Monetary Non-Neutrality in the Cross-Section

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Abstract

This paper models the heterogeneous effects of monetary policy on employment and consumption across households who participate in segmented labor markets, and earn and spend their income in different industries. Changes in the labor supply of households who are exposed to flex-price or capital intensive industries imply a larger price response, as captured by household-specific Phillips curves. Employment is more sensitive to changes in aggregate demand induced by monetary policy for the households with steeper Phillips curves. In the aggregate, the ability of producers and consumers to shift expenditure towards the workers with flatter Phillips curves increases the real effects of monetary policy. Calibrating the model to the US economy reveals significant heterogeneity in the impact response of employment to monetary policy across occupations, varying from 0.25% (food services) to 1.1% (construction) for a 1% increase in nominal GDP. Ignoring input-output linkages would reduce this cross-sectional range from 0.86% to 0.35%.

1 Introduction

Monetary policy has the mandate to stabilize aggregate inflation and aggregate employment. But, as a side effect of its action on aggregates, does monetary policy also have a differential impact on the labor income and cost of living of different households? At present, we have no established theory and little empirical evidence of how monetary policy affects different occupations, geographic regions, or demographic groups, depending on which industries they earn income and purchase goods from. In this paper I develop a rich model of the interaction between households and industries, where multiple industries hire workers, capital, and intermediate inputs in different proportions, while households participate in segmented labor markets and purchase different bundles of goods. I show that a monetary expansion reallocates demand towards price-rigid and supply-elastic goods, because their price increases

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by less. This in turn bids up the labor demand of households who make these goods, and the real income of those who buy them.

Compared with the HANK literature (Kaplan et al. (2018), Auclert (2019)) – which focuses on the households' consumption-saving problem while modeling cross-sectional differences in real income as exogenous to monetary policy – I adopt a stylized model of the consumption-saving decision, and highlight the endogenous effects of monetary policy on real incomes across households.

Before introducing the model, in Section 2 I present some stylized facts about household heterogeneity. I classify households according to the occupation of the primary earner. Merging employment data by industry and occupation from the American Community Survey (ACS) with industry-level measures of price adjustment frequencies from Pasten et al. (2019), I find that different occupations are employed by industries with different degree of price stickiness. By contrast, using data from the Consumer Expenditure Survey (CEX), I find little heterogeneity in the price stickiness of consumption baskets. I then revisit the traditional aggregate business cycle facts (Cooley and Prescott (1995)) for a cross-section of occupations. Using time series of employment and wages from the Current Population Survey (CPS), and industry-level prices from the Bureau of Labor Statistics (BLS), I find that occupations with more cyclical employment have less cyclical wages, and the prices of the goods that they produce are also less cyclical.

These findings are in line with the theoretical model, introduced in Section 3. The model features many heterogeneous industries and households. Households differ in their consumption preferences, labor endowments, ownership of industries and capital assets, wage rigidity, and labor supply elasticity. Industries hire different bundles of labor types, capital assets, and intermediate inputs, and they face different nominal rigidities and demand elasticities. Workers are allowed to move freely across industries, but they cannot change the type of labor they are endowed with. Labor types thus represent segmented labor markets, which could correspond to different occupations, or industry-occupation pairs, demographic groups, residents of different regions, etc. Capital assets in the model represent any fixed or semi-fixed factors of production (such as equipment, land, structures, but also entrepreneurial know-how, etc.).

In Section 3 I log-linearize the model, and show that the equilibrium can be summarized by three equations: the Phillips curves (a supply equation), a relative demand equation, and an aggregate demand equation. The Phillips curve and the relative demand equation relate industry-level prices with the relative employment of different primary factors and the aggregate output gap. These relations are valid regardless of the policy instruments or policy rules that determine the output gap, which instead is pinned down by the aggregate demand equation.

The Phillips curves relate inflation in the various industries with the employment gaps of all primary factors. In turn, the employment of primary factors depends on the aggregate output gap and on relative prices, as captured by the relative demand equation. Combining the Phillips curves with the relative demand equation allows me to isolate the impact and propagation effects of monetary policy on employment and consumption. I show that a monetary expansion increases the demand for all goods and factors proportionally on impact, because all final users increase spending proportionally on all goods.[1] This uniform impact effect, however, gives rise to different propagation effects across households in general equilibrium. This is because changes in aggregate demand affects relative prices (as captured by the Phillips curves), which in turn affect the relative demand for primary factors and the relative income of households (through the relative demand equation). Changes in relative demand then feed back into relative prices, and so on. The equilibrium fixed point is given by a cross-sectional multiplier, which depends on the product of the Phillips curve slope with the relative demand elasticity.

Like in the representative agent benchmark (Gali (2015); Woodford (2003)), the Phillips curve slope depends on sectoral nominal rigidities and factor supply elasticities. Heterogeus nominal rigidities or supply elasticities (in a network-adjusted sense) are necessary for monetary policy to affect relative prices. Factors that are supplied more elastically (or are complementary with elastically supplied factors), or that are employed by sectors with stickier prices (or whose customers have stickier prices, etc.), have flatter Phillips curves, so that the relative price of the goods that they produce falls after a monetary expansion. Therefore producers and consumers shift expenditure towards these goods, as governed by the relevant demand elasticities, thereby boosting the employment of flat-Phillips-curve factors.

By contrast, if all primary factors face the same supply elasticity and the same degree of (network-adjusted) nominal rigidity, to a first order there are no redistributive effect coming from home bias in consumption, or from heterogeneous size and centrality or demand elasticities across industries.

A key implication of the theory is that there is a negative cross-sectional correlation between the response of prices and employment to monetary policy, in line with the evidence in Section 2. This result has a clear parallel in the representative agent benchmark, where economies with a flatter (aggregate) Phillips curve exhibit more monetary non-neutrality. A flatter Phillips curve is like a flatter supply equation, so that prices respond less, and output responds more, to the demand shocks induced by monetary policy – both in the aggregate and in the cross section.

While the Phillips curves are sufficient to determine the direction of cross-sectional employment responses to monetary policy, the direction of income responses is ambiguous, as it depends on the workers' ownership share of primary factors and profits from the various industries, and on an appropriate notion of the aggregate elasticity of substitution between primary factors. In fact, factor and profit income depend on both quantities (employment, output) and prices (including wages), and we just established that the two are negatively correlated in the cross-section. The employment effect dominates if and only if primary factors are aggregate substitutes.

These cross-sectional effects of monetary policy also have aggregate implications. Proposition 5 shows that, like in models with a single primary factor, the aggregate monetary non-neutrality depends on the slope of the Phillips curve for the GDP deflator. However, with multiple heterogeneous primary factors, the GDP deflator is less sensitive

[1] The assumptions of homothetic preferences and production, uniform MPCs, and no portfolio heterogeneity are key to this result.

to the employment gaps of primary factors with flatter Phillips curves, while employment gets reallocated precisely towards these factors (the negative correlation between price and employment that we just discussed). Through a composition effect, this leads to a flatter relation between aggregate employment and the GDP deflator, compared to an economy with a single factor of production. In turn, a flatter aggregate Phillips curve means more aggregate monetary non-neutrality (the GDP deflator responds less, and aggregate employment responds more to monetary policy).

Nonetheless, the aggregate implications of household heterogeneity are much smaller than the cross-sectional ones. In fact, the difference between aggregate non-neutrality in the representative agent approximation vs the heterogeneous agents economy is a quadratic function of the cross-sectional dispersion in price rigidities and factor supply elasticities, which is bound to be small for reasonable parameter values.[2] By contrast, the cross-sectional effects are a linear function of the underlying parameter heterogeneity.

In Section 5 I show that, when there is no heterogeneity in the supply elasticity of primary factors, the cross-sectional effects of monetary policy on employment and income can be fully undone by using an appropriate combination of production and consumption taxes. First, zero-sum production subsidies must be set to replicate the change in relative marginal costs across industries that would result from a proportional increase in the wages of all primary factors. Second, zero-sum lump-sum taxes must be set so that the agents' incomes move proportionately. Under this tax and transfer scheme monetary policy has no cross-sectional effects, and the evolution of all aggregate variables is the same as in the representative-agent equilibrium. However this is a complex policy scheme, which requires knowledge of the full input-output structure and of the agents' ownership share in the various industries.

Section 6 calibrates the model to the US economy, finding large redistributive effects of monetary policy in the cross-section. I measure household-level non-neutrality based on the impulse response of employment to a 1% monetary shock. The impact response varies from 0.25% to 1.1% across occupations, and the cumulative impulse-responses exhibit a similar dispersion across occupations. Heterogeneity is not driven by differences in wage rigidity or labor supply elasticity directly, but rather by different nominal rigidities in the sectors that hire different occupations. Moreover, input-output linkages significantly amplify the cross-sectional effects. The range of impact responses falls from 0.86% in the full calibration to 0.35% in a counterfactual economy with no input-output linkages.[3]

1.1 Related Literature

Adopting a similar input-output framework, but with flexible prices, Baqaee and Farhi (2018) study the propagation and aggregation of exogenous productivity and markup shocks in economies with multiple agents and a general input-output network. Different from Baqaee and Farhi (2018), in my framework markups change endogenously due

[2] Note that price adjustment probabilities are bounded between 0 and 1, while estimates of labor supply elasticities across occupations fall into a relatively narrow range.

[3] Like in representative agent models, the presence of input output linkages also flattens the Phillips curve, thereby increasing aggregate non-neutrality.

to sticky prices. I also provide an explicit solution for changes in relative employment and income across households, while Baqaee and Farhi (2018) characterize them only implicitly.

In a related paper, La'O and Morrison (2023) study optimal monetary policy when earning inequality is (exogenously) correlated with aggregate demand. In this paper I show that monetary policy endogenously affects earnings inequality, but do not solve for the optimal policy.

The HANK literature (Kaplan et al. (2018), Auclert (2019), Auclert et al. (2021)) also considers New Keynesian frameworks with heterogeneous households, focusing on the effects of monetary policy across households through their consumption-saving decisions. In this paper instead I focus on the endogenous redistributive effects of monetary policy on the households' labor demand and consumption prices.

A large literature studies the implications of input-output networks for aggregate monetary non-neutrality in models with only one factor of production (Basu (1995), Carvalho (2006), Nakamura and Steinsson (2010), Pasten et al. (2019), LaO and Tahbaz-Salehi (2019), Rubbo (2023)), while the currency union literature (Aoki (2001); Benigno (2004), Devereux and Engel (2003), Huang and Liu (2007), Gali and Monacelli (2008)) allows for multiple primary factors – the workers in each country – in a simpler Armington-style production structure with symmetric nominal rigidities and labor supply elasticities. This paper jointly accounts for multiple primary factors in production and a quantitatively realistic input-output network, proposing a novel characterization of cross-sectional monetary non-neutrality which uses industry-by-primary factor Phillips curve slopes as sufficient statistics.

Based on detailed micro-level data from Denmark and Sweden, Huber et al. (2020) and Coglianese et al. (2021) empirically document heterogeneous effects of monetary policy when households derive income and consumption from different industries. Minton and Wheaton (2022) also document heterogeneous effects of monetary policy across states with different downward wage rigidity, as proxied by more or less binding minimum wages. The empirical results are in line with the theoretical predictions of this paper.

2 Stylized facts

Based on data for the US, this Section documents heterogeneity in the degree of nominal rigidity faced by workers in different occupations, both on the consumption side, through the price stickiness of the goods they buy, and on the employment side, through the price stickiness of the goods they make. It then moves on to connect these micro-level facts with the cyclical volatility of prices and employment. It first documents large heterogeneity in the cyclical volatility of industry-level prices, which – as expected – is strongly correlated with estimates of industry-level price flexibility based on micro data. It then documents significant heterogeneity also in the cyclicality of employment across different occupations, and shows that it is negatively correlated with the average price cyclicality of the goods that they produce.

2.1 Data

Throughout the analysis, we follow the BEA 3-digit industry classification and the IPUMS 2010 2-digit classification of occupations, excluding the non-employed and active military. This leaves us with 71 industries and 22 occupations.

Occupation level expenditure shares on different consumption goods are computed from the Consumer Expenditure Survey (CEX) for the year 2007, following the methodology in Borusyak and Jaravel (2021). The probability that workers in each occupation are hired by each industry is computed based on the American Community Survey (ACS) for the year 2012. Estimates of industry-level price adjustment probabilities are computed by Pasten et al. (2017), based on micro data underlying sectoral price series published by the BLS. Estimates of occupation-level wage rigidity are based on Grigsby et al. (2019). Grigsby et al. (2019) collect the best available data on wage adjustment frequencies, but nonetheless they can only report wage adjustment probabilities for five aggregated industries, and not by occupation. Occupation level wage adjustment probabilities are proxied for by averaging industry level probabilities, weighted by the probability that each occupation is hired by the given industry.

Inflation series at the industry level are based on the Producer Price Index (PPI) database published by the BLS. Time series of occupation-level employment, hours worked, hourly wages and weekly earnings are provided by the Current Population Survey (CPS).

2.2 Nominal rigidities across occupations

Figure 1 shows that there is substantial heterogeneity in the price adjustment frequency of the industries which hire different occupations. This is computed by averaging industry level frequencies, weighted by the probability that a given occupation is hired by each industry.

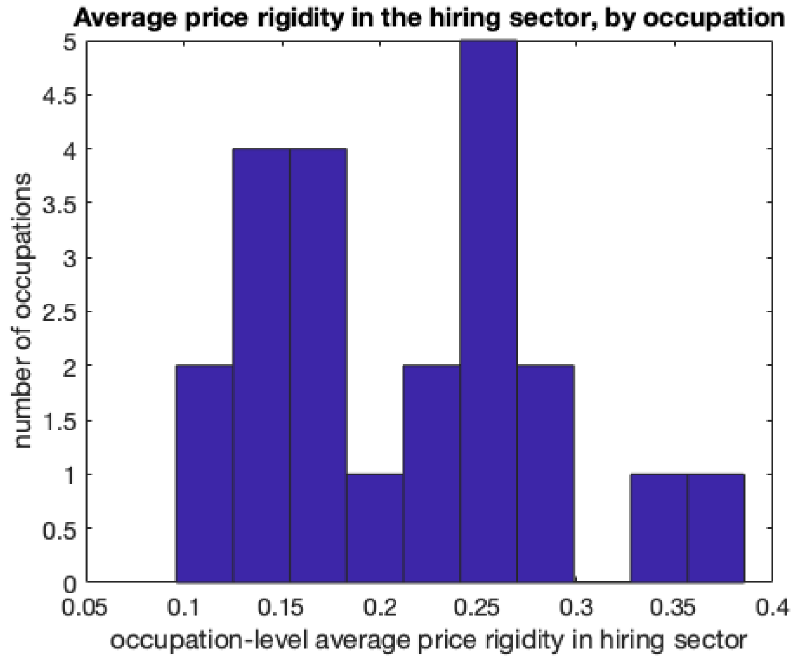

Figure 1: Average price adjustment probability in the hiring sector, by occupation

Based on the available data, other elements of heterogeneity seem less relevant. Figure 2 plots the average price adjustment probability of goods purchased by individuals in different occupations, weighted by consumption shares.

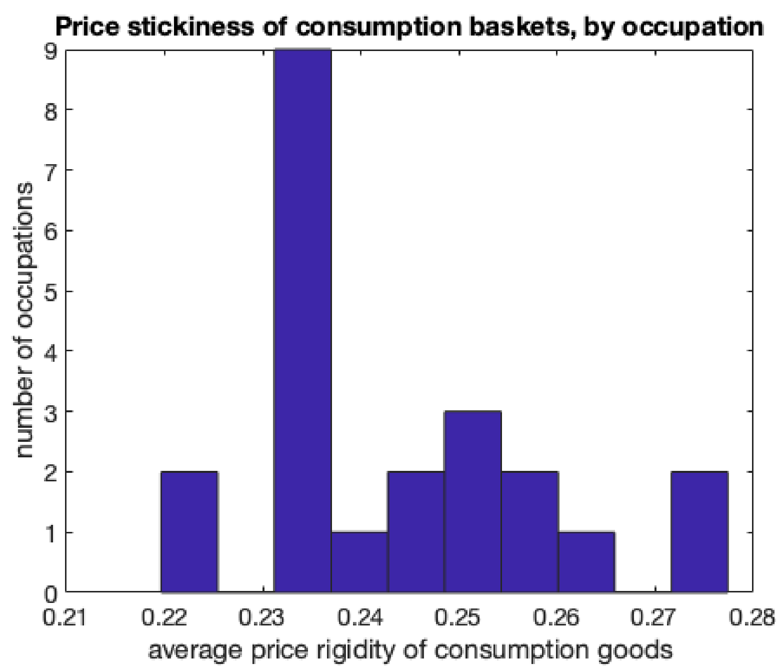

Figure 2: Average price stickiness of goods consumed, by occupation

The figure displays little heterogeneity, suggesting that this is not an important driver of cross-sectional non-neutrality from a quantitative point of view. Finally, Figure 3 plots the distribution of occupation-level wage rigidity computed based on Grigsby et al. (2019). Again, there is relatively little heterogeneity across occupations, likely due to lack of detailed occupation level data.

