\Pi_t^{R,w}}{P_t} - 1 \right) \frac{\Pi_t^{R,w}}{P_t} Y_t \right\} + \beta \varepsilon_t \left[ P_{t+1} \lambda_{t+1} \phi_w \left( \frac{\Pi_{t+1}^{R,w}}{P_{t+1}} - 1 \right) \frac{\Pi_{t+1}^{R,w}}{P_{t+1}} Y_{t+1} \right] = 0,
\]
\[
n_{n^R} (n_t^R, n_t^{IR}) = \lambda_t W_t^{IR},
\]
where \( n_{n^R} \equiv \partial n(n_t^R, n_t^{IR})/\partial n_t^{IR} \),
\[
\lambda_t R_t K_t = \mu_t \delta'(v_t) K_t,
\]

\(^7\)Given complicated functional forms of \( n(n_t^R(h), n_t^{IR}) \), it is hard to represent \( n(n_t^R(h), n_t^{IR}) \) recursively, which makes using Calvo-type nominal wage rigidities difficult.
with $\mu_t$ as a lagrangian multiplier on a capital accumulation process,

$$
\Lambda_t = \beta E_t \Lambda_{t+1} (1 + i_t),
$$

$$
\lambda_t = \mu_t Z_t \left( 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \left[ \mu_{t+1} Z_{t+1} \kappa \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right],
$$

$$
\mu_t = \beta E_t \left[ \Lambda_{t+1} R_{t+1} \nu_{t+1} + \mu_{t+1} (1 - \delta (\nu_{t+1})) \right].
$$

B.14.2 Habit Formation in Consumption

As is the case of CTW, the baseline model is flexible enough to introduce habit formation of consumption, as long as consumption habit is governed by the family-wide consumption, $C_{t-1}$. If we assume that utility from consumption for each type of worker is given by $\log (c_{tm} - bC_{t-1})$ for $m = R, IR$, and similarly for a non-worker’s utility, $\log (c_{tm}^{lR} - bC_{t-1})$ for $m = R, IR$, then the indirect utility of a family $h$ is now simplified as

$$
u(C_t, n_t^R(h), n_{t}^{lR}) = \log (C_t - bC_{t-1}) - n(n_t^R(h), n_t^{lR}),
$$

with the same functional form of $n(n_t^R(h), n_t^{lR})$ defined in Appendix B.3 and $b$ denotes the degree of habit formation in consumption.

Note that in the case of utility with habit formation, only the necessary condition regarding the choice of $C_t$ needs to be replaced with the one below:

$$
P_t \Lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta b E_t \left[ \frac{1}{C_{t+1} - bC_t} \right].
$$