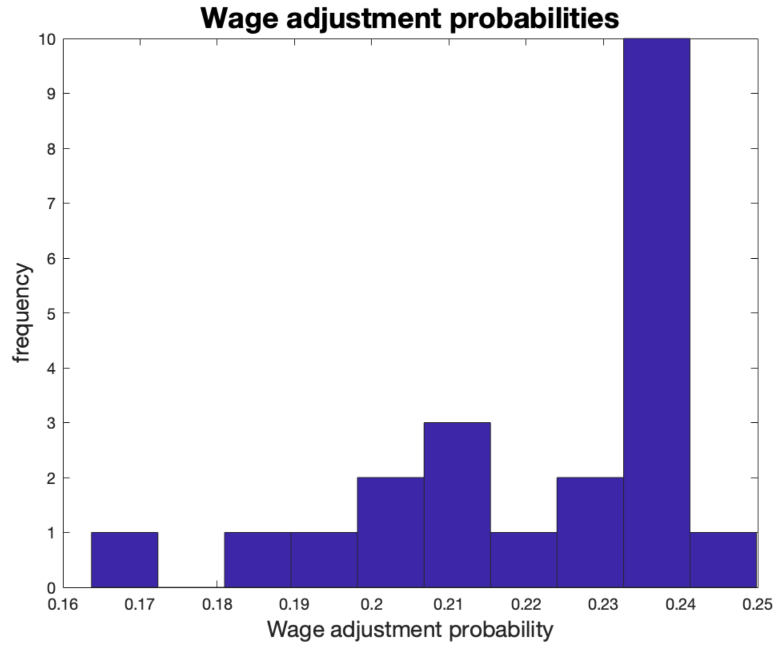

Figure 3: Distribution of wage adjustment probabilities across occupations

2.3 Cross-sectional cyclicality of prices and employment

This section presents a cross-sectional revisit of business cycle facts, in the spirit of Cooley and Prescott (1995). The first column of Tables 1 and 2 report standard deviations of hp-filtered employment and wages at the occupation level, while the remaining columns report their correlations with leads and lags of aggregate employment.

Figures 4 through 6 highlight some notable features of the tables.

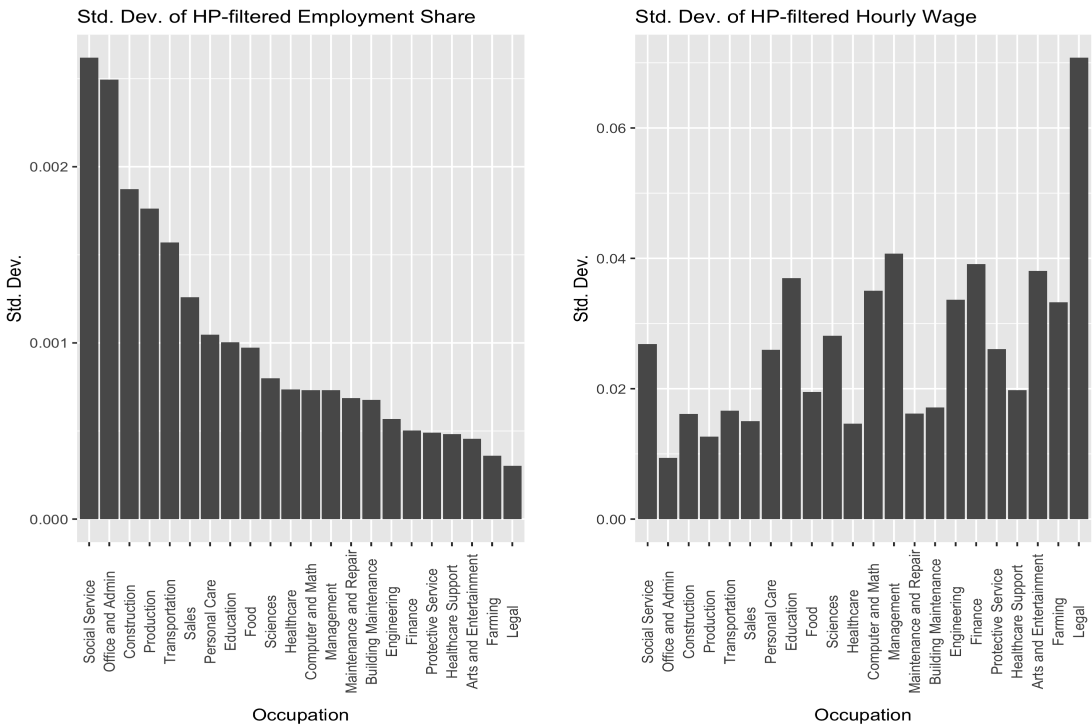
Std. Dev. of HP-filtered Employment Share
Std. Dev.
0.002
0.001
0.000
Social Service
Office and Admin
Construction
Production
Transportation
Sales
Personal Care
Education
Food
Sciences
Healthcare
Computer and Math
Management
Maintenance and Repair
Building Maintenance
Engineering
Finance
Protective Service
Healthcare Support
Arts and Entertainment
Farming
Legal
Occupation
Std. Dev. of HP-filtered Hourly Wage
Std. Dev.
0.06
0.04
0.02
0.00
Social Service
Office and Admin
Construction
Production
Transportation
Sales
Personal Care
Education
Food
Sciences
Healthcare
Computer and Math
Management
Maintenance and Repair
Building Maintenance
Engineering
Finance
Protective Service
Healthcare Support
Arts and Entertainment
Farming
Legal
Occupation

Figure 4:

Series	Occupation	Std Dev	Lag 4	Lag 3	Lag 2	Lag 1	Contemp	Lead 1	Lead 2	Lead 3	Lead 4
Employment Share	Construction	0.0019	0.30	0.47	0.59	0.72	0.82	0.81	0.73	0.62	0.50
	Production	0.0018	0.09	0.29	0.50	0.68	0.79	0.76	0.67	0.54	0.38
	Sales	0.0013	0.26	0.37	0.43	0.51	0.62	0.57	0.52	0.46	0.33
	Transportation	0.0016	0.37	0.41	0.43	0.45	0.43	0.38	0.24	0.08	-0.03
	Engineering	0.0006	0.36	0.44	0.49	0.44	0.41	0.33	0.18	0.07	0.02
	Maintenance and Repair	0.0007	0.14	0.25	0.32	0.37	0.40	0.39	0.38	0.36	0.29
	Sciences	0.0008	0.35	0.39	0.40	0.41	0.38	0.30	0.20	0.14	0.11
	Management	0.0007	0.14	0.23	0.30	0.35	0.38	0.31	0.26	0.17	0.02
	Social Service	0.0026	0.59	0.57	0.53	0.45	0.36	0.21	0.05	-0.10	-0.21
	Office and Admin	0.0025	-0.19	-0.06	0.07	0.17	0.24	0.30	0.33	0.35	0.32
	Building Maintenance	0.0007	0.02	0.09	0.11	0.15	0.21	0.22	0.24	0.27	0.26
	Arts and Entertainment	0.0005	0.02	0.02	0.05	0.13	0.17	0.15	0.10	0.05	-0.02
	Legal	0.0003	-0.18	-0.13	-0.05	0.00	0.05	0.10	0.18	0.23	0.26
	Education	0.0010	0.08	0.04	0.01	-0.01	-0.04	-0.06	-0.10	-0.12	-0.14
	Personal Care	0.0010	-0.13	-0.09	-0.06	-0.05	-0.04	-0.02	0.01	0.06	0.12
	Farming	0.0004	-0.15	-0.14	-0.12	-0.10	-0.07	-0.01	-0.01	0.05	0.07
	Food	0.0010	-0.19	-0.17	-0.10	-0.08	-0.08	-0.02	0.02	0.05	0.09
	Finance	0.0005	-0.08	-0.13	-0.16	-0.15	-0.12	-0.12	-0.08	-0.04	-0.07
	Computer and Math	0.0007	0.04	-0.03	-0.08	-0.10	-0.14	-0.13	-0.11	-0.11	-0.08
	Healthcare Support	0.0005	-0.06	-0.08	-0.13	-0.17	-0.19	-0.20	-0.14	-0.06	0.01
	Healthcare	0.0007	-0.08	-0.19	-0.26	-0.28	-0.27	-0.20	-0.17	-0.12	-0.10
	Protective Service	0.0005	-0.32	-0.35	-0.35	-0.35	-0.29	-0.22	-0.11	0.02	0.08

Table 1: Correlation values of employment share with FRED employment share.

Series	Occupation	Std Dev	Lag 4	Lag 3	Lag 2	Lag 1	Contemp	Lead 1	Lead 2	Lead 3	Lead 4
Hourly	Construction	0.0161	0.15	0.07	-0.01	-0.07	-0.18	-0.29	-0.37	-0.43	-0.47
Wage	Production	0.0127	0.06	0.01	0.00	-0.07	-0.10	-0.14	-0.19	-0.19	-0.22
	Sales	0.0150	-0.15	-0.11	-0.08	-0.05	-0.01	0.05	0.04	0.08	0.06
	Transportation	0.0166	-0.15	-0.14	-0.15	-0.19	-0.18	-0.16	-0.18	-0.17	-0.16
	Engineering	0.0336	-0.01	0.00	0.00	-0.01	-0.01	-0.02	0.01	0.02	0.02
	Maintenance and Repair	0.0162	-0.00	0.07	0.15	0.15	0.18	0.12	0.02	-0.06	-0.15
	Sciences	0.0281	-0.12	-0.07	0.01	-0.01	0.01	0.01	-0.01	-0.03	-0.05
	Management	0.0407	-0.03	0.03	0.04	0.07	0.11	0.11	0.09	0.08	0.12
	Social Service	0.0268	-0.14	-0.13	-0.10	-0.08	-0.05	-0.05	-0.02	-0.05	-0.04
	Office and Admin	0.0094	-0.37	-0.32	-0.23	-0.18	-0.16	-0.16	-0.17	-0.13	-0.14
	Building Maintenance	0.0171	-0.04	-0.02	0.05	0.04	-0.02	-0.06	-0.10	-0.11	-0.18
	Arts and Entertainment	0.0381	-0.03	-0.05	-0.04	-0.01	-0.03	0.01	-0.02	-0.02	-0.04
	Legal	0.0708	-0.03	-0.03	-0.02	-0.01	-0.03	-0.06	-0.08	-0.11	-0.09
	Education	0.0370	-0.07	0.01	0.05	0.13	0.15	0.16	0.15	0.12	0.08
	Personal Care	0.0259	-0.06	-0.03	0.01	0.04	0.05	0.05	0.07	0.06	0.00
	Farming	0.0333	-0.17	-0.12	-0.06	-0.08	-0.08	-0.12	-0.17	-0.17	-0.19
	Food	0.0195	-0.06	-0.01	0.00	0.03	-0.01	-0.04	-0.08	-0.04	-0.05
	Finance	0.0391	-0.04	0.05	0.09	0.06	0.05	0.05	0.09	0.08	0.11
	Computer and Math	0.0350	-0.05	-0.04	0.04	0.05	0.05	0.05	0.06	0.03	0.02
	Healthcare Support	0.0198	-0.03	-0.01	-0.05	-0.06	-0.08	-0.15	-0.15	-0.12	-0.11
	Healthcare	0.0147	-0.17	-0.16	-0.09	-0.09	-0.04	-0.08	-0.10	-0.10	-0.15
	Protective Service	0.0261	0.15	0.18	0.22	0.19	0.16	0.15	0.04	0.00	-0.08

Table 2: Correlation values of employment share with FRED employment share.

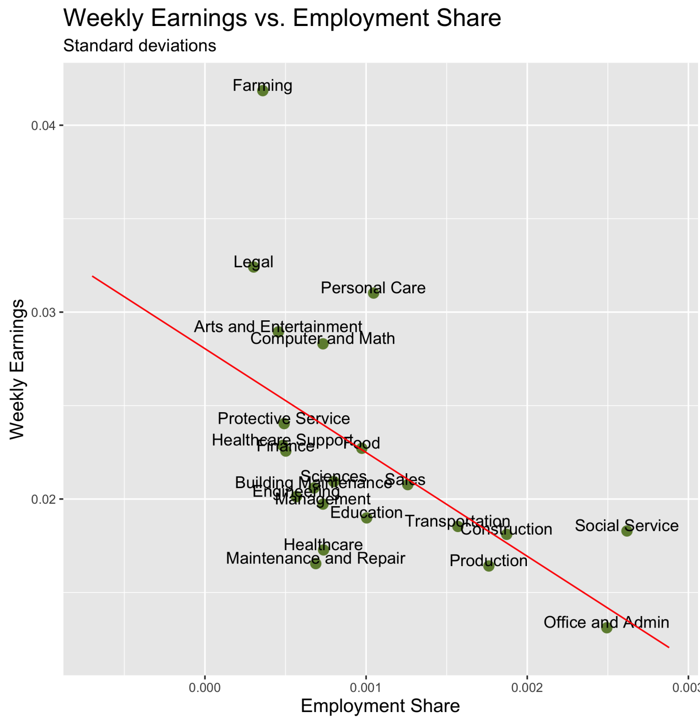

Figure 5:

Figure 4 shows that there is significant heterogeneity in the cyclical volatility of employment and wages at the occupation level, and the two are negatively correlated (figure (5)). A similar patterns emerges when measuring the correlation of the cyclical component of occupation-level employment and wages with aggregate employment. Occupations with more positive employment correlation tend to have more negative wage correlation (figure 6).

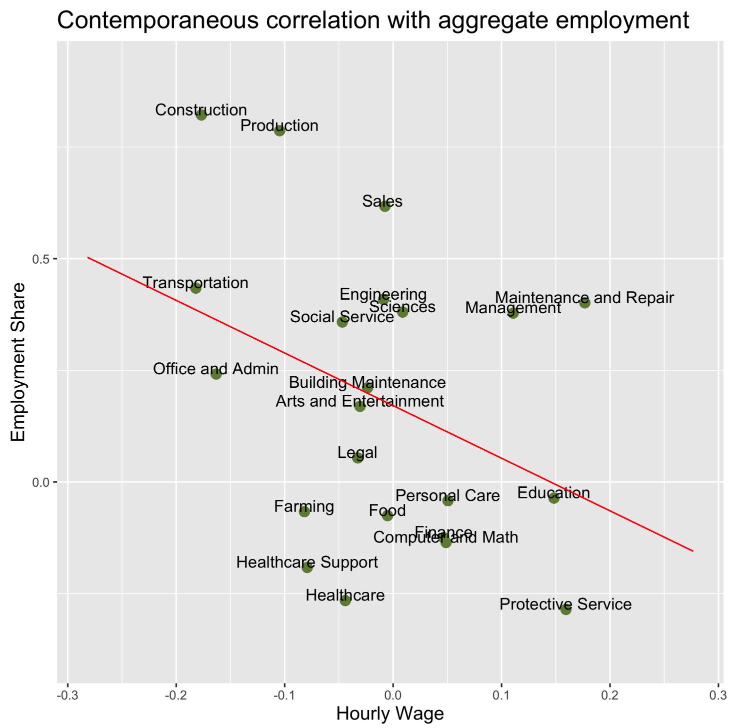

Figure 6:

The negative relation between the volatility of prices (wages) and quantities (employment) in Figure 4 and 6 presents an analogy with aggregate business cycle comovement, and it is consistent with the implications of the theoretical model presented below.

3 General environment

This section sets up a New Keynesian model with monopolistic competition and sticky prices, featuring multiple heterogeneous industries and primary factors. Sections 3.1 through 3.3 lay out the assumptions about preferences, production and policy instruments and derive optimality conditions for consumers and producers; section 3.4 defines the general equilibrium; and sections 3.5 and 3.6 describe the log-linearized model.

Figure 7 provides a schematic representation of the economy.

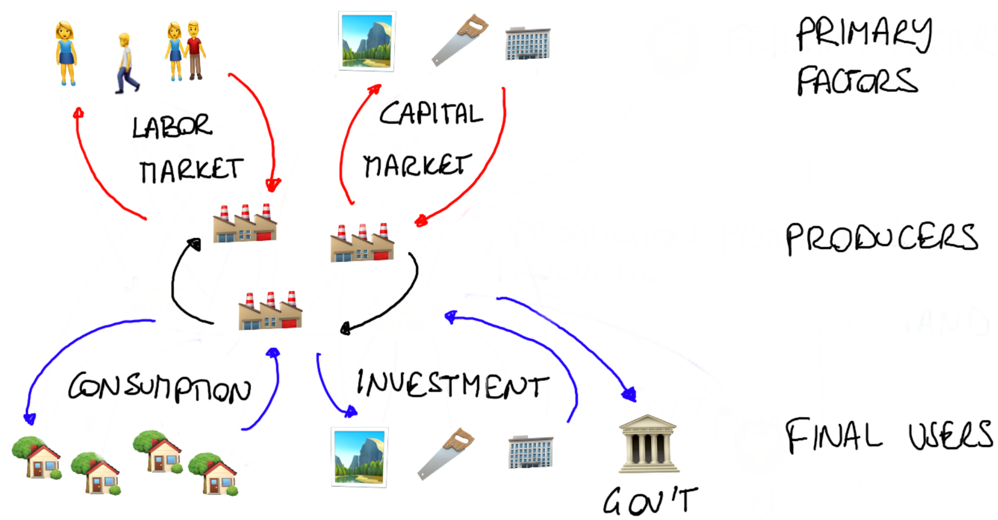

Figure 7: Schematic representation of the economy

3.1 Final users

The final users are households, investment providers, and the government. There are N_w household types and N_f capital asset types, each with a corresponding set of investment providers. Final users are indexed by h or $f \in \{1, ..., N_w + N_f + 1\}$. The set of households is denoted by $\mathcal{N}_w$, and the set of capital types is denoted by $\mathcal{N}_f$. The price of factor h is W_h, and the quantity supplied is L_h.

Remark 1. Different types of labor and capital need not be employed by different industries. Rather, they correspond to segmented factor markets. For example, worker types could represent regions or occupations between which there are no worker flows at business cycle frequencies. Likewise, capital assets could represent equipment and structures which cannot be repurposed in the short run.

3.1.1 Households

Preferences Each household type $h \in \mathcal{N}_w$ has a representative agent, who supplies a distinct labor type and whose per-period preferences are described by the utility function

$$U_{ht} = \frac{C_{ht}^{1-\gamma_h}}{1-\gamma_h} - \frac{1}{\mathcal{E}_h} \frac{L_{ht}^{1+\varphi_h}}{1+\varphi_h} \tag{1}$$

All households enjoy consumption (C) and dislike labor (L), with heterogeneous income effects in labor supply γ_h and Frisch elasticities $\frac{1}{\varphi_h}$. Households also face type-specific shocks $\mathcal{E}_h$ to their relative preference for leisure. Consumption aggregators $C_{ht} \equiv \mathcal{C}_{ht}\left(c_{1ht}, ..., c_{Nht}\right)$ are homothetic over the N goods produced in the economy, and can differ across households. Consumption preferences $\mathcal{C}_{ht}$ are time-varying, to account for changes in expenditure shares across goods at constant prices.

Budget constraints Households maximize the present discounted value of per-period utility flows, with a common discount factor ρ, subject to type-specific budget constraints

$$P_{ht}^{C} C_{ht} \leq W_{ht} L_{ht} + \sum_{f \in \mathcal{N}_f} \mathcal{W}_{fh} W_f L_f + \sum_{i=1}^{N} \Xi_{ih} \left(\Pi_{it} - \mathcal{T}_{it} \right) + T_{ht} \tag{2}$$

where P_{ht}^{C} is the price index implied by the consumption aggregator $\mathcal{C}_h$, W_{ht} is the nominal wage earned by workers of type h, and T_h is a lump-sum transfer received by type h.

Each household type h owns shares in all the N industries in the economy, as described by the ownership matrix Ξ. The elements Ξ_{ih} denote the share of profits from industry i accruing to type-h agents. Profits Π_{it}, net of lump-sum taxes $\mathcal{T}_{it}$ paid by firms to the government, are allocated according to ownership shares. Agents also own shares of the N_f capital assets in the economy, as described by the matrix $\mathcal{W}$. The elements $\mathcal{W}_{fh}$ denote the share of income from asset f that is rebated to households of type h.

Remark 2. We assume that agents cannot borrow and lend between each other, so that the transfers $\{T_h\}_{h \in \mathcal{N}_w}$ are exogenously chosen by the government. The inflation decomposition in Section **??** would remain valid if these transfers were determined endogenously by the agents' borrowing and lending decisions.

Consumption-leisure tradeoff The optimal consumption-leisure tradeoff satisfies the first order condition

$$C_{ht}^{\gamma_h} L_{ht}^{\varphi_h} = \mathcal{E}_h \frac{W_{ht}}{P_{ht}^{C}} \tag{3}$$

As explained in detail in Section 3.2 below, the flexible nominal wage W_{ht} is defined as the value W_{ht} which satisfies the consumption-leisure tradeoff (3) given consumption, labor demand, prices, and $\mathcal{E}_h$. Importantly, W_{ht} has no data counterpart. The model counterpart of wages in the data is the sticky nominal wage paid by the firms, introduced in Section 3.2 below.

3.1.2 Capital utilization

I adopt a stylized model of investment, with the objective to deliver capital supply curves with constant elasticity φ_f. Importantly, to simplify the analysis I abstract from the intertemporal dimension of the investment problem, which would determine a different sensitivity of investment and consumption demand to monetary policy. I assume that investment fully depreciates within one period, so that changes in interest rates have no direct effect on the relative price of consumption and investment goods.

Each capital asset f is produced by combining a fixed endowment ($\bar{K}_f$) with an investment good I_f. At each period, the supply of asset f is given by (omitting time subscripts for legibility)

$$L_f = \left[\left(1 + \varphi_f \right) I_f \right]^{\frac{1}{1+\varphi_f}} \mathcal{E}_f \bar{K}_f$$

where $\mathcal{E}_f$ is a shock to the supply of capital. For convenience I assume that the investment component I_f fully depreciates from one period to the next, while the endowment component $\bar{K}_f$ never depreciates.

In turn investment is produced with constant returns to scale, using as inputs a combination of primary factors (L_{fht}) and intermediate goods (X_{fit}), according to the production function

$$I_{ft} = G_{ft}(\{L_{fht}\}, \{X_{fit}\})$$

The production function G_{ft} is time-varying, to allow for changes in expenditure shares at constant prices.

There are N_f investment producing sectors, one for each asset type f, who sell the investment good at marginal cost P_f^I to capital retailers. Retailers purchase capital endowments from the agents, combine them with the investment good, and sell capital services to the firms at a rental rate W_f in a perfectly competitive market. Capital retailers are owned by the agents in proportion to their ownership shares $\mathcal{Z}$ in the capital endowments, and rebate their profits accordingly.

Profit maximization yields the capital supply curves

$$U_f^{\varphi_f} = \frac{W_f}{P_f} \mathcal{E}_f \bar{K}_f \tag{4}$$

where

$$U_f \equiv \left[(1+\varphi_f) I_f\right]^{\frac{1}{1+\varphi_f}}$$

can be interpreted as a measure of capital utilization. As a result, the payments $W_f L_f$ to factor f are divided between investment expenditures, given by

$$\frac{1}{1+\varphi_f} W_f L_f \tag{5}$$

and profits of the investment producers, given by

$$\frac{\varphi_f}{1+\varphi_f} W_f L_f \tag{6}$$

Profits in turn are rebated to households according to their ownership shares of investment firms, denoted by $\mathcal{Z}$. Hence the share of income from factor $f \in \mathcal{N}_f$ rebated to households of type $h \in \mathcal{N}_w$ is given by $\mathcal{W}_{fh} = \mathcal{Z}_{fh} \frac{\varphi_f}{1+\varphi_f}$, while the share of income from factor f that goes to investment into factor $f' \in \mathcal{N}_f$ is $\mathcal{W}_{ff'} = \frac{1}{1+\varphi_f}$ if $f = f'$ and $\mathcal{W}_{ff'} = 0$ otherwise.

Like flexible wages in the consumption-leisure tradeoff (3), flexible rental rates are defined as the value W_{ft} that satisfies the utilization equation (3) given capital demand and prices. These flexible rental rates have no data counterpart. The model counterpart of the rental rates measured in the data is the sticky rental rate paid by the firms, introduced in Section 3.2 below.

3.2 Production

There are N good-producing industries in the economy (indexed by $i, j \in \{1, ..., N\}$). Within each industry there is a continuum of firms, producing differentiated varieties.

All firms z in industry i have the same constant returns to scale production function

$$Y_{izt} = A_{it} G_i(\{L_{ihzt}\}_{h \in \mathcal{N}_h \cup \mathcal{N}_f}, \{X_{ijzt}\}_{j=1}^{N}; \{\mathcal{X}_{ijt}\})$$

where L_{ihzt} is the quantity of primary factor h hired by firm z in industry i at time t, and X_{ijzt} is the quantity of intermediate input j used by the firm. The Hicks-neutral productivity shifter A_{it} is industry-specific, and varies over time. The matrix $\mathcal{X}$ denotes shocks to the production functions which determine exogenous changes in expenditure shares at constant prices:

$$\mathcal{X}_{ijt} \equiv d \log \omega_{ijt} \mid_{\text{const. prices}}$$

Customers (consumers and other producers) buy a CES bundle of sectoral varieties, with elasticity of substitution ϵ_i. The industry output is given by

$$Y_i = \left(\int Y_{if}^{\frac{\epsilon_i - 1}{\epsilon_i}} df \right)^{\frac{\epsilon_i}{\epsilon_i - 1}}$$

and the implied sectoral price index is

$$P_i = \left(\int P_{if}^{1-\epsilon_i} df \right)^{\frac{1}{1-\epsilon_i}}$$

The government provides time-varying proportional input subsidies, fully financed through lump-sum taxes on profits. We follow a standard practice in the literature, and assume that steady-state subsidies eliminate the markup distortions arising from monopolistic competition. Subsidies are such that at time t sector i pays a share

$$(1 - \tau_{it}) \frac{\epsilon_{it} - 1}{\epsilon_{it}}$$

of its actual marginal cost. With this notation, $\tau_i^* = 0$ in steady state.

All producers minimize costs given input prices. With constant returns to scale marginal costs are the same for all firms within a sector i, and they all use inputs in the same proportions. The marginal cost of sector i, denoted by MC_i, is the solution of the cost minimization problem (omitting time subscripts for legibility)

$$MC_i = min_{\{X_{ij}\},\{L_{ih}\}} \sum_{h \in \mathcal{N}_h \cup \mathcal{N}_f} W_h L_{ih} + \sum_{j=1}^{N} P_j X_{ij} \quad s.t. \; A_i G_i \left(\{L_{ih}\}_{h \in \mathcal{N}_h \cup \mathcal{N}_f}, \{X_{ij}\}_{j=1}^{N}; \{\mathcal{X}_{ij}\} \right) = 1 \qquad (7)$$

Producers are subject to nominal rigidities, modeled à la Calvo. In every sector i and at each period, only a randomly-selected fraction δ_i of the firms can update their price. They set it to maximize the present discounted value of future profits, conditional on being unable to update their price:

$$P_{it}^{*}=\frac{\epsilon_{it}}{\epsilon_{it}-1}\frac{\mathbb{E}\sum_{s}\left[SDF_{t+s}\left(1-\delta_{i}\right)^{s}Y_{izt+s}(P_{it}^{*})\left(1-\tau_{it+s}\right)MC_{it+s}\right]}{\mathbb{E}\sum_{s}\left[SDF_{t+s}\left(1-\delta_{i}\right)^{s}Y_{izt+s}(P_{it}^{*})\right]} \tag{8}$$

where $SDF_{t+s}=\rho^{s}\frac{U_{ct+s}}{U_{ct}}\frac{P_{t}^{c}}{P_{t+s}^{c}}$ is the households' stochastic discount factor, and demand functions are given by $Y_{izt+s}(P_{zt})=Y_{it+s}\left(\frac{P_{zt}}{P_{it+s}}\right)^{-\epsilon_{it}}$. The firms z who cannot adjust their price instead keep it unchanged ($P_{izt}=P_{izt-1}$), and must absorb any cost changes in their markup $\mathcal{M}_{izt}$.

Sticky factor prices To model sticky factor prices I assume that primary factors (workers and capital assets) are first purchased by marketplaces, who then sell their services to producers in all the different sectors. Each marketplace deals with only one primary factor.

Marketplaces are treated like any other industry. In particular there is a continuum of marketplaces for each type, with fixed unit mass, facing Calvo-style price rigidities. The marketplaces rebate profits to final users according to their ownership shares of primary factors $\mathcal{W}$.

Primary factor types are fixed, hence wages differ across primary factors. Nonetheless workers and investment providers can freely move across marketplaces, therefore all units of a given primary factor earn the same flexible wage. By contrast, different marketplaces who hire the same factor type might charge different prices, due to the Calvo friction. We assume that ownership of each marketplace is equally shared among the relevant final users, so that in practice all units of each primary factor h earn the same (sticky) wage, equal to the average price charged by h-specific marketplaces.

Retailers To streamline the notation below, it is convenient to augment the set of industries with one retailer for each final user. Retailers assemble consumption, investment and government spending bundles and sell them to the relevant final user, to whom they also rebate their profits. Importantly, these fictitious retailer sectors are distinct from actual retailers in the data. Their price is equal to the price index for the relevant final user. Without loss of generality, we model relative preference shocks for consumers, investment producers, or the government, as relative demand shocks in the corresponding retailers' production function.

Remark 3. Given our modeling of retailers and factor marketplaces, the overall number of sectors in the economy is $\bar{N}=N+2N_{w}+2N_{f}+1$ (the actual production sectors, the factor marketplaces for labor and capital, and the retailers for consumption, investment and government spending).

3.2.1 Aggregation

Our goal is to relate changes in aggregate inflation, as measured by the GDP deflator, with changes in aggregate output, as measured by real GDP. These aggregate quantities are defined below. Changes in real GDP and the GDP deflator are measured relative to the initial steady-state, denoted by starred variables.

Definition 1. Nominal GDP is given by total final expenditures:

$$GDP = \sum_{h \in \mathcal{N}_w} P_h^C C_h + \sum_{f \in \mathcal{N}_f} P_f^I I_f + G_t$$

Definition 2. Infinitesimal changes in real GDP around the initial steady-state are denoted by $d \log Y_t$, and are equal to the share-weighted sum of changes in aggregate consumption, aggregate investment, and government spending:

$$d \log Y_t = \sum_{h \in \mathcal{N}_w} \frac{P_h^{*C} C_h^*}{GDP^*} d \log C_{ht} + \sum_{f \in \mathcal{N}_f} \frac{P_f^* I_f^*}{GDP^*} d \log I_{ft} + \frac{dG_t}{GDP^*}$$

Definition 3. Infinitesimal changes in the GDP deflator around the initial steady-state are denoted by $d \log P_t^Y$, and are equal to the share-weighted change in the price indices of the final users:

$$d \log P_t^Y = \sum_{h \in \mathcal{N}_w} \frac{P_h^{*C} C_h^*}{GDP^*} d \log P_{ht}^C + \sum_{f \in \mathcal{N}_f} \frac{P_f^{*C} I_f^*}{GDP^*} d \log P_{ft}^C + \frac{G}{GDP^*} d \log P_G$$

Remark 4. Around an efficient equilibrium, changes in real GDP equal the income-weighted sum of changes in the quantities of primary factors, plus the change in aggregate productivity:

$$d \log Y_t = \sum_{h \in \mathcal{N}_w \cup \mathcal{N}_f} \frac{W_h^* L_h^*}{\sum_{f \in \mathcal{N}_w \cup \mathcal{N}_f} W_f^* L_f^*} d \log L_{ht} + \sum_{i=1}^{N} \frac{P_i^* Y_i^*}{GDP^*} d \log A_{it}$$

3.3 Monetary policy

At each period the economy is subject to a cash-in-advance constraint, whereby the aggregate nominal GDP cannot exceed the money supply M_t chosen by the central bank:

$$P_t Y_t \leq M_t \tag{9}$$

Seignorage revenues are distributed in proportion to the agents' consumption shares, so that – to a first order – seignorage rebates are exactly equal to the amount of new money that the agents need to purchase in order to finance consumption, and the two cancel out from the budget constraint.

3.4 Equilibrium

The equilibrium concept adapts the definition in Baqaee and Farhi (2020) to account for the endogenous determination of markups given pricing frictions and shocks. Given sectoral markups, all markets must clear; and the evolution of markups must be consistent with Calvo pricing and the realization of monetary policy.

Definition 4. At each period t, for given sectoral probabilities of price adjustment δ_i and money supply M_t, the general equilibrium is given by a vector of firm-level markups $\mathcal{M}_{ift}$, a vector of sectoral prices P_{it}, a vector of factor-specific nominal wages W_{ht}, a vector of factor supplies L_{ht}, a vector of sectoral outputs Y_{it}, a matrix of intermediate input quantities X_{ijt}, and a matrix of final demands C_{iht} such that: (i) a fraction δ_i of firms in each sector i charges the profit-maximizing price given by (8); (ii) the markup charged by adjusting firms is given by the ratio of the profit-maximizing price and marginal costs, while the markups of non-adjusting firms are such that their price remains constant; (iii) agents maximize utility subject to their budget constraint; (iv) producers in each sector i minimize costs and charge the relevant markup; and (v) markets for all goods and factors clear.

Remark 5. This equilibrium concept nests the standard one with flexible prices, which is obtained as a special case when $\delta_i = 1$ for every sector i.

3.5 Log-linearized model

I now derive a log-linear approximation of the model around an initial efficient steady state, with no transfers ($T_h^* = 0\ \forall h$), and unit nominal GDP ($M^* = \sum_h P_h^* C_h^* + \sum_f P_f^* I_f^* = 1$). Starred variables denote the steady-state, lower case letters denote log-deviations from the steady-state, and vectors are in bold.

Remark 6. From now on, I assume that lump-sum transfers are $T_{ht} \equiv 0\ \forall h, t$. While this is not without loss of generality, in Appendix **??** I argue that the implications for cross-sectional employment remain almost unchanged.

Variables Table 3 introduces the endogenous variables.

Employment	$\boldsymbol{\ell}_t = \begin{pmatrix} \ell_{1t} & \dots & \ell_{N_w+N_f,t} \end{pmatrix}^T$
Consumption	$\boldsymbol{c}_t = \begin{pmatrix} c_{1t} & \dots & c_{N_w+N_f,t} \end{pmatrix}^T$
Factor prices	$\mathbf{w}_t = \begin{pmatrix} w_{1t} & \dots & w_{N_w+N_f,t} \end{pmatrix}^T$
Inflation rates	$\boldsymbol{\pi}_t = \begin{pmatrix} \pi_{1t} & \dots & \pi_{Nt} \end{pmatrix}^T, \ \pi_{it} \equiv p_{it} - p_{i,t-1}$

Table 3: Model variables

Remark 7. Table 3 defines employment for primary factors, rather than for sectors. In this environment, knowing factor-level employment is sufficient to characterize the evolution of prices, employment and output in all sectors.

Parameters I now introduce notation for four sets of parameters, which describe the log-linearized model: expenditure shares (α, β and Ω); income rebates ($\mathcal{W}$ and Ξ); demand and supply elasticities; and sectoral price adjustment probabilities. Tables 4 and 5, and Figure 8 below summarize this notation.

Recall the matrix $\mathcal{W} = (\mathcal{W}_{hf})$ introduced in Section 3.1.1, which tells us which share of each factor f's income is rebated to each final user h. I assume that households fully own their labor income, so that $\mathcal{W}_{hh'} = \mathbb{I}\,(h = h')$ for $h, h \in \mathcal{N}_w$.

Final users also receive profit rebates from the various industries, according to the ownership shares Ξ defined in Section 3.1.1. Our assumptions about retailers and factor marketplaces imply $\Xi_{hr_f} = \mathbb{I}(h = f)$ and $\Xi_{hm_f} = \mathcal{W}_{hf}$ for $h, f \in \mathcal{N}_w \cup \mathcal{N}_f$, where we denoted by r_f the retailer specific to final user f and by m_f the marketplace specific to factor f. We further assume that investment suppliers do not own shares in production industries ($\Xi_{fi} = 0$ for $i = 1, ..., N,\ f \in \mathcal{N}_f$).

Next, I define the matrix Ω of expenditure shares for producers and final users. Following Remark 3, I include factor marketplaces and retailers among the production sectors. I order the factor marketplaces first, then the good-producing sectors, and lastly the final consumption and investment retailers. The matrix Ω is then defined as

$$\Omega \equiv \begin{bmatrix} \mathbb{O}_{N_w+N_f} & \mathbb{O}_{N,N_w+N_f} & \mathbb{O}_{N,N_w+N_f} \\ \left(\Omega_{ih} \equiv \frac{W_h L_{ih}}{MC_i Y_i}\right) & \left(\Omega_{ij} \equiv \frac{P_j X_{ij}}{MC_i Y_i}\right) & \mathbb{O}_{N_w+N_f} \\ \mathbb{O}_{N_w+N_f} & \left(\Omega_{hi} \equiv \frac{P_i C_{ih}}{P_h^C C_h}\right) & \mathbb{O}_{N_w+N_f} \end{bmatrix}$$

I also define the $\bar{N} \times N_w + N_f + 1$ matrix of final expenditure shares β, and the $\bar{N} \times N_w + N_f$ matrix of primary factor shares α, as follows.

$$\alpha = \begin{bmatrix} I_{N_w+N_f} \\ \mathbb{O}_{N,N_w+N_f} \\ \mathbb{O}_{N_w+N_f} \end{bmatrix}, \quad \beta = \begin{bmatrix} \mathbb{O}_{N,N_w+N_f} \\ \mathbb{O}_{N_w+N_f} \\ I_{N_w+N_f} \end{bmatrix}$$

The matrix α has only one non-zero block, because only factor marketplaces hire primary factors directly. Moreover each marketplace only hires one primary factor, therefore the upper block of α is the identity matrix. Likewise, final consumers only purchase goods from their specific retailer, therefore β also has only one non-zero block, equal to the identity matrix.

Definition 5. The Leontief inverse is $\Psi \equiv (I - \Omega)^{-1}$. I denote the lowest $N_w + N_f \times \bar{N}$ block of the Leontief inverse by $\Psi_{C,:}$, and I denote the leftmost $\bar{N} \times N_w + N_f$ block by $\Psi_{:,L}$.

While the input-output matrix captures the *direct* elasticity of each sector i's cost to every other sector j's price, the Leontief inverse captures the *total* exposure, directly and indirectly through intermediate inputs. To see this, one can expand the Leontief inverse as a geometric sum, where the n-th term is the exposure of i to j through paths of length n:

$$\Psi_{ij} \equiv (I - \Omega)^{-1}_{ij} = I_{ij} + \Omega_{ij} + \left(\Omega^2\right)_{ij} + ...$$

Note that the lower block $\Psi_{C,:}$ of the Leontief inverse corresponds to the total content of good i in the bundle of final user h, while the leftmost block $\Psi_{:,L}$ denotes the total content of factor f in good i. In particular, the bottom-left block $\Psi_{CL} \equiv (\Psi_{hf})_{h,f=1}^{N_w+N_f}$ denotes the total content of factor f in h's final bundle.

Remark 8. In every sector, the expenditure shares on inputs and primary factors must sum to one: $(\Omega + \alpha)\mathbf{1} = \mathbf{1}$.

This implies the steady-state relation

$$\Psi_L \mathbf{1} = \mathbf{1}$$

To understand Remark 8 , note that $(\Psi_L)_{nh}$ is the total share (directly and through intermediate inputs) of primary factor h in n's cost, where n could be an industry or a final user. Remark 8 states that primary factors $(\Psi_L \mathbf{1})$ accounts for the entirety of costs paid by producers and final users, which must be true whenever producers price at marginal cost.

Definition 6 and Lemma 1 introduce and compute income shares for primary factors, industries, and final users.

Definition 6. The steady-state income shares of final users $\mathbf{s}$ and Domar weights $\bar{\boldsymbol{\Psi}}$ are implicitly defined by the following system:

$$\begin{cases} \bar{\boldsymbol{\Psi}}^T = \mathbf{s}^T \Psi_{C,:} \\ \mathbf{s} = \mathcal{W} \bar{\boldsymbol{\Psi}}_L + \mathbf{T} \end{cases} \tag{10}$$

where I denoted by $\Psi_{C,:}$ the rows of Ψ corresponding to the retailers, by $\Psi_{:,L}$ the columns of Ψ corresponding to the factor marketplaces, and by $\bar{\boldsymbol{\Psi}}_L^T \equiv \mathbf{s}^T \Psi_{CL}$.

Assumption 1. *Assume*

$$rank\left(I - \mathcal{W}\Psi_{CL}^T\right) = N_w + N_f - 1$$

Lemma 1. *Under Assumption 1 there is a unique solution for $\bar{\boldsymbol{\Psi}}$ and $\mathbf{s}$ in (**??**) such that $\sum_{h \in \mathcal{N}_H \cup \mathcal{N}_F} s_h + s_G = 1$. Moreover, final use shares are equal to*

$$s_h = \frac{P_h^C C_h}{nGDP}$$

and Domar weights are equal to

$$\bar{\Psi}_i = \frac{P_i Y_i}{nGDP}$$

for producers $i \in \{1, ..., N\}$ and

$$\bar{\Psi}_h = \frac{W_h L_h}{nGDP}$$

for primary factors $h \in \mathcal{N}_H \cup \mathcal{N}_F$.

The second equation in (10) states that, in a steady-state with zero profits, the income of final users consists of factor income rebates plus transfers. In turn, the first equation states that each producers' (primary factor's) income depends on the total content of this producer (primary factor) in final demand, weighting final users according to their income shares.

Note that the system (10) is not invertible.[4] Nonetheless, Assumption 1 guarantees that it has a unique solution

[4]The system implies

$$\left(I - \mathcal{W}\Psi_{CL}^T\right)\mathbf{s} = \mathbf{T}$$

The matrix $I - \mathcal{W}\Psi_{CL}^T$ is not invertible, because $\mathbf{1}^T\left(I - \mathcal{W}\Psi_{CL}^T\right) = \mathbf{1}^T - \mathbf{1}^T\Psi_{CL}^T)$, and, from remark 8, $\mathbf{1}^T - \mathbf{1}^T\Psi_{CL}^T = \mathbf{1}^T - \mathbf{1}^T = \mathbf{0}^T$.

such that $\sum_{h \in \mathcal{N}_H \cup \mathcal{N}_F} s_h = 1$. The assumption is equivalent to imposing that, for every pair (h, f) of a primary factor h and a final user f, there is a connected path $(h, a_1, ..., a_K, f)$ between h and f, where a_k is connected to a_{k-1} if there is an income flow from a_k to a_{k-1}. This guarantees that relative prices and income shares are all well defined.

Table 4 and Figure 8 summarize the notation for income flows and income shares.

Factor ownership	$\mathcal{W} \in \mathbb{R}^{N_w+N_f+1 \times N_w+N_f}$
Sector ownership	$\Xi \in \mathbb{R}^{\bar{N}, N_w+N_f+1}$
Input shares	$\Omega \in \mathbb{R}^{\bar{N} \times \bar{N}}$
Expenditure on primary factors	$\alpha \in \mathbb{R}^{\bar{N} \times N_w+N_f}$
Final use bundles	$\beta \in \mathbb{R}^{\bar{N} \times N_w+N_f+1}$
Leontief inverse	$\Psi \equiv (I - \Omega)^{-1}$
Income shares of final users	$\mathbf{s} \in \mathbb{R}^{N_w+N_f+1}$, $\mathcal{S} \equiv diag(\mathbf{s})$
Domar weights	$\bar{\mathbf{\Psi}}^T = \mathbf{s}^T \Psi_C$
Factor income shares	$\bar{\mathbf{\Psi}}_L^T \equiv \mathbf{s}^T \Psi_{CL}$, $\mathcal{L} \equiv diag\left(\bar{\mathbf{\Psi}}_L\right)$

Table 4: Input-output definitions and income rebates

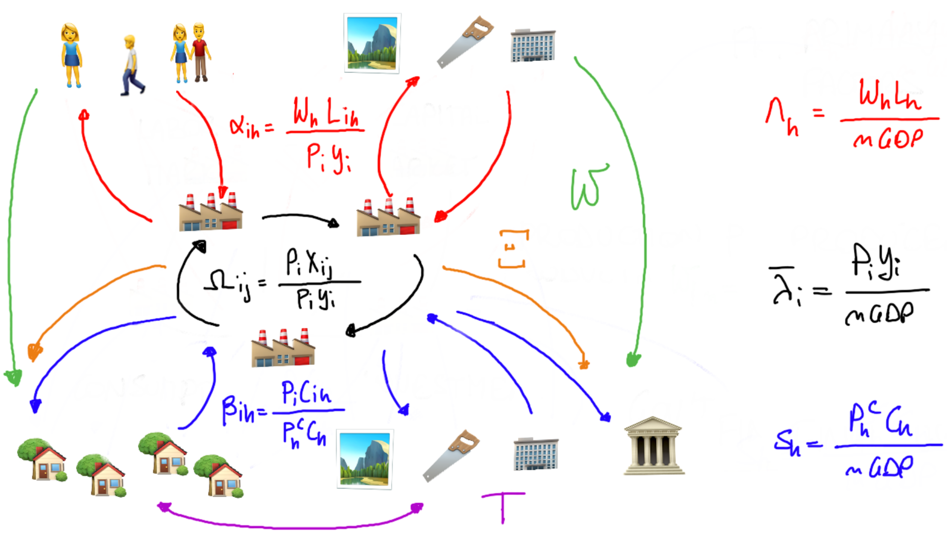

Figure 8: Log-linearized model

Finally, Table 5 summarizes the notation for price adjustment probabilities and for demand and supply elasticities. Note that in the matrix Δ the primitive Calvo parameters δ_i are replaced with the increasing and convex function

$$\hat{\delta}_i(\rho, \delta_i) \equiv \frac{\delta_i (1 - \rho(1 - \delta_i))}{1 - \rho\delta_i(1 - \delta_i)}$$

which appears in the derivation of the Phillips curve (see Gali (2015); Woodford (2003)).

Wealth effects	$\Gamma \in \mathbb{R}^{(N_w+N_f)\times(N_w+N_f)},\ \Gamma \equiv diag\left(\gamma_1, ..., \gamma_{N_w}, 0, ..., 0\right)$
Factor supply elasticities	$\Phi \in \mathbb{R}^{N_w+N_f},\ \Phi \equiv \Gamma\mathcal{S}^{-1}\mathcal{W}\mathcal{L} + diag\left(\varphi_1, ..., \varphi_{N_w+N_h}\right)$
Demand elasticities	$\theta^i_{jk} \equiv \frac{d\log\frac{X_{ij}}{X_{ik}}}{d\log\frac{P_j}{P_k}}$
Price adjustment probabilities	$\Delta \equiv diag\left(\hat{\delta}_1, ..., \hat{\delta}_N\right)$

Table 5: Input-output definitions

Aggregation Following definition 3, log-changes in the GDP deflator π^Y are a given by weighted average of sectoral inflation rates, with weights equal to the aggregate final expenditure shares $\bar{\boldsymbol{\beta}}$:

$$\pi^Y = \sum_{i=1}^{\bar{N}} \bar{\beta}_i \pi_i$$

In turn, aggregate final expenditure shares are a weighted average of the expenditure shares of the various final users, with weights given by final use shares:

$$\bar{\beta}_i \equiv \sum_{h\in\mathcal{N}_w\cup\mathcal{N}_f} s_h \beta_{ih}$$

Following Definition 2 and Remark 4 , aggregate output is equal to aggregate employment (including all factors, not just labor). It is given by a weighted average of factor-level employment, with weights equal to factor income shares:

$$\bar{y} = \sum_{h\in\mathcal{N}_w\cup\mathcal{N}_f} \bar{\Psi}_h \ell_h$$

3.6 Log-linearized equilibrium conditions

The equilibrium of the economy is characterized by three equations, describing the supply, the relative demand, and the aggregate demand for primary factors. A fourth equation describes the evolution of prices. I start by introducing and commenting these four equations. In Section 5 I combine the supply and pricing equations to express the evolution of prices in terms of sector-by-factor Phillips curves.

Supply equation The supply equation stacks together the log-linearized consumption-leisure tradeoff (3) and the optimal capital utilization (4), to express factor supply as a function of real wages:

$$\Phi\boldsymbol{\ell}_t = \mathbf{w}_t - \underline{\beta}^T \mathbf{p}_t \tag{11}$$

where

$$\underline{\beta}^T \equiv \beta^T - \Gamma\left(\beta^T - \mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\right)$$

and Φ is defined in Table 5.

Equation (11) tells us that factor supply is increasing in real wages, with elasticity Φ. Note that Φ is not diagonal, capturing wealth effects in labor supply from capital income. Moreover, the relevant price index for real wages is weighted according to $\underline{\beta}^T$, rather than by expenditure shares β^T. This is because with heterogeneous households we need to account for the discrepancy between the income exposure to different industries through consumption prices (β^T) versus profits ($\mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}$). While in a representative agent economy the two would cancel out, with heterogeneous agents some households might purchase from firms that they do not own and vice versa.

Relative demand equation The relative demand equation (12) expresses factor-level employment as a function of aggregate employment $\bar{y}$, and sectoral prices.

Lemma 2. *The vector* $\boldsymbol{\ell}$ *of factor-specific employment is given by*

$$\boldsymbol{\ell}_t = \mathbf{1}\bar{y}_t + \mathcal{D}_p \mathbf{p}_t \tag{12}$$

where

$$\mathcal{D}_p \equiv -\mathcal{L}^{-1}\left(I - Cov_{\mathbf{s}}\left(\Psi_{CL}^T, S^{-1}\mathcal{W}\right)\right)^{-1}\left[Cov_{\mathbf{s}}\left(\Psi_{CL}^T, \beta^T - \mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\right) + \Theta\left(\Psi_{:,L}, I\right)\right]$$

It holds that $\bar{\boldsymbol{\Psi}}_L^T \mathcal{D}_p = \mathbf{0}^T$ *and* $\mathcal{D}_p \mathbf{1} = \mathbf{0}$. *Definition 7 introduces the substitution operator* Θ.

Proof. See Appendix. □

Definition 7. Given any two matrices $X \in \mathbb{R}^{\bar{N}\times H}$ and $Y \in \mathbb{R}^{\bar{N}\times M}$ (where M can take any value in $\mathbb{N}$), the substitution operator $\Theta\left(X, Y\right) : \mathbb{R}^{N_w + N_f \times \bar{N}} \times \mathbb{R}^{\bar{N}\times M} \mapsto \mathbb{R}^{H\times M}$ is defined as

$$\left[\Theta\left(X,Y\right)\right]_{hm} \equiv \frac{1}{2}\sum_{n=1}^{\bar{N}} \bar{\Psi}_n \sum_{i,j=1}^{\bar{N}} \Omega_{ni}\Omega_{nj}\theta_{ij}^n \left(X_{ih} - X_{jh}\right)\left(Y_{im} - Y_{jm}\right) \tag{13}$$

for all $h \in \{1, ..., N_w + N_f\}$ and $m \in \{1, ..., M\}$.

In Equation (12), increasing the output gap raises the demand for all primary factors proportionately, as captured by the vector $\mathbf{1}$.[5] Because of homothetic preferences and unit marginal propensities to consume, as aggregate employment and output increase, all final users – who were optimizing their consumption baskets prior to the shock – increase expenditures on all goods proportionally. Given that production functions are also homothetic, this raises the demand for all primary factors proportionally (for constant relative prices).

[5] It is immediate to verify that, $Cov_s\left(\Psi_{CL}, \mathcal{S}^{-1}\mathcal{W}\right)\mathbf{1} = \mathbf{0}$, so that $\left(I - Cov_s\left(\Psi_{CL}, \mathcal{S}^{-1}\mathcal{W}\right)\right)^{-1}\mathbf{1} = \mathbf{1}$.

In Equation (12) employment also responds to changes in relative prices, as described by the matrix $\mathcal{D}_p$. Note that changes in the price level have no effect on factor demand, for constant relative prices ($\mathcal{D}_p \mathbf{1} = \mathbf{0}$). Moreover, changes in relative prices have a direct effect only on relative factor demand, not on aggregate demand ($\bar{\boldsymbol{\Psi}}_L^T \mathcal{D}_p = \mathbf{0}^T$).

The matrix $\mathcal{D}_p$ isolates two channels through which changes in relative prices affect relative factor demand. First, producers and final users substitute towards the goods and primary factors whose relative price declines. This is captured by the substitution operator Θ. In equation (13), each primary factor h is affected by a change in the price of good j according to j's network-adjusted expenditure share on h, Ψ_{jh}. The decline in the demand for h depends on how much j's customers substitute away from it. This is governed by the elasticities θ^i_{jk}, which tell us how much each customer i of sector j substitutes away towards other inputs k. Customer sectors i are weighted by their sales shares $\bar{\Psi}_i$. Note that the substitution operator also accounts for changes in the (sticky) factor prices, which are included in the price vector $\mathbf{p}$.

It is useful to write explicitly the effect of a change in prices $\mathbf{p}$ on the demand for factor h, using the definition of Θ in the special case where sectors have CES production functions with elasticities $\{\theta_i\}_{i=1}^N$:

$$\Theta\left(\Psi_{:,h}, \mathbf{p}\right) = \sum_i \bar{\Psi}_i \theta_i Cov_{\Omega_{(i,:)}}\left(\Psi_{:,h}, \mathbf{p}\right) \tag{14}$$

In equation (14), the subscripts $(i,:)$ and $(:,h)$ denote the i-th row and h-th column of a matrix, and the covariance $Cov_{\Omega_{(i,:)}}(\cdot,\cdot)$ uses sector i's input shares, $\{\omega_{ij}\}_{j=1}^{\bar{N}}$, as probability weights. Equation (14) shows how relative prices changes affect the demand for factor h through the sourcing decisions of each sector i. On net, sector i will substitute away from factor h if the relative price of h-intensive inputs increased, that is, if the covariance $Cov_{\Omega_{(i,:)}}\left(\Psi_{:,h}, \mathbf{p}\right)$ between factor contents $\Psi_{j,h}$ and price changes p_j across sectors j is positive. The importance of each sector i depends on its size, as captured by the Domar weight $\bar{\Psi}_i$, and on its ability to substitute across inputs, θ_i.

Second, the matrix $\mathcal{D}_p$ also captures how changes in relative prices reallocate real income across final users with different consumption baskets and ownership shares. Higher prices mean both higher profits and lower real incomes. While the two effects cancel out if all prices change proportionately, changes in relative good and factor prices imply that some households have higher real income due to higher profits, while others have lower real income due to higher consumption prices. Relative demand increases for primary factors who produce the goods consumed by the richer households. This is captured by the covariance between factor contents in consumption (Ψ_{CL}^T) and changes in real income ($\mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1} - \beta^T$).

Finally, the multiplier $\left(I - Cov_s\left(\Psi_{CL}, \mathcal{S}^{-1}\mathcal{W}\right)\right)^{-1}$ captures a feedback loop. Changes in relative factor demand affect the relative income of the final users who own these factors, and – if final users consume different baskets – this in turn feeds back into relative factor demand, and so on.

Aggregate demand equation The aggregate demand equation follows immediately from the cash-in-advance constraint. It tells us that changes in final expenditure ($\pi_t^Y + \bar{y}_t$) must be equal to changes in money balances

$(m_t - p^Y_{t-1})$:

$$\pi^Y_t + \bar{y}_t = m_t - p^Y_{t-1} \tag{15}$$

Pricing equation The pricing equation relates good prices with factor prices $\mathbf{w}$, lagged prices $\mathbf{p}_{t-1}$, and expected inflation $\mathbb{E}\boldsymbol{\pi}_{t+1}$:

$$\mathbf{p}_t = \mathcal{P}_L \mathbf{w}_t + \left[I - \mathcal{P}\Psi^{-1}\right](\mathbf{p}_{t-1} + \mathbb{E}\boldsymbol{\pi}_{t+1}) \tag{16}$$

Definition 8. The pass-through matrix $\mathcal{P}$ in equation (16) is defined as

$$\mathcal{P} \equiv \Delta\left(I - \Omega\Delta\right)^{-1}$$

$\mathcal{P}$ is an $\bar{N} \times \bar{N}$ matrix whose (i, j)-th element describes the propagation of a cost shock to sector j into the price of sector i. I also denote by

$$\mathcal{P}_L \equiv \mathcal{P}\alpha$$

the $\bar{N} \times N_H + N_F$ block of this matrix corresponding to primary factors, and by

$$\mathcal{P}_{C,:} \equiv \beta^T \mathcal{P},\ \underline{\mathcal{P}}_{C,:} \equiv \underline{\beta}^T \mathcal{P}$$

the $N_H + N_F \times \bar{N}$ block of this matrix corresponding to final users.

In the definition of $\mathcal{P}$, the price adjustment probability Δ governs the response of prices to changes in marginal costs. The matrix $(I - \Omega\Delta)^{-1}$ is similar to the Leontief inverse, although it discounts supplier sectors by their price adjustment probability. Its (i, j)-th element corresponds to the propagation of a cost shock in sector j into the marginal cost of sector i, either directly or through intermediate suppliers, suppliers' suppliers, etc.

The matrix $\mathcal{P}_L$ describes the pass-through of factor prices into good prices (directly, and through input-output linkages). The response to lagged prices and expected inflation instead is mediated by the matrix $\mathcal{P}\Psi^{-1}$. The matrix Ψ^{-1} maps price wedges into the underlying changes in marginal costs, while the matrix $\mathcal{P}$ describes the pass-through of desired price changes into actual prices, directly and indirectly through the input-output network. If prices were fully flexible we would have $\mathcal{P}\Psi^{-1} = \mathbb{I}$, which means that current prices do not depend on past prices or expected future inflation. If instead prices are fully rigid we have $\mathcal{P} = \mathbb{O}$, so that prices are constant over time ($\mathbf{p}_t = \mathbf{p}_{t-1}$, and $\mathbb{E}\boldsymbol{\pi}_{t+1} = \mathbf{0}$).

In Section 4 I derive sector-by-factor Phillips curves by combining the pricing equation (16) with the consumption-leisure tradeoff (3).

4 Sector-by-factor Phillips curves

Proposition **??** derives sector-by-factor Phillips curves by combining the pricing equation (16) with the consumption-leisure tradeoff (3).

Proposition 1. *Sectoral inflation evolves according to the sector-by-factor Phillips curves*

$$\boldsymbol{\pi}_t - \rho \mathbb{E} \boldsymbol{\pi}_{t+1} = \kappa \boldsymbol{\ell}_t - \mathcal{V} \left(\mathbf{p}_{t-1} + \rho \mathbb{E} \boldsymbol{\pi}_{t+1} \right) \tag{17}$$

where

$$\begin{aligned} \kappa &\equiv \mathcal{P}_L \left(I - \underline{\mathcal{P}}_{CL} \right)^{-1} \Phi && \in \mathbb{R}^{\bar{N} \times (H+F)} \\ \mathcal{V} &\equiv \mathcal{P} \Psi^{-1} - \kappa \Phi^{-1} \underline{\beta}^T \left(I - \mathcal{P} \Psi^{-1} \right) && \in \mathbb{R}^{\bar{N} \times \bar{N}} \end{aligned} \tag{18}$$

The matrix $\mathcal{V}$ is such that $\sum_j \mathcal{V}_{ij} = 0$, $(I - \mathcal{V})_{ij} \in [0, 1]$ and, as long as no sector has fully flexible prices ($\delta_i \neq 1$ $\forall i$), the matrix $I - \mathcal{V}$ is invertible.

In an environment with multiple sectors and factors, inflation in each sector depends on the employment gap of all the factors. This is captured by the slope κ in equation (17). Current inflation also responds to lagged prices and expected future inflation, with elasticity $-\mathcal{V}$.

The slope κ is key to the cross-sectional effects of monetary policy, because it determines the effect of changes in employment on relative prices. The elements κ_{ih} capture the response of prices in sector i to the employment of factor h. Equation (18) decomposes the slope κ in three parts:

$$\kappa = \text{(factor price pass-through)} \, \text{(factor price Phillips curves)} \, \text{(supply elasticity)} \tag{19}$$

I discuss each of the three terms below, highlighting the dimensions of heterogeneity that determine which primary factors tend to have steeper or flatter Phillips curves.

Factor supply elasticities The matrix Φ captures heterogeneity in factor supply elasticities. The elements Φ_{hk} tell us by how much the real wage[6] of factor h needs to increase if the employment of factor k increases by 1%. Φ_{hk} depends on Frish elasticities and wealth effects in labor supply. Both can vary across primary factors, depending on the households' preferences and ownership of capital assets, and on the investment technology.

Factor price pass-through The pass-through $\mathcal{P}_L$ of factor prices into good prices captures heterogeneity in the nominal rigidities of primary factors, and of the sectors which employ them. The price pass-through of factor h into

[6] Given our notation (see Section 3.5), nominal wages mean the counterfactual flexible wages that satisfy equation (3) given factor demand and good prices. These are different from the actual (sticky) factor prices, which correspond to the first $H + F$ entries in the vector $\boldsymbol{\pi}$.

industry i is given by $(\mathcal{P}_L)_{ih} \equiv \left(\Delta\left(I-\Omega\Delta\right)^{-1}\alpha\right)_{ih}$. The matrix of factor input shares α captures the direct effect of factor prices on sectoral marginal costs, while the geometric sum $(I-\Omega\Delta)^{-1} = \Omega\Delta + (\Omega\Delta)^2 + ...$ describes the indirect effect through intermediate inputs. The n-th term of the geometric sum, $(\Omega\Delta)^n_{ij}$, tells us the effect of an increase in the marginal cost of sector j on sector i's marginal cost, through all supply chains of length n starting with j and ending with i. Along the path, each supplier is weighted according to its input shares Ω, and discounted by the probability of price adjustment Δ. Thus, factors h which are used intensively by industries (including labor unions) with flexible prices, or whose customers have flexible prices, etc., have a higher price pass-through and stepper slopes $\kappa_{:,h}$

Factor price Phillips curves The slope $(I-\underline{\mathcal{P}}_{CL})^{-1}$ of the factor price Phillips curves captures heterogeneity in the nominal rigidly of the sectors which different final users own and buy from. These Phillips curves map changes in real factor prices into changes in nominal factor prices. As factor prices increase, good prices also increase, so that the nominal change in factor prices must be larger than the real change. The effect of factor prices on final prices is given by $\underline{\mathcal{P}}_{CL}$. For a given increase in factor prices, consumption (investment) prices increase more for final users who spend more on flex-price goods (or goods that are produced using flex-price intermediates, etc), so that the primary factors that they supply will have steeper Phillips curves. In addition, through a wealth effect in labor supply, households who own a smaller share of profit income also have steeper wage Phillips curves.[7]

4.1 Examples

This section introduces three simple economies, to illustrate how different dimensions of heterogeneity shape sector-by-agent Phillips curves. In the examples below I use a static version of the model, with $\rho = 0$, to provide intuition. I assume that the economy enters period 0 in steady-state, so that pre-set prices from $t = -1$ are uniform across producers within each sector, but only a fraction δ_i of producers in each sector can update prices after observing the money supply.

Example 1. Heterogeneous nominal rigidity

Consider and economy with two agents and one final good, as in Figure 9. The two agents supply different types of labor, and face different wage rigidity. We denote their wage adjustment probabilities by $\delta_{sticky} < \delta_{flex}$. Final good prices are flexible.

[7] This is because, with sticky prices, profits are negatively correlated with employment. Therefore, as employment rises, firm owners become poorer and are willing to supply labor at a lower wage.

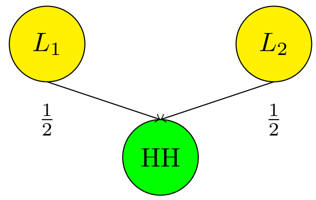

Figure 9: Two-workers, one-good economy

The elasticity of substitution between the two labor types is θ. Both agents have the same inverse Frish elasticity φ, and there are no wealth effects in labor supply ($\Gamma = \mathbb{O}$).

According to our notational convention, there are three sectors in the economy: the two labor unions, and the final good producers. The input-output primitives are given by

$$\alpha = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \ \Omega = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{sticky} & 1-\alpha_{sticky} & 0 \end{pmatrix}, \ \beta = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \mathbf{1}^T$$

Following Proposition 1, the slope of the Phillips curve for the GDP deflator is given by

$$\begin{pmatrix} \kappa^Y_{sticky} & \kappa^Y_{flex} \end{pmatrix} = \begin{pmatrix} \alpha_{sticky}\delta_{sticky} & (1-\alpha_{sticky})\,\delta_{flex} \end{pmatrix} \frac{\varphi}{1-\bar{\delta}}$$

where

$$\bar{\delta} \equiv \alpha_{sticky}\delta_{sticky} + (1-\alpha_{sticky})\,\delta_{flex}$$

Example 2. Heterogeneous labor supply elasticity

Consider the same economy as in Figure 9, but assume now that the two workers face the same wage rigidity ($\delta_{flex} = \delta_{sticky} = \delta$), while facing different inverse Frish elasticities ($\varphi_I > \varphi_E$).

In this economy, the slope of the Phillips curve is steeper for the inelastic worker (I) than for the elastic one (E):

$$\kappa^Y_I = \frac{\delta}{1-\delta}\alpha_I\varphi_I > \kappa^Y_E = \frac{\delta}{1-\bar{\delta}}\,(1-\alpha_I)\,\varphi_E$$

Example 3. Input-output linkages

In example 1, workers face different degrees of nominal rigidity because they work in sticky vs flexible sectors. However, workers may also face different degrees of nominal rigidity because of their different positions along the production chain. The example illustrates this in an economy with a single vertical chain, two stages (intermediate and final goods), and two agents. Agent type I produces the intermediate input, while agent type F assembles the final good. The economy is depicted in Figure 10.

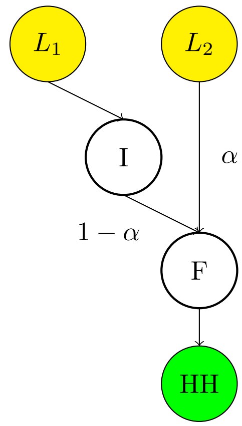

Figure 10: Two-stage vertical chain

Wages are flexible, while both the intermediate and final good sectors have sticky prices, with the same Calvo parameter δ. The elasticity of substitution between intermediate inputs and labor in the production of final goods is θ. Agents have the same Frish elasticity and there are no wealth effects in labor supply.

The production primitives are given by

$$\alpha = \begin{pmatrix} 1 & 0 \\ 0 & \alpha_F \end{pmatrix}, \; \Omega = \begin{pmatrix} 0 & 0 \\ 1-\alpha_F & 0 \end{pmatrix}, \; \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathbf{1}^T$$

Following Proposition 1, sector-by-agent slopes are

$$\kappa = \frac{\delta}{1-\bar{\bar{\delta}}} \begin{pmatrix} 1-\alpha_F\delta & \alpha_F\delta \\ (1-\alpha_F)\,\delta & \alpha_F \end{pmatrix} \varphi, \quad \bar{\bar{\delta}} \equiv \delta \left[(1-\alpha_F)\,\delta + \alpha_F \right]$$

Inflation in the final sector, which also coincides with consumer price inflation, responds to the employment gap of the final good worker F more than proportionately to its input share. This is because worker F only faces one round of nominal rigidity, while worker I faces two rounds (the intermediate and the final sector). Thus the impact of worker I's employment on inflation is muted relative to worker F, even though the intermediate and final sector have the same Calvo parameter.

Also note that, even if the intermediate good sector only hires worker I, inflation in this sector depends on the employment gap of both workers, through a general equilibrium spillover. When worker F has a positive employment gap ($l_F > 0$), his wage must increase. This bids up consumer prices, therefore the wage of worker I must also increase to keep his employment gap l_I constant.

5 Monetary non-neutrality: cross-section and aggregate

This section characterizes the response of employment to monetary policy, in the cross-section and in the aggregate. To this end, it is convenient to gather together the three equations which govern the dynamics of the economy. Lagged prices ($\mathbf{p}_{t-1}$) are a state variable, and the evolution of the economy is summarized by the sector-by-factor Phillips curves (a supply equation), the relative factor demand equation, and the cash-in-advance constraint (an aggregate demand equation):

$$\begin{cases} \boldsymbol{\pi}_t - \rho \mathbb{E} \boldsymbol{\pi}_{t+1} = \kappa \boldsymbol{\ell}_t - \mathcal{V}\left(\mathbf{p}_{t-1} + \rho \mathbb{E} \boldsymbol{\pi}_{t+1}\right) & \text{Phillips curves} \\ \boldsymbol{\ell}_t = \mathbf{1} \bar{y}_t + \mathcal{D}_p \mathbf{p}_t & \text{relative demand} \\ \pi_t^Y + \bar{y}_t = m_t - p_{t-1}^Y & \text{cash-in-advance} \end{cases} \tag{20}$$

The Phillips curves relate the vector of industry-level inflation rates $\boldsymbol{\pi}$ with the vector of factor-level employment $\boldsymbol{\ell}$. The relative demand equation instead expresses factor-level employment as a function of aggregate employment $\bar{y}$ and relative prices. In Section 5.1, combining these two equations will allow us to relate factor-specific employment with the aggregate employment, which in turn is pinned down by monetary policy through the cash-in-advance constraint. Section 5.2 instead illustrates the determinants of aggregate monetary non-neutrality, contrasting the heterogeneous agent model with the representative agent benchmark.

Remark 9. The Phillips curves and the relative demand equation hold regardless of the policy rule. In an economy where there is a risk-free asset and monetary policy follows a Taylor rule, we would replace the cash-in-advance constraint with (i) an aggregate Euler equation; and (ii) a set of cross-sectional Euler equations (i.e. Backus-Smith conditions). The cross-sectional Euler Equations would determine a path of endogenous zero-sum transfers $\{\mathbf{T}_t\}_{t=0}^{\infty}$ across agents, which in turn would also appear in the Phillips curve and relative demand equations. In Appendix **??** I argue that the effect of transfers on the Phillips curve and relative demand is small, because households have similar consumption baskets and similar wealth effects in labor supply.

5.1 Cross-sectional non-neutrality

Proposition 2 combines the Phillips curves (17) with the relative demand equation (12), to express cross-sectional employment as a function of the aggregate output gap, past prices, and inflation expectations.

Proposition 2. *It holds that*

$$\boldsymbol{\ell}_t = \left(I - \mathcal{D}_p \kappa\right)^{-1} \left[\mathbf{1} \bar{y}_t + \mathcal{D}_p \left(I - \mathcal{V}\right)\left(\mathbf{p}_{t-1} + \rho \mathbb{E} \boldsymbol{\pi}_{t+1}\right)\right] \tag{21}$$

Proof. Equation (21) follows immediately from equations (17) and (12). □

A central element in equation (21) is the *cross-sectional multiplier* $(I - \mathcal{D}_p\kappa)^{-1}$, which captures a feedback loop between changes in relative factor demand and changes in factor prices. The effect of factor demand on relative prices is captured by the slope κ of sectoral Phillips curves. As we will see, heterogeneity in κ across sectors is key to the cross-sectional effect of monetary policy. In turn, the effect of relative price changes on cross-sectional labor demand is captured by the matrix $\mathcal{D}_p$ from equation (12).

In commenting Proposition 2 I focus on the aggregate employment term in (21), which is more directly connected with current monetary policy (through the cash-in-advance constraint). The remaining terms describe the employment response to relative price changes coming from past monetary shocks (as captured by lagged prices $\mathbf{p}_{t-1}$) and expected future inflation.

The loading on aggregate employment has two components: a direct effect, captured by the vector $\mathbf{1}$, and a propagation effect, captured by the cross-sectional multiplier $(I - \mathcal{D}_p\kappa)^{-1}$. To understand the direct effect, note that a monetary expansion increases the demand for all final users proportionately on impact (i.e. parallel to the vector $\mathbf{1}$). This is because preferences and production are homothetic, and agents were optimizing their initial expenditures.

However, when Phillips curve slopes differ across primary factors, due to heterogeneous nominal rigidities or supply elasticities, even a proportional increase in labor supply will affect relative factor prices. Changes in relative prices in turn affect relative real incomes, and trigger expenditure switching by producers and consumers. This then feeds back into relative factor demand, as captured by the cross-sectional multiplier $(I - \mathcal{D}_p\kappa)^{-1}$. Overall, demand falls for factors who have steeper Phillips curves, or who are employed by the same sectors as steep-Phillips curve factors, etc.

Corollary 1 highlights the necessary conditions for monetary policy to affect cross-sectional employment.

Corollary 1. *Changing the aggregate output gap does not affect relative employment across primary factors if either:*

1. sectors use primary factors in equal proportions ($\alpha = \alpha\mathbf{1}\mathbf{s}^T$);

2. a proportional increase in the employment of all primary factors does not affect relative prices ($\kappa\mathbf{1} \propto \mathbf{1}$).

If there is no variation in factor intensity across sectors (condition 1), then changes in relative prices have no effect on factor demand and $\mathcal{D}_p = \mathbb{O}$. Conversely, if a proportional increase in the employment of all primary factors has no effect on relative prices (condition 2), then we also have no feedback effect on relative demand ($\mathcal{D}_p\mathbf{1} = \mathbf{0}$). Note that Corollary 1 holds regardless of demand elasticities. Hence heterogeneous demand elasticities per-se do not imply a cross-sectional effect of monetary policy.

In this section I focused on the effect of monetary policy on cross-sectional employment. Cross-sectional consumption instead does not only depend on employment, but also on income from primary factors, profit rebates, and consumption or investment prices. I discuss cross-sectional consumption in Section **??**.

5.1.1 Response of consumption

Proposition 2 characterized the effect of monetary policy on the relative employment of different primary factors. Proposition 3 instead characterizes the response of consumption across final users. Importantly, we maintain the assumption of financial autharchy and no exogenous income transfers across households.

Proposition 3. *The final users' consumption is given by*

$$\mathbf{c} = \left(I - \mathcal{D}_p^C \kappa^C\right)^{-1} \left[\mathbf{1}\bar{y} + \mathcal{D}_p^C \left(I - \mathcal{V}^C\right)\left(\mathbf{p}_{t-1} + \rho \mathbb{E}\boldsymbol{\pi}_{t+1}\right)\right] \tag{22}$$

where

$$\begin{aligned}
\underline{\beta}_C^T &\equiv \beta^T + diag\left(\boldsymbol{\varphi}\right)\left(\mathcal{W}\mathcal{L}\right)^{-1}\left(\mathcal{S}\beta^T - \Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\right)\\
\Phi_C &\equiv \Gamma + diag\left(\boldsymbol{\varphi}\right)\left(\mathcal{W}\mathcal{L}\right)^{-1}\mathcal{S}\\
\kappa^C &\equiv \mathcal{P}_L\left(I - \underline{\beta}_C^T \mathcal{P}_L\right)^{-1}\Phi_C\\
\mathcal{V}^C &\equiv \mathcal{P}\Psi^{-1} - \kappa_C \Phi_C^{-1}\underline{\beta}_C^T\left(I - \mathcal{P}\Psi^{-1}\right)\\
\mathcal{D}_p^C &\equiv \mathcal{S}^{-1}\left(I - Cov_s\left(\mathcal{W}\Psi_{CL}^T, \mathcal{S}^{-1}\right)\right)^{-1}\left[Cov_{\mathbf{s}}\left(I, \beta^T - \mathcal{S}^{-1}\Xi^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\right) + \mathcal{W}\Theta\left(\Psi_L, I\right)\right]
\end{aligned}$$

Proposition 3 and equation (22) show that consumption can be expressed as a function of aggregate real GDP in a similar way as employment (see Proposition 2 and equation (21)), using an appropriate cross-sectional multiplier. Like for employment, the cross-sectional multiplier combines the relevant sector-by-final user Phillips curve slope κ^C with a relative demand elasticity $\mathcal{D}_p^C$. The Phillips curve slope and the demand elasticity take a similar form for employment and consumption, with two differences: (i) the relevant supply elasticity Φ_C and the relevant price index $\underline{\beta}^T\mathbf{p}$ for the consumption-leisure tradeoff are different; and (ii) the income reallocation component of the demand elasticity is equal to the difference between consumption prices vs profit income, rather than to its effect on the demand for primary factors.

Like for employment, changing aggregate demand (i.e. changing $\bar{y}$) has no effect on cross-sectional consumption whenever a proportional increase in consumption across all final users does not affect relative prices. Thus for monetary policy to affect cross-sectional consumption is essential that the Phillips curve slopes are heterogeneous (i.e. households face different supply elasticities or different nominal rigidities in consumption and/or employment) This is stated in Corollary 2. Importantly, and different from the employment case, monetary policy can affect cross-sectional consumption even when all sectors use primary factors in the same proportions. This happens because, after a monetary expansion (contraction) all final users suffer from higher consumption prices, but the households who earn larger industry shares are compensated by higher profits.

Corollary 2. *Changing the aggregate output gap does not affect relative consumption across final users if either:*

1. sectors use primary factors in equal proportions ($\alpha = \alpha \mathbf{1}\mathbf{s}^T$) and sector ownership is proportional to consumption shares ($\mathcal{S}^{-1}\Xi^T diag\left(\bar{\mathbf{\Psi}}\right)\Psi^{-1} = \beta^T$);

2. a proportional increase in the consumption of all final users does not affect relative prices ($\kappa_C \mathbf{1} \propto \mathbf{1}$).

5.1.2 Examples

The examples below return to the simple economies introduced in Section **??**, to illustrate how various dimensions of heterogeneity shape cross-sectional monetary non-neutrality. To simplify the exposition, I focus on a static setting with $\rho = 0$ and $\mathbf{p}_{t-1} = \mathbf{0}$.

Example 4. Nominal rigidity

Let's return to the economy in Example 1, with two agents and one final good. For simplicity, let's assume that the two workers have equal shares in production ($\alpha_{sticky} = \frac{1}{2}$). In this economy we shut down the relative income channel by assuming equal consumption shares. Thus monetary policy affects relative employment only through expenditure switching. A monetary expansion (contraction) increases (reduces) the relative employment of the worker with stickier wage:

$$l_t^{sticky} - l_t^{flex} \propto \frac{\varphi\theta\bar{\delta}}{1+\varphi\theta\bar{\delta}}\left(\delta_{flex} - \delta_{sticky}\right)\bar{l}_t, \quad \bar{\delta} \equiv \frac{\delta_{flex} + \delta_{sticky}}{2}$$

Intuitively, during a monetary expansion the sticky worker becomes cheaper, because his wage takes longer to respond. Therefore producers demand relatively more of his labor. The effect is larger when workers are more substitutable (i.e. when θ is larger), while monetary policy has no cross sectional effect when production is Leontief ($\theta = 0$).

The behavior of cross-sectional incomes depend on both employment and wages, which move in opposite directions. The sticky worker's relative income increases if and only if workers are substitutes in production:

$$c_t^{sticky} - c_t^{flex} \propto \frac{\varphi\left(\theta - 1\right)\bar{\delta}}{1+\varphi\theta\bar{\delta}}\left(\delta_{flex} - \delta_{sticky}\right)\bar{y}$$

Example 5. Labor supply elasticity

Consider the economy in Example 2, assuming $\alpha_I = \alpha_E = \frac{1}{2}$. Again, we shut down the relative income channel by assuming equal consumption shares, so that monetary policy affects relative employment only through expenditure switching. A monetary expansion increases the relative employment of the elastic workers, as their wages increase by less, and therefore producers increase demand for their labor. The effect is proportional to the substitution elasticity θ:

$$l_E - l_I = \left(\varphi_I - \varphi_E\right)\frac{\theta\delta}{1+\bar{\varphi}\theta\delta}\bar{l}$$

Like in Example 1, the response of cross-sectional incomes depends on whether workers are complements or substitutes:

$$c_E - c_I = (\varphi_I - \varphi_E)\,\frac{(\theta - 1)\,\delta}{1 + \bar{\varphi}\theta\delta}\bar{y}$$

Example 6. Fixed assets

Consider the economy in Figure 11, with two sectors ("NYC housing" and "Boise housing") and two agents (construction workers in the two cities).

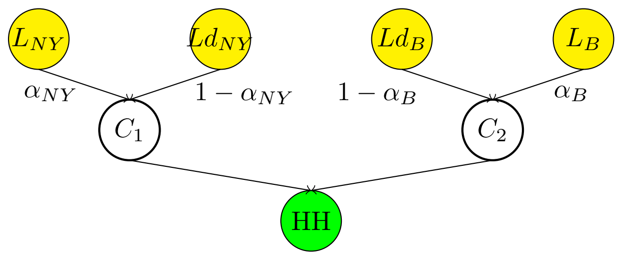

Figure 11: Housing economy

Housing is produced using labor and land, which is supplied inelastically ($\varphi_{land} \to \infty$). Land is scarce in NYC, so NYC housing has a larger expenditure share on land ($\alpha_{NY} < \alpha_B$). For simplicity, all agents consume housing in both cities, in equal shares. We want to study whether employment is more sensitive to monetary policy in NYC or in Boise.

In this economy, cross sectional employment is given by

$$l_B - l_{NY} \propto \theta\,(\sigma\delta - \theta)\,(\alpha_B - \alpha_{NY})\,\bar{l}$$

So, employment in NYC is more cyclical if and only if $\sigma\delta < \theta$, that is, if housing across the two cities is less substitutable than labor and land in production. In this case, agents don't want to change their "location mix", but they'd rather improve the housing stock in its current location, by using labor more intensively on the same amount of land.

Example 7. Input-output linkages

Let's return to the vertical economy in Example 3, with $\alpha_F = \frac{1}{2}$. The response of cross-sectional employment to monetary policy is given by

$$l_I - l_F = \delta\,(1 - \delta)\,\frac{\varphi\theta}{1 + \varphi\theta\bar{\delta}}\bar{l}$$

The formula is the same as in Example 4, replacing $\delta_{flex} - \delta_{sticky}$ with the difference $\delta\,(1 - \delta)$. Like in Example 1, this difference corresponds to the relative price change of type I-intensive vs type F-intensive goods, when the employment gap of both agents increases by the same amount. As usual, the effect on relative employment is governed by the elasticity of substitution θ.

Example 8. Network-adjusted elasticity of substitution

The insights from the previous examples extend to more complicated network structures, where there is no one-to-one mapping between factors and sectors. In this case, however, it is not obvious how to define an aggregate counterpart of the elasticity of substitution θ between factors. We illustrate this in an economy with two consumption goods and two workers, as in Figure 5.1.2. Both consumption goods use both workers, although in different proportions. Workers have different wage adjustment probabilities, δ_{flex} and δ_{sticky}, while final good prices are sticky, both with the same adjustment probability δ. All workers have the same labor supply elasticity φ. The substitution elasticity between worker types is θ, and the elasticity of substitution between final goods is σ.

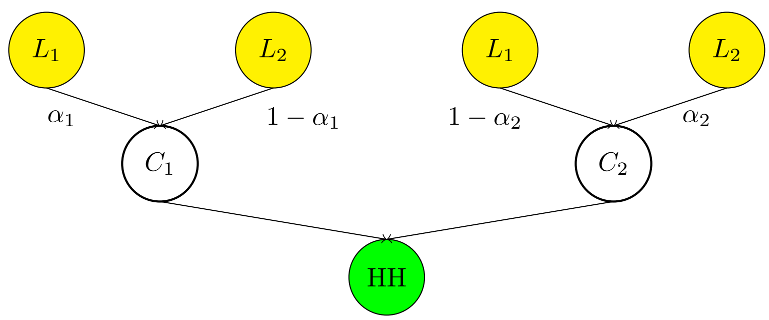

Figure 12: Two-good, two-workers economy

Cross-sectional employment is the same as in Example 1, replacing the elasticity θ with an appropriate aggregate elasticity $\bar{\theta}$:

$$l_{sticky} - l_{flex} = \frac{\varphi\bar{\theta}}{1+\varphi\bar{\theta}} \left(\delta_{flex} - \delta_{sticky}\right) \bar{l}$$

where

$$\bar{\theta} \equiv \theta \left(1 - \frac{\beta_1 \left(1-\beta_1\right) \left(\alpha_1 - \alpha_2\right)^2}{s_1 \left(1 - s_1\right)}\right) + \sigma\delta \frac{\beta_1 \left(1-\beta_1\right) \left(\alpha_1 - \alpha_2\right)^2}{s_1 \left(1 - s_1\right)}$$

The aggregate elasticity $\bar{\theta}$ combines substitution elasticities in production and consumption, discounting the consumption elasticity by the price adjustment probability in the final good sectors (because this dampens the response of final good prices to changes in the workers' relative wage, and hence the amount of substitution by final consumers). If both goods use workers in the same proportions $\bar{\theta}$ equals the production elasticity θ, because substitution across final goods has no effect on the relative demand for different workers.

5.2 Aggregate non-neutrality

5.2.1 Static case

In Section 5.1 I discussed how monetary policy affects the relative employment and consumption of different households. I now consider the implications of households heterogeneity for aggregate monetary non-neutrality. I start by considering a static economy ($\rho = 0$), starting from a steady-state with $\mathbf{p}_{t-1} = \mathbf{0}$. Section 5.2.3 extends the analysis to a dynamic economy.

I measure monetary non-neutrality as the elasticity of aggregate real GDP ($\bar{y}$) with respect to the money supply m. To derive it, I combine the Phillips curve (17) with the cash-in-advance constraint (9). Let's start from a benchmark model with a single household and a single primary factor ($N_w = 1$, $N_f = 0$). Denote by

$$\boldsymbol{\kappa}^Y \equiv \mathbf{s}^T \beta^T \kappa \in \mathbb{R}^{1,N_w+N_f}$$

the slope of the Phillips curve for the GDP deflator. Combining equations (17) and (9) yields

$$\bar{y}_t = \frac{m_t}{1+\kappa^Y} \tag{23}$$

Equation (23) highlights the key role of the Phillips curve slope κ^Y. When the Phillips curve is steep prices respond more, and therefore output responds less, to any given increase in m.

The heterogeneous agents counterpart of equation (23) is

$$\bar{y}_t = \frac{m_t}{1+\bar{\kappa}^Y + Cov_s\left(\frac{\boldsymbol{\kappa}^Y}{\mathbf{s}}, \frac{\boldsymbol{\ell}}{\bar{y}}\right)} \tag{24}$$

where $\bar{\kappa}^Y = \sum_h \kappa_h^Y$ denotes the response of the GDP deflator to a 1% increase in the employment gap of all factors. In equation (24), aggregate monetary non-neutrality depends both on the aggregate slope $\bar{\kappa}^Y$, and on the cross-sectional covariance between the Phillips curve slopes $\frac{\boldsymbol{\kappa}^Y}{\mathbf{s}}$ and factor employment $\frac{\boldsymbol{\ell}}{\bar{y}}$. The aggregate Phillips curve is flatter – and monetary policy has larger real effects – when employment increases more for factors h with a flatter slope κ_h^Y.

While it is hard to determine the sign of this covariance in general, in Section 5.1 I argued that the demand for primary factors with steeper Phillips curves tends to decline after a monetary expansion. Thus we can expect the covariance to be negative. The examples in Section 5.1.2 illustrate that this is the case in some relevant special cases. Hence the presence of heterogeneous nominal rigidities and labor supply elasticities tends to increase the aggregate monetary non-neutrality.

5.2.2 Examples

The examples below specialize equation (24) to the simple economies in Sections 4.1 and 5.1.2, showing how different dimensions of heterogeneity across agents affect aggregate non-neutrality. Like in Section 5.1.2, we assume $\rho = 0$ and $\mathbf{p}_{t-1} = \mathbf{0}$.

Example 9. Nominal rigidity

Consider the economy in Examples 1 and 4, with two agents and one final good. In the aggregate, the ability to shift demand towards the worker with stickier wages increases monetary non-neutrality, compared to an economy where both workers have the same wage rigidity, equal to the average $\bar{\delta}$. The response of aggregate employment to a 1% increase in the money supply is given by

$$\bar{l} = \frac{1}{1 + \varphi \frac{\bar{\delta}}{1-\bar{\delta}} \left[1 - \frac{\varphi \theta \bar{\delta}}{1+\varphi \theta \bar{\delta}} \left(\frac{\delta_{flex} - \delta_{sticky}}{\bar{\delta}} \right)^2 \right]} > \frac{1}{1 + \varphi \frac{\bar{\delta}}{1-\bar{\delta}}}$$

Aggregate monetary non-neutrality is increasing in the elasticity of substitution θ, which governs the ability of the economy to reallocate demand towards the sticky worker, and it is the same as with a representative agent only when production is Leontief.

Example 10. Labor supply elasticity

Consider now the economy in Example 2. The response of aggregate employment to a 1% increase in the money supply is given by:

$$\bar{l} = \frac{1}{1 + \bar{\varphi} \frac{\delta}{1-\delta} \left[1 - \frac{\bar{\varphi} \theta \delta}{1+\bar{\varphi} \theta \delta} \left(\frac{\varphi_I - \varphi_E}{\bar{\varphi}} \right)^2 \right]} > \frac{1}{1 + \bar{\varphi} \frac{\delta}{1-\delta}}$$

The intuition is very similar to Example 9: the ability of the economy to reallocate demand towards the more elastic worker increases aggregate non-neutrality, compared to an economy where both workers have the average labor supply elasticity $\bar{\varphi}$. Again, heterogeneity increases aggregate non-neutrality by more when workers are more substitutable, and it has no effect when production is Leontief.

5.2.3 Dynamic case

Inflation and employment impulse responses In the dynamic case, monetary non-neutrality can be measured from the impulse response of aggregate employment to monetary shocks. In the one-sector, representative agent model, inflation and employment decay over time at the same rate as the monetary shock. With multiple industries and primary factors, instead, inflation and employment in general do not decay at constant rates, even when the monetary shocks do. This makes it harder to compute and compare the impulse responses of inflation and employment.

To address the issue, Proposition 4 derives a set of inflation indices which do decay at constant rates. This result is useful to characterize the dynamic evolution of all other inflation indices as well, as they can be written as linear combinations of these special indices.

Proposition 4. *Let*

$$\mathcal{M} \equiv \left(I - \alpha \underline{\Psi}_{C,:}\right) \Psi^{-1}$$

Denote by $\{\boldsymbol{\zeta}_i\}_{i=1}^{N}$ *the left eigenvectors of the matrix* $\mathcal{M}$*, and its eigenvalues by* $\{\nu_i\}_{i=1}^{N}$*. Consider the price indices* $\pi_t^{\zeta_i} \equiv \boldsymbol{\zeta}_i^T \boldsymbol{\pi}_t$*, defined by the eigenvectors* $\boldsymbol{\zeta}_i$*. The impact response* $\pi_0^{\zeta_i}$ *to future output gaps* ℓ_t *decays at constant rate* ξ_i *with time* t*, and the law of motion for* $\pi_t^{\zeta_i}$ *is given by:*

$$\pi_t^{\zeta_i} = \xi_i \pi_{t-1}^{\zeta_i} + \sum_{s=0}^{\infty} (\rho \xi_i)^s \, \boldsymbol{\kappa}^{\zeta_i} \mathbb{E} \boldsymbol{\ell}_{t+s}$$

where κ^{ζ_i} *is the slope of the Phillips curve defined by the index* $\pi_t^{\zeta_i}$*. The slope* κ^{ζ_i} *is equal to*

$$\boldsymbol{\kappa}^{\zeta_i} \equiv \frac{\xi_i}{\nu_i} \boldsymbol{\zeta}_i^T \Delta \left(I - \Delta\right)^{-1} \mathcal{W} \alpha \left(\Gamma + \Phi\right)$$

Moreover it holds that $\nu_i \geq 0 \ \forall i$*, and the constant* ξ_i *satisfies*

$$\xi_i = \frac{1 + \rho + \nu_i - \sqrt{\left(1 + \rho + \nu_i\right)^2 - 4\rho}}{2\rho} \in (0, 1)$$

Building on Proposition 4, one can characterize the impact response of any price index $\pi_t^{\phi} \equiv \boldsymbol{\phi}^T \pi_t$, where $\boldsymbol{\phi} \in \mathbb{R}^{\bar{N}}$ is a vector of sectoral weights. For example, for $\boldsymbol{\phi} = \mathbf{s}^T \beta^T$ the index π^{ϕ} corresponds to the GDP deflator π_t^Y, while for $\boldsymbol{\phi} = \mathbf{e}_i$ the index π^{ϕ} corresponds to inflation in industry i. Note that any index π^{ϕ} can be written as a linear combination of the eigen-indices $\left\{\pi_t^{\zeta_i}\right\}_{i=1}^{N}$, with some coefficients $\{x_i\}_{i=1}^{N}$. Hence, assuming $\pi_{-1} = \mathbf{0}$, we have

$$\pi_0^{\phi} = \sum_t \rho^t \left[\sum_i \xi_i^t x_i \kappa^{\zeta_i}\right] \mathbb{E} \boldsymbol{\ell}_t \tag{25}$$

In equation (**??**), the response of current inflation π_0^{ϕ} to employment gaps t-periods ahead is given by

$$\kappa_t^{\phi} \equiv \sum_i \xi_i^t x_i \kappa^{\zeta_i} \tag{26}$$

With some abuse of notation I denote by $\{\kappa_t\}_{t=0}^{\infty}$ the t-periods ahead slope, whose elements $(\kappa_t)_{ih}$ are the impulse response of inflation in sector i at time 0 to the employment of factor h at time $t \geq 0$.

Definition 9. Given any sectoral weights ϕ, the elasticity of current inflation π_0^{ϕ} to a proportional increase in the employment gap of all factors t periods ahead is

$$\bar{\kappa}_t^{\phi} = \sum_h \kappa_{ht}^{\phi}$$

Remark 10. Two special cases are worth mentioning. First, in the static economy with $\rho = 0$ and $\mathbf{p}_{-1} = \mathbf{0}$, the slope κ derived in Proposition 1 coincides with the coefficient κ_0 derived from the eigendecomposition (25). Second, in a representative agent economy with only one good ($N_w = \bar{N} = 1$, $N_f = 0$) it holds that $\kappa_t = \kappa_0\ \forall t$.

Leveraging on these results, Proposition 5 derives a dynamic analog of equation (24). Start by expressing the impact response of the GDP deflator as a function of expected factor-level employment:

$$\pi_0^Y = \sum_{t=0}^{\infty} \rho^t \kappa_t^Y \boldsymbol{\ell}_t \tag{27}$$

Using equation (27) and the cash-in-advance constraint (**??**) we can derive the impact response of aggregate output to a change in m_0, as a function of the coefficients $\left\{\kappa_t^Y\right\}_{t=0}^{\infty}$ and of the rate of decay of the employment impulse-response, $\frac{\boldsymbol{\ell}_t}{\bar{y}_0}$:

$$\bar{y}_0 = \frac{m_0}{1 + \sum_{t=0}^{\infty} \rho^t \kappa_t^Y \frac{\boldsymbol{\ell}_t}{\bar{y}_0}} \tag{28}$$

Proposition **??** moves from equation (28) to derive a dynamic counterpart of equation (24).

Proposition 5. *The impact response of employment to an increase in the money supply m_0 is given by*

$$\bar{y}_0 = \frac{m_0}{1 + \sum_t \rho^t \left(\bar{\kappa}_t^Y + Cov_s\left(\frac{\kappa_{ht}^Y}{s_h}, \frac{\ell_{ht}}{\bar{y}_t}\right)\right) \frac{\bar{y}_t}{\bar{y}_0}} \tag{29}$$

As a special case, in the one-sector, representative-agent economy ($N_w = \bar{N} = 1$, $N_f = 0$, see Gali (2015); Woodford (2003)) the slopes κ_t^Y are constant over time ($\kappa_t^Y \equiv \kappa^Y\ \forall t$). Moreover, employment decays at the same rate as the monetary shock. Therefore if the shock decays at a constant rate η we also have $\frac{y_t}{y_0} = \eta^t$, and equation (28) simplifies to

$$y_0 = \frac{m_0}{1 + \frac{\kappa^Y}{1 - \rho\eta}}$$

In an economy with multiple primary factors, instead, there is more aggregate non-neutrality when employment increases relatively more for factors with flatter Phillips curves, at each given period ($Cov_s\left(\frac{\kappa_{ht}^Y}{s_h}, \frac{\ell_{ht}}{\bar{y}_t}\right) < 0$ in equation (29)).

5.3 Eliminating the cross-sectional effects of monetary policy

Proposition 6 characterizes a set of taxes and subsidies which eliminates the cross-sectional effects of monetary policy on employment and income, when agents have the same wealth effects and Frish elasticities in labor supply. As it is clear from equation (30), this scheme requires industry-specific subsidies, along with agent-specific transfers, which depend on the details of the input-output structure. Therefore, in practice it may not be feasible to implement this policy.

Proposition 6. *Suppose that all agents have the same wealth effects and Frish elasticities in labor supply ($\Gamma = \gamma I$, $\Phi = \varphi I$). Then there exists a set of input subsidies $\{\tau_i\}_{i=1}^N$ and lump-sum taxes $\{T_h\}_{h=1}^H$, such that monetary policy has no effect on cross sectional employment and incomes when complemented with these taxes and subsidies. The relative input subsidies paid to industries i and j are proportional to the differential exposure of prices in the two industries to total labor costs:*

$$\tau_i - \tau_j \propto \left(\Delta\left(I - \Omega\Delta\right)^{-1}\bar{\alpha}\right)_i - \left(\Delta\left(I - \Omega\Delta\right)^{-1}\bar{\alpha}\right)_j \tag{30}$$

where $\bar{\alpha}_i = \sum_h \alpha_{ih}$. Lump-sum taxes are then set so that all the agents' incomes, including labor and profit rebates, change proportionately. The level of the subsidies is pinned down by imposing that lump-sum taxes sum to 0 and the government budget is balanced. Under these taxes and subsidies, the impulse responses of sectoral inflation and aggregate real GDP to monetary policy are the same as with a representative agent, with sectoral labor shares given by $\bar{\alpha}$.

6 Calibration

This section calibrates the model to the US economy. All results are based on the cash-in-advance economy with financial autarcky. Results for the Taylor rule economy are coming soon.

To quantify the cross-sectional effects of monetary policy, Section **??** computes impulse-responses of employment to a 1% money growth shock, at the occupation-level. The model predicts substantial heterogeneity. In order to isolate its origins, Sections **??** and **??** compare the baseline model with alternative calibrations, which shut down direct heterogeneity in wage rigidity and labor supply elasticities, and indirect heterogeneity through the input-output network. Finally, Section **??** highlights that cross-sectional non-neutrality increases proportionally to the amount of substitutability between inputs.

6.1 Data

To calibrate the model, one needs to assign values to the preference and production parameters in Tables 5 and **??**. Throughout the analysis, we follow the BEA 3-digit industry classification and the IPUMS 2010 2-digit classification of occupations, excluding the non-employed and active military. This leaves us with 71 industries and 22 occupations.

Let's begin from the preference parameters. The discount factor is $\rho = 0.9975$, and the wealth effect in labor supply is set to $\gamma = 1$ for all agents. Occupation-specific inverse Frish elasticities are calibrated based on Webber (2018). Final consumption shares β for each occupation are computed by combining data from the BEA input-output accounts and the Consumer Expenditure Survey (CEX) for the year 2007, following the methodology in Borusyak and Jaravel (2021).

Let's now turn to the production parameters. Sectoral input shares Ω are derived from the Make and Use tables provided by the BEA,[8] for the year 2012, at the 405-sector level. The BEA input-output accounts also report total expenditures on labor in each industry. These are then allocated across occupations to construct the matrix α, based on data from the American Community Survey (ACS), again for the year 2012. In the ACS data, respondents report the industry where they work, their occupation and their wage. Under the assumption that the sample is representative of the overall population, this allows us to infer the share of each industry's labor expenditures on the various occupations.

Sector-level probabilities of price adjustment are provided by Pasten et al. (2017), who compute them based on firm-level price series underlying the PPI data published by the BLS. There are no estimates for personal services, repair and maintenance and government. In the baseline calibration, their adjustment probability is set equal to the mean.[9] Occupation-specific wage adjustment probabilities are calibrated based on the data in Grigsby et al. (2019), which is the most detailed available wage adjustment data. Nonetheless, their dataset only reports wages adjustment probabilities for five broad industries, and not by occupation. We proxy for occupation level wage adjustment frequencies by averaging industry level frequencies, weighted by the probability that each occupation is hired by the given industry (computed based on ACS data).

The elasticities of substitution in production and consumption are set to consensus values in the literature. The substitution elasticity between consumption goods is $\sigma = 0.9$,[10] the elasticity of substitution between labor and intermediate inputs is $\theta_L = 0.5$, and [11] the elasticity of substitution across intermediate inputs is $\theta = 0.1$.

6.2 Impulse responses

Figure 13 plots impulse responses of employment to a 1% increase in the money supply, at the occupation-level. The blue lines correspond to employment in each given occupation, while the red lines plot the impulse-response of aggregate employment (they are the same in all subplots). The calibrated employment responses have very different magnitudes across occupations. The next subsections study how different dimensions of heterogeneity (wage rigidity, labor supply elasticity, position in the input-output network) affect cross-sectional non-neutrality.

[8] I follow the methodology in Horowitz and Planting (2006) to compute expenditure shares.

[9] The missing sectors account for 10% of total sales, of which 8% comes from the government. None of the results changes significantly in an alternative calibration which excludes these sectors.

[10] Atalay (2017), Herrendorf et al. (2013), and Oberfield and Raval (2014) estimate it to be slightly less than one.

[11] This is consistent with Atalay (2017), who estimates this parameter to be between 0.4 and 0.8.

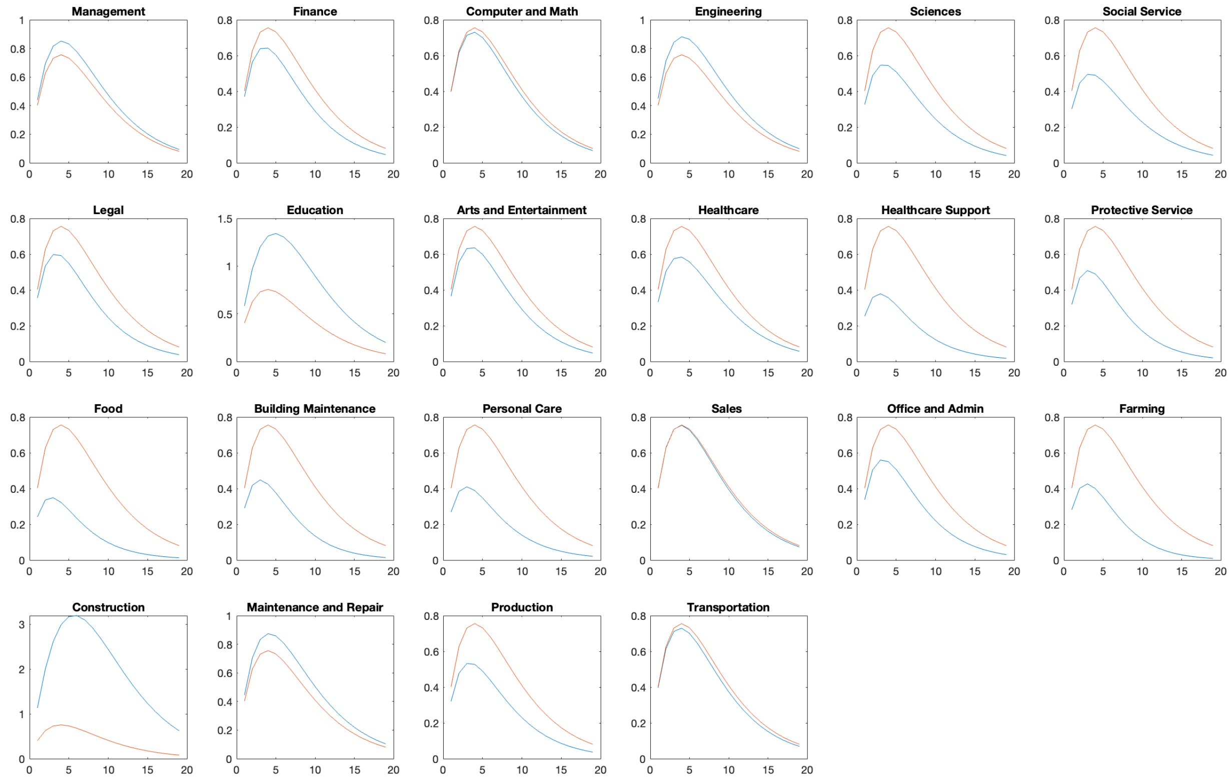

Figure 13: Impulse-responses of occupation-level employment to a 1% increase in the money supply. The red lines correspond to aggregate employment.

6.3 Direct heterogeneity

Figure **??** explores the role of direct heterogeneity coming from wage rigidity. The blue line corresponds to the baseline calibration. The yellow line instead shows the impact response of occupation-level employment when all occupations have the same wage adjustment probability, equal to the average. The available data displays little heterogeneity in wage rigidity across occupations (see Figure **??**), therefore it is not surprising that eliminating heterogeneity altogether has almost no effect on cross-sectional non-neutrality. A more surprising result is shown by the red line, which instead corresponds to a counterfactual calibration that scales up the occupations' relative wage adjustment probabilities by a factor of XX (keeping the mean constant). Even in this calibration, heterogeneity in wage rigidities seems to have a small effect on cross-sectional non-neutrality, suggesting that most of the heterogeneity is due to network effects.

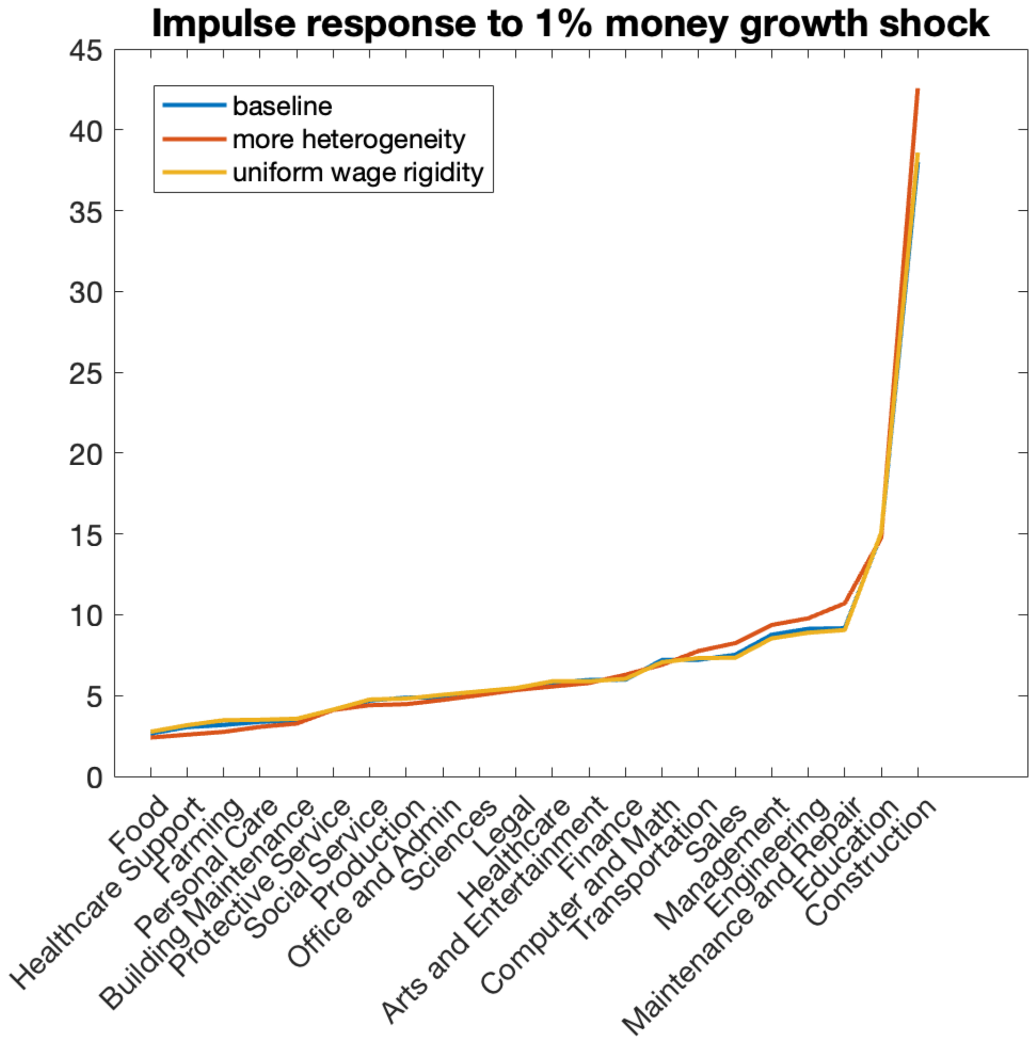

Figure 14: Impact responses of employment to a 1% increase in the money supply.

6.4 Input-output linkages

Figure 15 illustrates the role of the input-output network. The blue lines and the values on the left axis correspond to the baseline calibration, while the red lines and the values on the right axis correspond to an economy with no input-output linkages (but still accounting for multiple sectors and occupations).

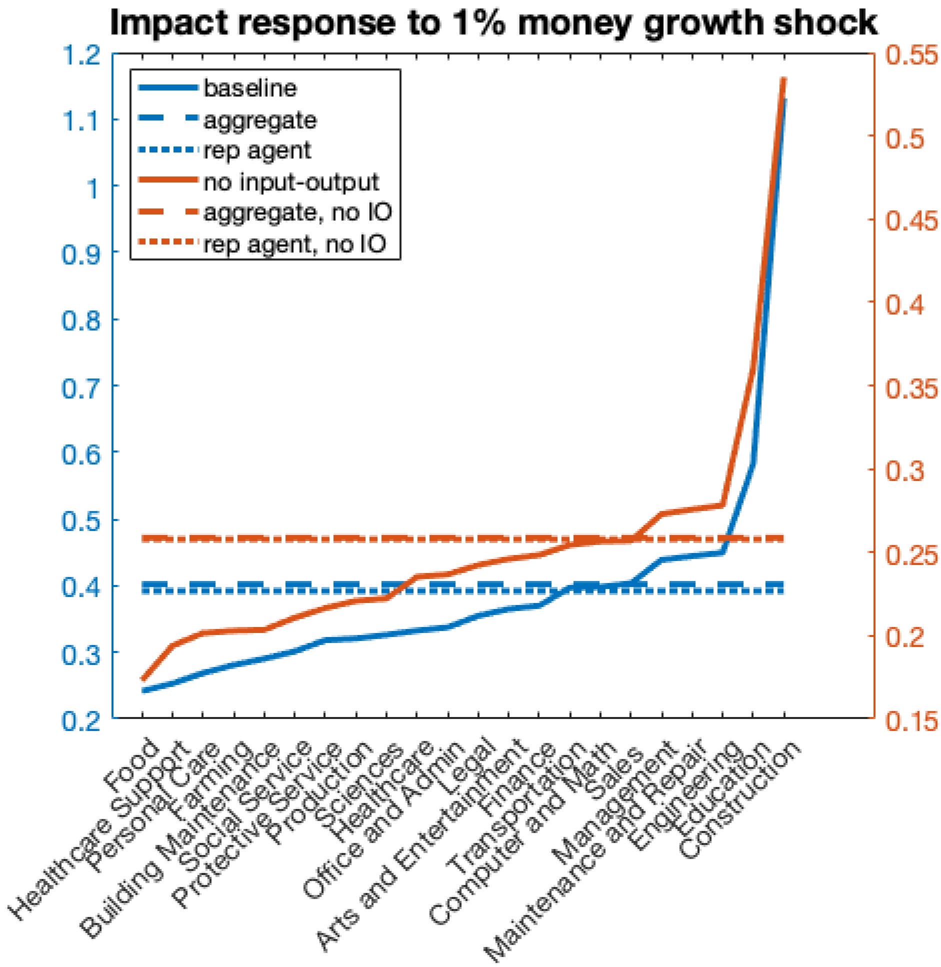

Figure 15: Impact-responses of employment to a 1% increase in the money supply

Like in the representative agent case, the presence of input-output linkages increases monetary non-neutrality (Basu (1995); Carvalho (2006); Nakamura and Steinsson (2010); **?**), so that the employment response to monetary policy is larger for every occupation. Moreover, input-output linkages also increase cross-sectional non-neutrality. As discussed in Section 5 the input-output network has two countervailing effects on cross-sectional non-neutrality. On the one hand, it amplifies heterogeneity in nominal rigidities; on the other hand, it makes the factor contents of different goods more uniform, thereby limiting the extent to which expenditure switching can affect the relative demand for different workers. The calibration shows that the first channel dominates quantitatively.

Finally, the dashed horizontal lines plot the impact response of aggregate employment in the heterogeneous agent economy, while the dotted lines plot the same impact response in a counterfactual representative-agent economy. Consistent with the theory, aggregate non-neutrality is larger in the heterogeneous agent economy, but the difference is small. As explained in Section 5, the difference is small because it depends quadratically on cross-sectional heterogeneity in price rigidities and labor supply elasticities.

6.5 Substitutability

Figure 16 illustrates how substitutability between inputs affects cross-sectional non-neutrality. The red line corresponds to the baseline calibration, where the elasticity of substitution between inputs in production is set to 0.5. The blue and yellow lines show results for a higher and a lower elasticity. Following the argument in Section **??**, the cross-sectional effect of monetary policy on employment increases proportionately to the substitution elasticities in production and consumption. The relative demand for different workers changes as producers and consumers substitute towards sticky-price goods, whose price responds less to a monetary expansion. If instead producers and consumers cannot substitute, monetary policy is neutral in the cross-section.

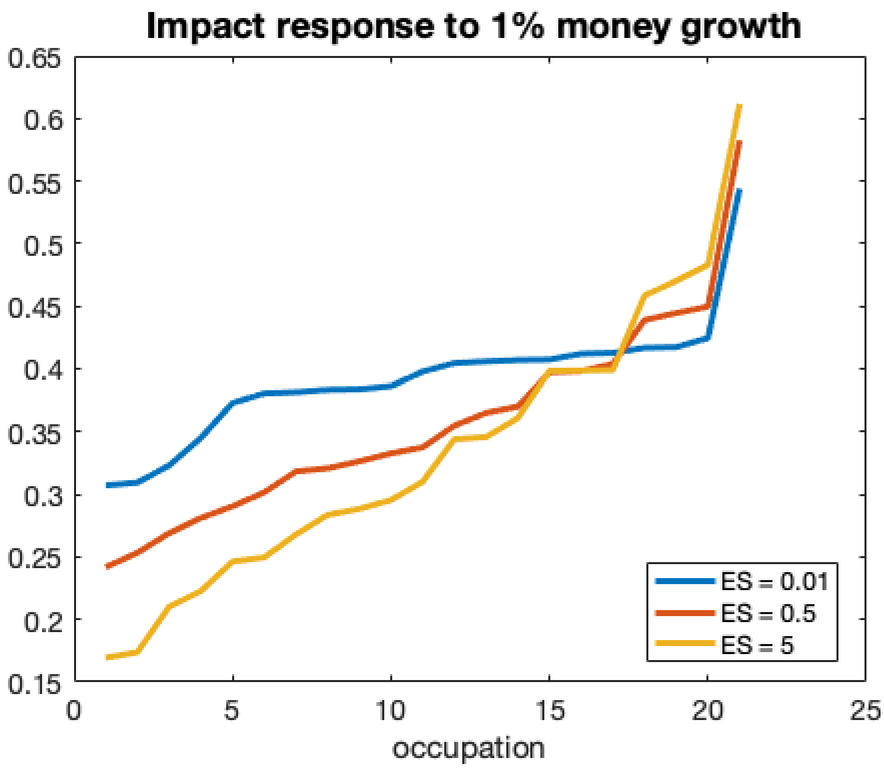

Figure 16: Substitution elasticities and monetary non-neutrality

7 Conclusion

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